ECON3014 : GAME THEORY: Oct 2008

Exercise 1

1. Consider the game with payoffs as depicted in the table below. Player 1 is the row player and her payoff is written first in every cell, and player 2 the column player.

	L	C	R
T	2,0	3,7	3,0
M	1,0	2,1	1,8
B	0,4	6,2	7,1

a) Eliminate the strictly dominated strategies for both players. After this elimination, are there any strategies for a player i which are strictly dominated given that the player's opponent j will not use strictly dominated strategies? Answer this question for $i = 1$ and $i = 2$.

b) Show that this game does not have a pure strategy Nash equilibrium.

2. What are the strictly dominated strategies for each player in the following game? Can you solve the game by iteratively deleting strictly dominated strategies? Solve for a pure strategy Nash equilibrium.

	L	C	R
T	2,1	1,2	6,1
M	3,3	5,6	3,5
B	2,1	4,2	1,9

3. Consider a second price auction for one unit of an indivisible good, with two bidders, where bidder i has valuation $v_i, i \in \{1, 2\}$. That is each bidder has submits a bid, and the object is allocated to the highest bidder, and the price that this bidder pays equals the second highest bid. Show that it is a weakly dominant strategy for a bidder to bid his valuation. That is, show that $b_i = v_i$ weakly dominates any other bid b_i'.

4. Consider a road which is represented by the interval $[0, 1]$. Let a be a number such that $0 < a < 1$. Vendor 1 can locate at any point on the interval $[0, a]$ (that is, he can locate at any point x such that $0 \leq x \leq a$). Vendor 2 can locate at any point on the interval $[a, 1]$. A unit mass of onsumers are uniformly distributed on $[0, 1]$ and each consumer buys one unit of the good from the vendor who is closest to him. If the two vendors locate at the same point a, then each gets one-half of the consumers.

The game is as follows. Vendors choose locations simultaneiously, and a vendor's payoff is given by the number of consumers who purchase from him.

a) Write down the strategy sets and payoff functions in this game.

b) Suppose $a = 0.5$. Show that this game has a unique Nash equilibrium in pure strategies. That is, you need to show (i) there is a Nash equilibrium, and (ii) there is no other Nash equilibrium.

c) Suppose $a < 0.5$. Show that the game does not have a Nash equilibrium in pure strategies.

5. Two individuals must decide how much to contribute to a public good. Each individual has wealth $w_i > 1/4$ and she must choose to contribute a sum c_i, where $0 \leq c_i \leq w_i$. The amount of the public good, y, is given by

$$y = \sqrt{c_1 + c_2}.$$

Individual i's utility is given by

$$u_i(c_1, c_2) = y + w_i - c_i.$$

Find all the (pure strategy) Nash equilibria of the game where each contributor simultaneously chooses c_i.