Kings Cross St.Pancras Inter-Continental

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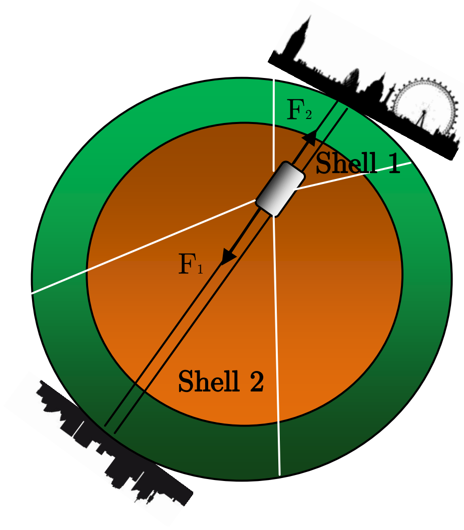
1. Intentions of this report

Worldwide, 27 billion tonnes of carbon dioxide are produced every year, with at least 40% of that being accounted for by transport. With the ominous shadow of global warming looming over modern civilisation, radical new changes are needed in how we travel if Earth is to circumvent what seems to be a dry, barren and diseased future. Originally proposed in a letter sent to Isaac Newton, Robert Hooke outlined the mathematics governing how objects might fall, if they were dropped through a hole bored to the centre of the Earth. Although initially intended as an intellectual exercise, this idea was revisited in Robert Cooper's 1966 paper: "Through the Earth in Forty Minutes" whereby the idea of 'gravity trains' was put forward as a serious suggestion for future transportation. Despite being regarded as just a physicist's pipe dream, this report examines the theory behind such a mode of transportation and the plausibility that gravity trains will supersede other methods of transport in the new millenia.

2. The Journey and Travel Times

Consider a tunnel drilled from Euston Square, London to Central Station, Sydney, passing through the centre of the Earth. The first question that arises is, what about the gravitational effect of the Earth that surrounds you?

Figure 1 - Forces acting on the 'gravity train'



Newton's law of gravitation tells us that:

$$F = \frac{GMm}{R^2} \qquad (1)$$

If 'M' is the mass of each shell encompassed within the arcs, and 'R' is the distance between the train and the surface of the Earth, then it can be shown that the two forces F_1 and F_2, in figure 1 exactly balance. This is because although shell 2 encompasses a larger mass, shell 1 is closer to the surface of the Earth, and since the gravitational force is inversely proportional to the square of the separation, the two forces balance.

The corollary of this is that only the mass beneath the train influences its motion. By treating the Earth as a perfect sphere with uniform density, ρ, and replacing 'M' with $4\pi r^3\rho/3$, the equation of motion of the train becomes:

$$mx'' = 4\pi\rho Gmx \qquad (2)$$

It can therefore be seen that the train will execute simple harmonic motion, with a half period, $T_{0.5}$, equal to:

$$T_{0.5} = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{\frac{4\pi\rho G}{3}}} = \frac{\pi}{\sqrt{\frac{g}{R}}} \quad (3)$$

Assuming the effects due to air resistance and friction to be negligible, substituting the Earth's radius and the acceleration due to gravity into equation (3) shows that every journey would take approximately 42 minutes. Regardless of whether it was from Tokyo to Turin, or from Warren Street to Tottenham Court Road. The travel time depends only on the density of the medium being travelled through and the gravitational constant.

Analogous to a child on a swing, with each oscillation of the train, it would come to rest at maximum displacement, i.e. the destination, and achieve maximum velocity as it passed through the equilibrium point. The greatest velocity that could be achieved would occur as the train passed vertically through the centre of the Earth and by considering the conservation of energy, has been shown to be: $7900ms^{-1}$. At such high velocities the passengers would experience weightlessness, and most probably motion sickness.

3. Why is this 400-year old idea still not a reality?

The number of obstacles facing such an idea are plentiful and with current technology seem insoluble. The journey-times calculated are made on the assumptions that the Earth is a perfect sphere with uniform density. It is clear that this assumption is not true from just glancing out of the window. What's more is, from studying the propagation of longitudinal and transverse waves through the Earth's mantle and outer core, the Earth's inner core is thought to consist of a molten iron-nickel alloy at a theoretical temperature of 5,700K. The enormous temperatures, fuelled by radioactive decay, and large pressures with increasing proximity to the centre of the Earth would mean that any tunnels built would have to be made from a material able to withstand such hostile conditions. Although composite ceramic materials offer potential candidates, these materials are still in development and are difficult to manufacture in bulk.

The 42 minute travel time was also deduced on the assumption that the train experiences no frictional forces, when in reality; friction would have a significant effect on a train moving at such high velocities. In order to achieve a near frictionless path, the tunnel would need to be evacuated of air to eliminate air resistance, which in turn would require the displacement of millions of cubic metres of rock. A possible means to minimise friction with the rail, could involve magnetic levitation. This in itself raises several issues, as the use of electromagnets or storing of liquid nitrogen along a track 12, 756km would increase its carbon footprint by several shoe sizes.

4. How much would this project cost?

It seems the closest feat of engineering Britain has to compare with the Gravity train is the Channel Tunnel. Although by no means does it work on the same physical principles, it can be used to anticipate the cost of the gravity train's construction. The Channel tunnel is approximately 50km long, fitted with 37km of track, and costing in total 9.62 billion pounds to complete. Following the same logic, at £260 million per kilometre, a similar tunnel spanning the diameter of the globe would equate to a price of over £3.3 trillion – potentially enough to fund a NASA project to hunt for a new hospitable planet!

5. Conclusion

'The gravity express' still seems to be condemned to mechanics textbooks and is not likely to replace any local rail network any time soon. Its main flaws involve its cost of production, and are attributed to:

- The upheaval of such large quantities of rock and storage of this rock.
- The earth's plate tectonics, high temperatures and pressures of the innermost core, and the fact that this core is liquid and would vaporise most materials it came into contact with.
- The friction experienced by the train would be significant, and measures to reduce friction would compromise its energy saving expenditure.
- The extremely high velocities that the passengers would experience would not give a pleasurable riding experience.

The idea however should not be completely lost as grey matter in a long dead physicist's brain. Planets which possess no liquid core or atmosphere, such as the moon, provide suitable sites. Unfortunately rapid transit is probably less in demand in such places. Though humans may never experience a 42-minute inter-continent commute, it's possible with the development of ceramic materials and government interest, that cargo could be exported across the globe by such a means.

6. Citations and References

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Figure 1

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