Handout: Hyperbolic Functions

We will now introduce a new family of functions, the hyperbolic functions. They are related to trigonometric functions, and are defined in terms of exponentials.

$$\begin{aligned}\sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2} \\ \tanh x &= \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}.\end{aligned}$$

We can show from these definitions that $\cosh x$ is an even function and $\sinh x$ and $\tanh x$ are odd functions. See Figure 1 for the graphs of these three functions. We can also differentiate these functions by using their definitions in terms of exponentials.

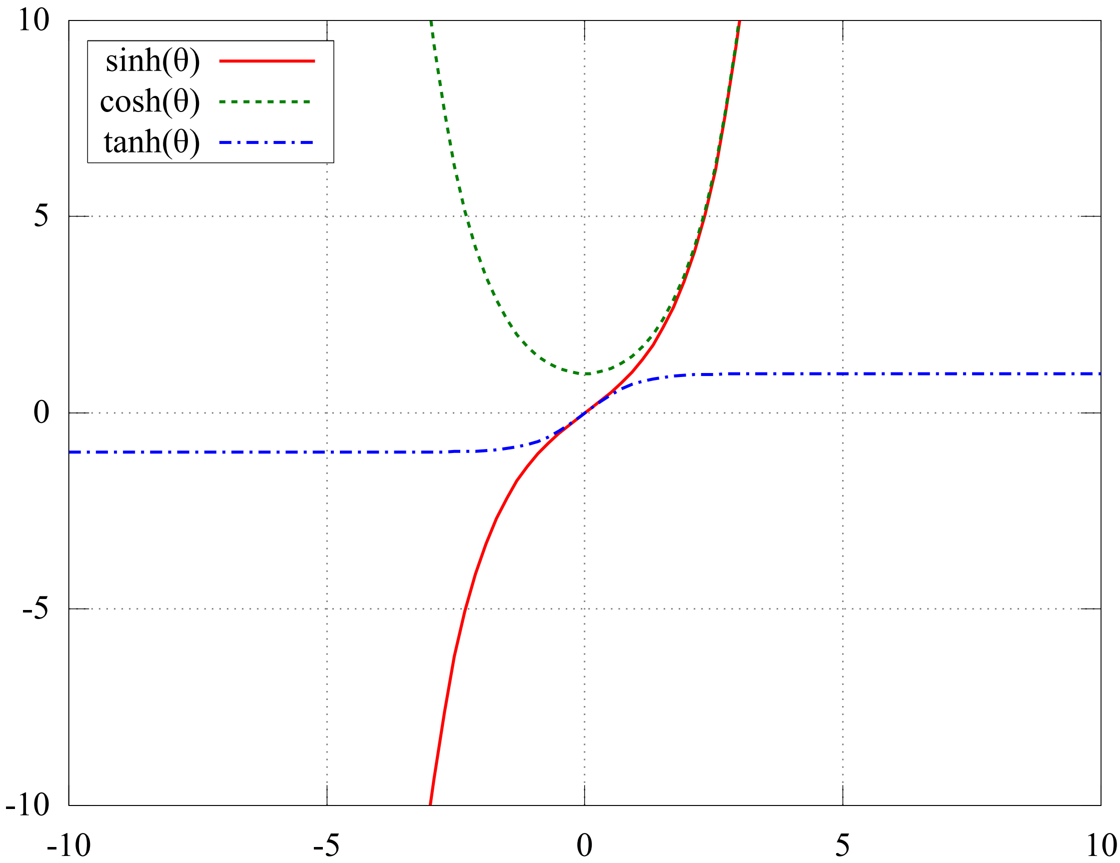
$$\begin{aligned}\frac{d}{dx}(\sinh(x)) &= \cosh(x) \\ \frac{d}{dx}(\cosh(x)) &= \sinh(x).\end{aligned}$$

We can derive several identities from these functions that are analogous to trigonometric identities. The most important of these to remember is

$$\cosh^2 x - \sinh^2 x \equiv 1.$$

In the table below, we have lots of hyperbolic identities, together with their trigonometric counterparts. Notice that they are very similar, but with some different signs! This is called **Osborne's rule**. The identities are the same, except when we have a product of sinhs, we flip the sign. This includes $\operatorname{cosech}^2 x$, $\tanh^2 x$ and $\coth^2 x$ as well as $\sinh^2 x$!

Figure 1: The graphs of sinh, cosh *and* tanh.



Hyperbolic	Trigonometric
$\coth x \equiv 1/\tanh x$	$\cot x \equiv 1/\tan x$
$\mathrm{sech}\, x \equiv 1/\cosh x$	$\sec x \equiv 1/\cos x$
$\mathrm{cosech}\, x \equiv 1/\sinh x$	$\mathrm{cosec}\, x \equiv 1/\sin x$
$\cosh^2 x - \sinh^2 x \equiv 1$	$\cos^2 x + \sin^2 x \equiv 1$
$\mathrm{sech}^2 x \equiv 1 - \tanh^2 x$	$\sec^2 x \equiv 1 + \tan^2 x$
$\mathrm{cosech}^2 x \equiv \coth^2 x - 1$	$\mathrm{cosec}^2 x \equiv \cot^2 x + 1$
$\sinh 2x \equiv 2 \sinh x \cosh x$	$\sin 2x \equiv 2 \sin x \cos x$
$\cosh 2x \equiv \cosh^2 x + \sinh^2 x$	$\cos 2x \equiv \cos^2 x - \sin^2 x$
$\cosh 2x \equiv 1 + 2\sinh^2 x$	$\cos 2x \equiv 1 - 2\sin^2 x$
$\cosh 2x \equiv 2\cosh^2 x - 1$	$\cos 2x \equiv 2\cos^2 x - 1$