Problem Sheet 6: More Integration

Assessed questions are marked with a star. You are welcome to check your answers by differentiating.

1. Evaluate the following integrals by parts:

   (a) $\int x \sin x \, \mathrm{d}x$

   (b*) $\int x^2 e^x \mathrm{d}x$

   (c*) $\int \cos^{-1} x \, \mathrm{d}x$

2. Evaluate the following integrals using partial fractions:

   (a*) $\int \dfrac{x}{x^2 + x - 6} \, \mathrm{d}x$

   (b) $\int \dfrac{2x + 2}{x^2 - 2x + 1} \, \mathrm{d}x$

   (c*) $\int \dfrac{2x}{x^2 + 2x + 1} \, \mathrm{d}x$

3. Which of the following improper integrals exist and which diverge? Remember to show all your working.

   (a*) $\displaystyle\int_0^\infty \frac{1}{x^2 + 1} \, \mathrm{d}x$

   (b*) $\displaystyle\int_1^\infty \frac{1}{x} \, \mathrm{d}x$

4.* **Maths "applied":** Find the area bounded by the curves $y = \cos^2 x$ and $y = \frac{1}{2}$ and by the lines $x = 0$ and $x = \frac{\pi}{4}$.

Due in by the start of the lecture on **Friday 25th November, 11am**. On the front page, please clearly write your name with your surname underlined and your student number. All pages must be **stapled together**, otherwise you will lose a mark!