

Problem Sheet 9: Numerical Methods and Probability

1. **Maths applied:** Integrals such as

$$\int_0^2 e^{-x^2} dx \quad (1)$$

are important in probability and in the theories of heat flow and signal transmissions. Unfortunately, it is impossible to find a nice expression for these integrals, hence the only option is to compute them numerically, as you will do in this question.

- (a) Use the rectangular rule with 4 equal intervals to estimate the integral (1).
(b) i. Use the trapezium rule with 4 equal intervals to estimate the integral (1).
ii. You are given that for all x with $0 \leq x \leq 2$,

$$\left| \frac{d^2}{dx^2} (e^{-x^2}) \right| \leq \frac{4}{e^{3/2}}$$

Using this fact, compute an estimate for the error of your Trapezium Rule approximation, to three significant figures.

- (c) i. Use Simpson's rule with 4 equal intervals to estimate the integral (1).
ii. You are given that for all x with $0 \leq x \leq 2$,

$$\left| \frac{d^4}{dx^4} (e^{-x^2}) \right| \leq 12$$

Using this fact, compute an estimate for the error of your Simpson's rule approximation, again to three significant figures.

2. Three absent-minded professors attend a Christmas party and, upon leaving, each picks one of their three coats at random. Find and plot the probability distribution of the number of professors who end up properly dressed. What is the expected number of well-dressed professors?
3. A company manufactures red and green Christmas baubles in equal numbers. It fills each box with 16 randomly selected baubles. What is the probability that a box contains at most 2 green baubles? What would be the probability if the company manufactured 3 times as many red baubles as green ones?
4. Fairy lights are manufactured on a machine that produces, on average, 1 defective light per 100. Use the Poisson distribution to estimate the probability of finding at most 3 defective lights in a box of 500.

This homework is **not assessed**, hence it is not due. However it is good practice, so **you should attempt these questions**. Solutions will be posted by the beginning of the next term. You already have all the material needed to tackle Question 1; by the end of the course you will also be ready to start handling Questions 2, 3 and 4.