ECONOMICS C31 : GAME THEORY

Answer any THREE questions. All questions carry equal weight.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the students first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored

1. Consider the game with payoffs as depicted in the table below. Player 1 is the row player and her payoff is written first in every cell, and player 2 the column player.

	L	C	R
T	2,0	3,7	3,0
M	1,0	2,1	1,8
B	0,4	6,2	7,1

a) Eliminate the strictly dominated strategies for both players. After this elimination, are there any strategies for a player i which are strictly dominated given that the player's opponent j will not use strictly dominated strategies? Answer this question for $i = 1$ and $i = 2$. (7 points).

b) Show that this game does not have a pure strategy Nash equilibrium. Solve for a mixed strategy Nash equilibrium. (16 points)

c) Consider an incomplete information game, where there is incomplete information about player 1's payoffs. Specifically, payoffs to player 2 are as given above. With probability 0.25, player 1's payoffs are as given in the table above. With probability 0.75, player 1's payoffs depend only on his own action and are 100 if he chooses B and 0 if he chooses T or M (i.e. player 1 has dominant action, B). Solve for a Bayes Nash equilibrium of this game. (10 points).

2. A committee consisting of three members, 1,2 & 3 has to choose an alternative from the set $\{a, b, c, d\}$. Each member's strict preference ordering over the set of alternatives is depicted in the table below, where alternatives are listed in order of decreasing preference (i.e. for example, d is the most preferred alternative for player 2).

1	2	3
a	d	c
b	a	d
c	b	a
d	c	b

a) The committee adopts the following *binary agenda* to select an alternative,

where alternatives are considered in lexicographic (alphabetical) order. First, the members vote on whether or not to choose a. If a is selected (by majority vote), the procedure ends. If a is not selected, members vote on whether or not to choose b. In the third and final stage, if neither a nor b have been selected, the committee chooses between c and d. Solve for an equilibrium where members engage in sophisticated voting. (That is solve for a subgame perfect equilibrium where each member does not at any stage vote for a weakly dominated choice). (10 points)

b) Design a binary agenda allowing any alternative to be chosen in principle (if the members vote appropriately), under which d is chosen under these preferences with sophisticated voting. (10 points)

c) What are the implications of the assumption that a player does not vote for weakly dominated choices? Specifically, show that under the procedure set out in (a), there exists a subgame perfect equilibrium where players may vote for dominated choices, such that alternative c is selected. (6 points)

d) Suppose that the situation is modified so that there is incomplete information about player 1's preferences. Specifically, with probability 0.5, his preferences are as set out in the table above, and with probability 0.5, his preference ordering is reversed, i.e. he prefers d to c to b to a. Player 2 and 3's preferences are unaffected. What additional information would you need to know in order to analyze this situation? (You do not have to analyze this situation − a purely verbal answer to this question will suffice.) (7 points).

3. Consider an auction for a single indivisible good with two bidders, $\{1, 2\}$. Each bidder's valuation v_i is independently and uniformly distributed on the interval $[0, 1]$, and this is common knowledge among the players. A bidder observes his own valuation, but not the valuation of his opponent. Consider an auction where the object is allocated to the highest bidder, where the price that this bidder pays is a weighted average of his bid and that of his rival. That is, if the bids are b and b' with $b > b'$, then the amount paid by the winner (the bidder of b) equals $\lambda b + (1 - \lambda)b'$, where $0 < \lambda < 1$. The person bidding lower pays nothing. If the bids are equal, the object each bidder gets the object with probability one-half, and in this case, pays his bid.

a) Suppose that bidder 1 assumes that bidder 2 bids a fraction γ of her valuation. Write down the expected payoff to bidder 1, as a function of his own valuation and his bid. (Ignore ties, where both bidders bid the same amount.) Solve for bidder 1's optimal bid, as a function of his valuation. Use this to solve for γ and thereby, for Bayes Nash equilibrium of this game, where each bidder bids a constant fraction γ of his valuation. (25 points).

b) Consider λ close to 0 and λ close to 1, and relate this auction and its equilibrium to the equilibria of other standard auction formats. (8 points).

4. Consider the following strategic situation, involving an incumbent firm (player 1) and a potential entrant (player 2):

i) Nature chooses the incumbent firm to be either a high cost firm (type H), or a low cost firm (type L), where the high cost firm is chosen with probability 0.6. The incumbent observes nature's choice, while the entrant firm does not (it is common knowledge that the probability of H is 0.6).

ii) The incumbent chooses a price from the set $\{P_H, P_L\}$. P_H yields a payoff in this stage of 2 to type H of incumbent and 2.5 to type L. P_L yields a payoff in this stage of 0 to type H and 2 to type L. The incumbent's choice has no direct payoff implications for the entrant.

iii) The entrant observes the incumbent's price choice and chooses from the set {IN,OUT}. If the entrant chooses OUT, his payoff is zero and the incumbent's payoff in this stage is 1, for both types of incumbent. If the entrant chooses IN, his payoff is 1 if the incumbent is type H and -1 if the incumbent is type L, and the payoff to both types of incumbent in this stage are zero.

The total payoff to each type of incumbent in this game is given by the sum of payoffs over stages (ii) and (iii). The payoff to the entrant is that which accrues in stage (iii) alone.

a) Set out the extensive form of this game. (13 points).

b) Solve for a pooling weak sequential equilibrium of this game, specifying clearly the beliefs of the entrant at each information set. (10 points)

c) Does this game have a separating equilibrium? If so, solve for a separating weak sequential equilibrium of this game, specifying the beliefs of the entrant at each information set. If not, explain why there is no separating equilibrium. (10 points)

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