ECON3014: Game Theory. Affiliate Exam, December 2007

Answer any three questions. All questions have equal weight.

1.

	L	C	R
U	1,-2	-2,1	0,0
M	-2,1	1,-2	0,0
D	0,0	0,0	1,1

Consider the game depicted above.

a) Show that the game depicted has a pure strategy Nash equilibrium. Show also that this pure strategy Nash equilibrium is unique, i.e. there is no other pure strategy Nash equilibrium.

b) Does this game have a mixed strategy Nash equilibrium where player 1 randomises between U and M only and player 2 randomises between L and C only.

Hint: solve for a mixed strategy equilibrium where the players randomize between the two strategies, as a first step. Then see if this is an equilibrium when player 1 can play his third strategy, given that the other player so mixes.

c) Consider a sequential move version of the game with the same payoff matrix depicted above. Player 1 moves first and chooses his action. Player 2 observes player 1's choice and then chooses his action. Solve for a subgame perfect equilibrium of this game, specifying clearly the action of player 2 at each information set.

2. Two people select a policy that affects them both by alternately vetoing policies until only one remains. First person 1 vetoes a policy. If more than one policy remains, person 2 then vetoes a policy. If more than one policy still remains, person 1 then vetoes another policy. The process continues until only one policy has not been vetoed.

a) Suppose there are three possible policies, X, Y, and Z. Person 1 prefers X to Y to Z, and person 2 prefers Z to Y to X. Model this situation as an extensive form game and find its subgame perfect equilibrium.

b) Suppose that there are three possible policies, X, Y, and Z. Person 1 prefers X to Y to Z, and person 2 prefers Y to X to Z. Find the subgame perfect equilibrium.

3. Consider the following Bayesian game. Nature chooses between states ω and ω', where ω is chosen with probability π. Player 1 (the row player) observes the state, while player 2 has no information regarding nature's choice. The two players then play a simultaneous move game with payoffs as given below, where player 1 chooses between T and B, and 2 chooses between L and R.

	L	R
T	2,2	1,3
B	3,1	0,0

payoffs at ω

	L	R
T	4,4	0,0
B	0,0	2,2

payoffs at ω'

Solve for the pure strategy Bayesian Nash equilibrium of this game when $\pi = 0.75$ and $\pi = 0.25$.(There may or may not be more than one equilibrium.)

4. Consider an auction for a single indivisible good with two bidders, $\{1, 2\}$. Each bidder's valuation v_i is independently and uniformly distributed on the interval $[0, 1]$, and this is common knowledge among the players. A bidder observes his own valuation, but not the valuation of his opponent. Consider an auction where the object is allocated to the highest bidder. The price that this bidder pays is determined by the toss of a coin. Witn probability one-half, the bidder pays her own bid. With probability one-half, the bidder pays the losing bid. The person bidding lower pays nothing. If the bids are equal, the object each bidder gets the object with probability one-half, and in this case, pays his bid.

Suppose that bidder 1 assumes that bidder 2 bids a fraction γ of her valuation. Write down the expected payoff to bidder 1, as a function of his own valuation and his bid. (Ignore ties, where both bidders bid the same amount.) Solve for bidder 1's optimal bid, as a function of his valuation. Use this to solve for γ and thereby, for Bayes Nash equilibrium of this game, where each bidder bids a constant fraction γ of his valuation.