Problem Sheet 3: Hyperbolic functions

Assessed questions are marked with a star.

1.* Starting with the equations

$$\begin{aligned}\cosh x + \sinh x &\equiv e^x \\ \cosh x - \sinh x &\equiv e^{-x},\end{aligned}$$

prove the following hyperbolic identity:

$$\cosh 2x \equiv \cosh^2 x + \sinh^2 x.$$

2. Starting with the definitions of $\sinh x$ and $\cosh x$, prove the following hyperbolic identities:

(a*)

$$\sinh 3x \equiv 3\sinh x + 4\sinh^3 x$$

(b)

$$\cosh 3x \equiv 4\cosh^3 x - 3\cosh x$$

3.* (a) Show that if $y = \coth^{-1} x$, then

$$y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right). \tag{1}$$

Hint: Start with $x = \coth y$, then solve for e^y.

(b) Use Equation (1) to express the number $\coth^{-1}(5/4)$ in terms of logarithms.

4.* **Maths applied:** If a rope or cable hangs between two points of equal height, it assumes the shape of a *catenary*, given by the equation

$$y(x) = a\cosh\left(\frac{x}{a}\right).$$

A live, uninsulated power cable is of length such that $a = 20$ in the equation above. It hangs from the tops of poles 40 feet apart (at $x = \pm 20$), where each pole is of height 16 feet 6 inches, above a pavement. Carl (height: 6 feet $1\frac{1}{2}$ inches) and Amanda (height: 5 feet $3\frac{1}{2}$ inches), distracted by a debate over doughnuts, walk under the cable. Do either of them get electrocuted by the low-hanging cable?

You are reminded that there are 12 inches in a foot, and you can use $\cosh(1) \approx 1.54$.

Due in by the start of the lecture on **Friday 28th October, 11am**. On the front page, please clearly write your name with your surname underlined and your student number. All pages must be **stapled together**, otherwise you will lose a mark!