Practical 3 & Exercise 4

Practical Questions (for 25 Nov 2008)

1. Whether candidate 1 or candidate 2 is elected depends on the votes of two citizens. The economy may be in one of two states, A and B. The citizens agree that candidate 1 is best if the state is A and candidate 2 is best if the state is B. Each citizen's preferences are represented by the expected value of a Bernoulli payoff function that assigns a payoff of 1 if the best candidate for the state wins (obtains more votes than the other candidate), a payoff of 0 if the other candidate wins, and payoff of 1/2 if the candidates tie. Citizen 1 is informed of the state, whereas citizen 2 believes it is A with probability 0.9 and B with probability 0.1. Each citizen may either vote for candidate 1, vote for candidate 2, or not vote.

a. Formulate this situation as a Bayesian game.

b. Show that the game has exactly two pure Nash equilibria, in one of which citizen 2 does not vote and in the other of which she votes for 1.

c. Show that one of the player's actions in the second of these equilibria is weakly dominated.

d. Why is the "swing voter's curse" an appropriate name for the determinant of citizen 2's decision in the second equilibrium?

2. Two firms compete a-la Cournot. Demand and costs are linear: $C_i(q_i) = cq_i$, $i = 1, 2$ and $P(q_1 + q_2) = \alpha - (q_1 + q_2)$ if $\alpha > q_1 + q_2$ and $P(q_1 + q_2) = 0$ if $\alpha \leq q_1 + q_2$. They both know that firm 1's unit cost is c. Only firm 2 knows its own unit cost; firm 1 believes that firm 2's cost is c_L with probability θ and c_H with probability $1 - \theta$, where $0 < \theta < 1$ and $c_L < c_H$. Find the Bayesian Nash equilibrium.

Tutorial Questions (for tutorial 4)

1. Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; she assigns probability α to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Each person's preferences are represented by the expected value of a Bernoulli payoff function that assigns the payoff of 0 if she yields (regardless of the other person's action) and a payoff of 1 if she fights and her opponent yields; if both people fight then their payoffs are (-1, 1) if person 2 is strong and (1, -1) if person 2 is weak. Formulate this situation as a Bayesian game and find its Nash equilibria if $\alpha < 1/2$ and if $\alpha > 1/2$.

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2. Firm A (the "acquirer") is considering taking over firm T (the "target"). It does not know firm T's value; it believes that this value, when firm T is controlled by its own management, is at least \$0 and at most \$100, and assigns equal probability to each of the 101 dollar values in this range. Firm T will be worth 50% more under firm A's management than it is under its own management. Suppose that firm A bids y to take over firm T, and firm T is worth x (under its own management). Then if T accepts A's offer, A's payoff is $3/2x - y$ and T's payoff is y; if T rejects A's offer, A's payoff is 0 and T's payoff is x. Model this situation as a Bayesian game in which firm A chooses how much to offer and firm T decides the lowest offer to accept. Find the Nash equilibria of this game. Explain why the logic behind the equilibrium is called adverse selection.

3. Consider a public goods provision game, with n individuals. Each individual must choose whether or not to contribute to the public good, and the public good is provided if and only if at least one individual contributes. The value of the good is v_i to individual i. The quantity v_i is independently and identically distributed across individuals, and is uniformly distributed on $[0, 1]$. The total payoff to an individual is the value of the good (if provided) minus the cost of provision (which is c if the individual provides the good, and zero otherwise). Solve for a symmetric Bayesian Nash equilibrium of this game where each individual provides the good if and only if v_i exceeds a critical threshold v^*. How does the probability that the good is provided at all vary with n? Explain how this relates to the "Kitty Genovese case" discussed in lecture 3.