Problem Sheet 4: Partial Differentiation

Assessed questions are marked with a star.

1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:

   (a*) $f(x,y) = \ln(x^2 + y)$

   (b*) $f(x,y) = \dfrac{x+y}{xy-1}$

   (c) $f(x,y) = y^x$

   (d*) $f(x,y) = \sinh(y - 2x)$

   (e) $f(x,y) = e^{xy} \ln y$

   (f) $f(x,y) = (2x - 3y)^3$

2.* Let $g(x,y) = xf(x-y)$ where f is any differentiable function. Show that

$$\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} = \frac{1}{x} g.$$

3.* Find $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$ for the function

$$f(x,y) = xe^{xy}.$$

   Check that they are, as I promised, the same.

4. **Maths applied:** In fluid dynamics, the function $u(x,y)$ represents the velocity of the fluid. We say that our fluid is "incompressible and irrotational" if $u(x,y)$ satisfies

$$u_{xx} + u_{yy} = 0. \qquad (*)$$

   Are the following fluid flows incompressible and irrotational? Show your working.

   (a) $f(x,y) = \ln(x^2 + y^2)$

   (b*) $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$

   (c*) $f(x,y) = 2xy - y + 3$

   (d) $f(x,y) = e^{-2y} \cos x$

   Note: Equation (∗) is the two-dimensional version of perhaps the world's most important partial differential equation. It's called Laplace's equation *and has its own notation:* $\nabla^2 u = 0$. *It governs heats, electromagnetism and many other things as well as fluid flows. Any function which satisfies it is called* harmonic, *and you found a few above.*

Due in by the start of the lecture on **Friday 4th November, 11am**. On the front page, please clearly write your name with <u>your surname underlined</u> and your student number. All pages must be **stapled together**, otherwise you will lose a mark!