Problem Sheet 1: Differentiation

Assessed questions are marked with a star.

1. Differentiate the following with respect to x. Simplify your answers as much as you can.

   (a*) $\ln\left(\sin\left(x^{2016}\right)\right)$,

   (b*) $(\cos x)^x$,

   (c) $\cos^3(\exp x)$,

   (d*) $\dfrac{\sin x}{x^2+\sin x}$

   (e) $e^{7x}\left(2\sin 10x+\cos 10x\right)$

   (f) $\sin\left(\cos\left(e^x \ln x\right)\right)$

2.* To differentiate $f(x)=\frac{u(x)}{v(x)}$ we use the quotient rule

$$f'(x)=\frac{v(x)u'(x)-u(x)v'(x)}{[v(x)]^2}.$$

By writing $f(x)=u(x).\frac{1}{v(x)}$, derive the quotient rule using the product rule and the chain rule.

3. (a) Show, in a similar way to how we showed for $\tanh x$ in class, that

$$\frac{d}{dx}(\tan x)=\sec^2 x.$$

(b*) Show, in a similar way to how we showed for $\sin^{-1} x$, that

$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}.$$

You are allowed to use the result from Question 3a for free.

4.* Find $\frac{dy}{dx}$ for $x^3-3xy^2+2y-e^x+1=\cos y$.

5. **Maths applied:** A manufacturer of clown shoes finds that the sales of their Big Red branded shoes is a function $f(p)$ of the selling price p (in pounds) for a pair of shoes. Suppose that $f(120)=9000$ pairs of shoes, and $f'(120)=-60$ pairs of shoes per pound. The revenue that the manufacturer will receive for selling $f(p)$ pairs of shoes at £p per pair is $R(p)=p\cdot f(p)$.

   (a*) Find $R'(120)$.

   (b) What impact would a small increase in price have on the manufacturer's revenue?

Due in by the start of the lecture on **Friday 14th October, 11am**. On the front page, please clearly write your name with your surname underlined and your student number. All pages must be **stapled together**, otherwise you will lose a mark!