

The Value of Private Schools: Evidence from Pakistan*

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Abstract

Using unique data from Pakistan we estimate a model of demand for differentiated products in 112 rural education markets with significant choice among public and private schools. Our model accounts for the endogeneity of school fees and the characteristics of students attending the school. As expected, central determinants of school choice are the distance to school, school fees, and the characteristics of peers. Families are willing to pay on average between 75% and 115% of the average annual private school fee for a 500 meter reduction in distance. In contrast, price elasticities are low: -0.5 for girls and -0.2 for boys. Both distance and price elasticities are consistent with other estimates in the literature, but at odds with a belief among policy makers that school fees deter enrollment and participation in private schooling. Using the estimates from the demand model we show that the existence of a low fee private school market is of great value for households in our sample, reaching about 25% to 100% of monthly per capita income for those choosing private schools. A voucher policy that reduces the fees of private schools to \$0 (from an average annual fee of \$13) increases private school enrollment by 7.5 percentage points for girls and 4.2 percentage points for boys. Our demand estimates and policy simulations, which account for key challenges specific to the schooling market, help situate ongoing debate around private schools within a larger framework of consumer choice and welfare.

Keywords: Education, School Choice, Pakistan, Characteristics Model

JEL Classification:I20,I21

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1 Introduction

Even as debates over school choice in the OECD countries continue, some of the most vibrant schooling markets with significant school choice are emerging in the poorest areas of the world. Driven by the surprising rise of low-fee for-profit schools, the private school share of enrollment in low and middle-income countries has increased from 10% in 1990 to almost 25% in 2015 (Baum et al. (2014)).¹ In Pakistan, the focus of this study, the share of private primary education in 2015 stood at 39 percent. Policy makers and researchers have been rightly concerned about the quality and costs of private schooling, leading to a series of papers on test-scores and civic values in private compared to public schools (see Muralidharan and Sundararaman (2015), Andrabi et al. (2015a), Rao (2013), and Angrist et al. (2002)).

What has received less attention is a second line of research, based on an established tradition in industrial organization, which estimates parents' valuations of different school attributes, and uses these to infer the welfare impacts of counterfactual school policies affecting the choices parents face. For example, currently we do not have estimates of the welfare gains (or losses) due to the introduction of private schools, nor for several other school programs. This contrasts with a large literature evaluating school policies in terms of their impacts on test scores.

Yet, estimating the welfare impacts of education policies is important. As Hausman (1996) points out, any new good with a substantial market share will significantly increase consumer welfare unless its price elasticity is very high. In his example, the introduction of a new brand of cereal, which attained a market share of only 1.6%, nevertheless increased consumer welfare by \$78 million per year (but see also Bresnahan and Gordon (1997) for a discussion). Given that the market share of private schools in our setting is 39 percent, the computation of the welfare benefits of private schools would appear to be a first-order issue.

This paper addresses that gap. Using unique data that we have collected from villages in the province of Punjab, Pakistan, we first estimate a model for the demand of differentiated products, applied to the local education market (Berry et al. (1995) and Berry et al. (2004)). Our model accounts both for the endogeneity of school prices and peer group characteristics. For the latter, we use an instrumental variables strategy proposed by Bayer and Timmins (2007).² To our knowledge, these are the first such estimates in the literature from a population of poor households with low levels of education, which has nevertheless seen a large expansion in private schooling over the last two decades (Andrabi et al. (2008)). Using our

¹Official schooling statistics often undercount such schools as they fall outside the ambit of “registered” and “recognized” schools. Detailed street-by-street enumerations, for instance, by Tooley and Dixon (2005), Dixon (2012), and Andrabi et al. (2010) highlight the growing importance of such schools in a number of countries.

²Such rich data has not been available in other studies of school choice, both in high and low-income countries. The dataset includes several characteristics of every school in each of the villages surveyed, including data on running costs and teacher salaries. It also includes a detailed household survey where we observe which school is attended by each child. Finally, it includes the distance between each school and each household in the survey. The school level data allow us to include a long list of school attributes potentially valued by parents, with less information unobserved to the econometrician. The cost data provides us with potentially exogenous sources of variation in tuition fees at the school level, especially after we control for school attributes and village fixed effects. These two features of the data allow us to credibly estimate the parental willingness to pay for a large set of school attributes and to examine how this willingness to pay for different school attributes varies with household characteristics.

preferred estimates, we compute (a) the welfare cost of abolishing the private school market (which is equivalent to the valuation of private schools); (b) the value of having multiple and differentiated private schooling options in the market (as opposed to a single type of public school and a single type of private school); and (c) the enrollment impacts of a voucher scheme.

What makes our setting particularly attractive for the analysis is that the villages in our sample each constitute a separate education market with parents choosing among the schools in the village, and most enrolment in schools drawn from the village where the school is located. Parents can choose among public and private schools in a very competitive setting with close to 7 schools in each village. This simplifies the issues that arise when markets are not as clearly defined, or when school nominations are affected by strategic considerations due to assignment mechanisms (see for instance, Burgess et al. (2015)).

Moreover, we are able to explore a very rich dataset with information on parental characteristics and their school choices, and a detailed set of school attributes and costs, for all schools in multiple villages. Many of the variables that we use here, ranging from teacher test-scores to detailed cost information on private schools are not readily available in administrative data, but as we will show, are important for understanding this market. We document several noteworthy findings.

First, the central determinant of school choice in this setting is the distance between a student's residence and each school (we do not endogenize the residential location choice explicitly, but we discuss below how our estimates are robust to this issue). The average distance between home and school (for those enrolled) is (only) 510 meters for girls and 680 meters for boys. A 500 meter increase in distance decreases the likelihood that a school is chosen by 5.4 percentage points for girls, and 4.1 percentage points for boys. For a 500 meter reduction in distance, parents of girls are willing to pay three quarters of the average annual value of private school fees (about \$13 per year), while parents of boys are willing to pay more than a full year of school fees. Although these estimates at first glance appear to be very high, we note that similar results are found in experimental studies of enrollment and distance to school in similar contexts (see Burde and Linden (2013), Jacoby and Mansuri (2015), and Glewwe and Muralidharan (2015) for a summary).

Second, price elasticities are quite low: -0.5 for girls and -0.2 for boys. These elasticities imply that an increase in annual tuition by one percent (the average private school fee is \$13 in our sample) decreases the likelihood that a school is chosen by 0.5 and 0.2 percentage points for girls and boys, respectively. These estimates are inconsistent with a general perception that prices act as a significant barrier to the use of private schools. We were surprised to discover that our estimates are consistent with price elasticities in several studies that range from -0.19 (Dynarski et al. (2009)) to -1.44 (Muralidharan and Sundararaman (2015)), with most estimates well below -1. These low price elasticities raise fundamental questions about the cost effectiveness of programs such as vouchers, which we discuss below.

Third, in terms of peer attributes, parents of boys and girls are willing to pay (on average) \$17-\$23 per year for a 1 standard deviation increase in the test scores of the other students in the school. This is around 150% of average annual tuition in a private school, but is imprecisely

estimated for girls. At the same time they are willing to pay (on average) \$5 per year for a 10% reduction in the proportion of students whose mothers have ever attended some schooling. To interpret the second result, notice that: 1) the specification conditions on the average test performance of peers and 2) only about 25% of mothers in these villages have ever attended any school. Conditional on the quality of students in the school, uneducated mothers, who are the majority, may prefer to sort into schools where other mothers have similar levels of education. Indeed this coefficient becomes less negative as mother's education increases.

Fourth, parents also value other school attributes such as teacher characteristics, but their value is lower than the value placed on distance. For instance, a 10 percentage point increase in the proportion of teachers with 3 years of experience from an average of about 60%, or a 10 percentage point increase in the proportion of teachers with a university degree (from an average of 25% for girls and 31% for boys), each has less than a 1 percentage point impact on the probability that a school is chosen.

Using our model we then estimate the welfare cost of abolishing the private school market. Our simulations are equivalent to computing the value of private schools for this population, and to a large extent, the value of school choice. To our knowledge, these are the first such estimates for education in the literature. Our estimates imply that the existence of a low fee private school market is of great value for households living in these locations. For those choosing private schools, the value of private schooling is USD\$3.5 for girls and USD\$12.2 for boys, which corresponds to about 25% and 100% of monthly per capita income. These figures are lower (USD\$1.8 and USD\$5.4, which is 18% and 52% of monthly per capita income) when we take the whole sample of households, including those choosing private schools, those choosing public schools, and those not sending their children to school. If our estimates from rural Punjab are applicable to the entire country, the total value of private schools in Pakistan would be \$114 million per year. We regard this as a lower bound for their true value, since the value of private schooling is likely to be larger in urban areas.

The value of low cost private schools could reflect the fact that the presence of one private school allows students to opt-out of a (perhaps) poorly performing public school. In addition, there could be value to having multiple private schools offering differentiated products, thereby expanding the choice set available to parents (Hausman (1996)). These two sources of valuation are related, but subtly different. We emphasize this difference because it is important to understand whether private schools provide valuable product variety over and beyond the potential of "opting-out" from the public system.

We therefore investigate the welfare loss of limiting choice in each village to public schools and a single hypothetical private school, the latter with the average attributes of all private schools in the village. Under this scenario, the standard deviation of private school enrollment across villages is reduced by at most 25%. Furthermore, we simulate the compensation required to keep the average household indifferent between having the available set of private schools versus this single private school. We obtain values that are only about 20% as high as those needed to compensate household for the total elimination of the private school market and only about 25% as high if we restrict attention to the households more likely to be affected by

this change, i.e., those currently using private schools. We conclude that in this context the rich product variety provided by multiple private schools adds little to consumer welfare, over and beyond the value of opting out from the public option.

Our model also help us further understand the potential impact of vouchers, which we simulate as a reduction to zero in the price of attending any private school. Such a voucher would cost \$13 for each student who uses it and would increase private school enrollment for girls from 19% to 26% and for boys from 23% to 27%. This is not an insubstantial amount, but it is also inconsistent with the view that private school fees are the primary obstacle to their higher use. The lower than expected impact on private school enrollment from a voucher program follows directly from the low price elasticity in our demand estimates and limits the potential cost savings (Muralidharan and Sundararaman (2015)) of using such vouchers to shift enrollment from higher cost public to lower cost private schools. The implied total cost of a voucher is about \$4 for both girls and boys relative to a valuation of around \$2 and we estimate a further cost saving of \$0.8 by shifting children from public schools (whose cost per student is higher) to private schools. This simulation highlights the key difference between using vouchers as an experiment to identify the test-score gains of children attending private schools (Muralidharan and Sundararaman (2015) and Angrist et al. (2002)), and the policy objective of a voucher system. One way to interpret these estimates is to note that the market failure that such programs are designed to address must have a shadow value at least as large as the deadweight loss that we compute.

We situate our contributions within a growing literature in economics on the determinants of school choice in a variety of settings. A few papers estimate simple discrete choice models for the decision between public and private schools (see Alderman et al. (2001), for Pakistan, or Checchi and Jappelli (2004), for Italy). Bayer et al. (2007) estimate more sophisticated residential choice models using US data where individuals choose location based on school quality, among other location attributes. Hastings et al. (2009) estimate an exploding mixed logit with information about the first and second school choices for a large set of household in a particular area, and Gallego and Hernando (2009) estimate a model similar to the one we present here (which is the standard model used to estimate the demand for differentiated products) to understand school choice in the metropolitan area of Santiago, the capital city of Chile. Neilson (2013) also estimates a model of this type for all of Chile, and uses it to understand the mechanisms through which targeted vouchers affected the school performance of poor children.

Relative to the model and data used in Gallego and Hernando (2009) and Neilson (2013), our dataset is smaller, which affects the precision of our estimates. However, we have much richer data on school and household attributes, which allows for a more complete picture of what guides parental choices of schools. In addition, Pakistan is a poorer country than Chile, and there are substantial differences in household preferences. In the Chilean case, for instance, children from richer households are willing to travel farther to school, and in Pakistan, the relationship is reversed. In addition, in Chile, there are a number of administrative and official policies such as vouchers and free-lunch programs that affect the choice and price equilibrium.

While these policies are important to understand in their own right, in our setting private schools are free to choose their prices and capacities, and we do not have instances of private schools receiving vouchers or subsidies during our data collection.

Finally, in a recent paper, Bau (2015) estimates a model similar to the one in this paper as an ingredient in a model of school entry, which asks how incumbent schools react when faced with new entrants. Her implementation of the demand model is however quite different, relying on less rich household data, and ignoring the endogeneity of peer attributes. In addition, it does not present any of the simulations performed in this paper.

The remaining of this paper is organized as follows. Section 2 presents the Data. Section 3 describes the econometric model used to study the determinants of parents choices among different schools. Section 4 presents the estimates from the model and Section 5 provides the results from the simulations. Finally, Section 6 concludes.

2 Data

This paper uses data from the Learning and Education Achievement in Punjab School (LEAPS) project. The LEAPS data are collected from 112 villages in the Punjab province, located in three districts: Attock (North), Faisalabad (Center), and Rahim Yar Khan (South). Villages were randomly chosen from a list of villages with at least one private school according to the 2000 census of private schools; in the first year of the survey, 50% of the rural population of the province lived in such villages. The first wave of data, which we use in this paper and which was collected in 2004, covered 823 schools (government and private) and close to 1800 households (with almost 6000 children). Private schools in these villages face virtually no de facto regulation and do not receive any subsidies from the government or other bodies (Andrabi et al. (2015b)). Therefore, the prices and attributes that they choose provide a relatively unadulterated view of what a market with public and unregulated private schools would look like in similar settings around the world.

The LEAPS project administered surveys to both households and schools, in addition to testing students in three basic subjects: Mathematics, English and the vernacular, Urdu. The household survey includes information on household demographics, expenditure data, and school attendance by children in the household. The schools attended are separately identified for each child, allowing us to link household and school attributes. The school survey has comprehensive information on school characteristics including teacher characteristics (sex, education, experience and performance in Mathematics, English and Urdu tests), basic and extra school facilities, and school costs. These include teacher salaries, the cost of utilities, school materials, and other items. Since it is possible to match households and schools, we are also able to construct the characteristics of the student body of each school, namely average test scores, average parental education, and average household assets for the typical student in the school. Finally, all households and schools were geo coded allowing us to construct the distance from each household's place of residence to each school in the village. This variable is a central determinant of school choice. The unusual combination of very rich household and

school level data make this a particularly attractive dataset to address the estimation issues in this paper.

Table 1 shows means and standard deviations of several school level variables. Each variable is described in Table A.1 in the Appendix. We present three sets of columns: one for all the schools in the sample, one for public schools, and one for private schools. In addition, because we separate our analyses for boys and girls, and because not all schools are attended by children of both genders we also distinguish schools depending on whether they enter the boys or the girls' analysis (with some schools entering both). There are 511 schools attended by girls and 520 schools attended by boys.

Table 1: Summary statistics - school characteristics

School Characteristics	Total		Public		Private	
	Girls	Boys	Girls	Boys	Girls	Boys
Private School (%)	53.6	51.0	-	-	-	-
School fees	-	-	-	-	13.3 (9.3)	13.1 (9.0)
School with toilets	0.85 (0.36)	0.74 (0.44)	0.73 (0.44)	0.52 (0.50)	0.95 (0.22)	0.95 (0.22)
School with permanent classroom	0.87 (0.33)	0.86 (0.34)	0.91 (0.28)	0.88 (0.32)	0.84 (0.37)	0.85 (0.36)
Number of extra facilities	3.0 (1.6)	2.7 (1.7)	2.1 (1.4)	1.7 (1.5)	3.7 (1.2)	3.7 (1.2)
Student test score (average)	0.36 (0.13)	0.35 (0.13)	0.29 (0.11)	0.27 (0.11)	0.42 (0.11)	0.42 (0.11)
Percentage of female teachers	0.82 (0.31)	0.44 (0.44)	0.87 (0.34)	0.09 (0.28)	0.77 (0.28)	0.78 (0.28)
Percentage of teachers with 3 years of experience	0.61 (0.35)	0.62 (0.34)	0.87 (0.24)	0.84 (0.24)	0.39 (0.27)	0.40 (0.26)
Percentage of teachers with university degree	0.25 (0.25)	0.31 (0.27)	0.32 (0.30)	0.42 (0.30)	0.20 (0.19)	0.20 (0.19)
Teacher absenteeism	2.0 (3.7)	1.9 (2.9)	3.0 (4.7)	2.6 (3.4)	1.1 (2.0)	1.2 (2.1)
Teacher test score (average)	0.86 (0.09)	0.87 (0.09)	0.86 (0.08)	0.88 (0.09)	0.86 (0.09)	0.86 (0.08)
Percentage of Mother with some education (school level)	0.27 (0.27)	0.24 (0.26)	0.18 (0.21)	0.12 (0.16)	0.36 (0.29)	0.36 (0.29)
Asset index (school level)	-0.35 (1.05)	-0.59 (1.14)	-0.79 (1.02)	-1.23 (0.99)	0.04 (0.92)	0.03 (0.91)
Number of Schools	511	520	237	255	274	265

Notes: Means and the standard deviations of different school characteristics. The standard deviation is in brackets.

Each variable is described in Table A.1 in the Appendix. We present three sets of columns: one for all the schools in the sample, one for public schools, and one for private schools. In addition, because we separate our analyses for boys and girls, and because not all schools are attended by children of both genders we also distinguish schools depending on whether they enter the boys or the girls' analysis (with some schools entering both).

School fees in US dollars. 1 US dollar = 85.6 Pakistani Rupees.

Private schools are more likely to be coeducational (about half the schools serving both boys or girls in the sample are private) and report better infrastructure, with more toilets, and extra facilities such as gyms, libraries or computer labs. More than 80% of the schools have permanent classrooms, and almost all of them have a blackboard. Public schools do not charge tuition while private schools do with an average tuition of \$13 per year, which is around 10% of annual per capita income. Student test scores (which have approximately a mean of 0.35

and a standard deviation of 0.13 in the sample) are significantly higher by about 1 standard deviation in private compared to public schools. The proportion of female teachers in public schools attended by girls is higher than in schools attended by boys, whereas in private schools there is little difference. Teachers in public schools are more educated and experienced than teachers in private schools, but report higher absenteeism. Teacher test scores are about the same in both types of schools. Finally, the proportion of mothers who have ever attended any school is much higher for students in private schools, as are their household assets.³

Table 2 reports individual and household characteristics for children between 5 and 15 years old in the sample, again distinguishing between boys and girls. There are 2244 girls and 2317 boys in the sample. On average children in the sample are about 10 years old, and their mothers have around 1.3 years of education. There are no basic differences in the characteristics of families of boys and girls. However, girls attend schools closer to their residence and are also much less likely to attend school than boys in general (see also Reis (2015)). The population in our sample is poor, with an average per capita monthly income of about \$10.

Table 2: Summary statistics - individual and household characteristics

Variables	Girls	Boys
Age (years)	9.9 (3.1)	9.7 (2.8)
Mother Education (years)	1.4 (2.7)	1.3 (2.7)
Income per capita	9.9 (10.6)	10.3 (14.1)
Household distance to facilities (Kms)	1.23 (2.96)	1.24 (2.86)
Distance to current school (Kms)	0.51 (0.63)	0.68 (0.88)
Distance to all schools (Kms)	1.09 (1.11)	1.25 (1.34)
Attending school (%)	66.8 (47.1)	79.8 (40.2)
Attending private school (% of attending school)	28.0 (44.9)	28.7 (45.2)
Number of children	2244	2317
Number of Households	1242	1292

Notes: Means and standard deviations of children between 5 and 15 years old, and their household characteristics. The standard deviation is in brackets.

Variables are described in Table A.1 in the Appendix. We present two sets of columns: one for girls and one for boys.

Tables 3 and 4 are analogous to Table 1, showing characteristics of schools attended by boys and girls, but distinguishing families with different levels of maternal education, household income, and average distance between each household and other important facilities in the village, such as hospitals and health clinics. These are often located in the center of the village. It is striking that the average tuition levels of girls attending private schools does not vary much with family background characteristics. However, both the proportion of girls attending any

³Even though we observe family expenditure in the household survey, which we use to construct family background characteristics, we do not observe it in the school census, which we use to construct the average characteristics of students in the school. The census only allows us to construct a simple measure of wealth, which we use as a school attribute.

Table 3: Summary statistics - school characteristics by type of household (girls)

Variables	Mother Education			Log of income per capita			Household distance to facilities	
	Illiterate	At least some education	\leq perc.25	$>$ perc.25 and \leq perc.50	$>$ perc.50 and \leq perc.75	$>$ perc.75 and \leq perc.100	below median	above median
School fees	10.3 (5.5)	11.9 (6.2)	10.9 (5.4)	12.0 (7.8)	10.6 (5.1)	10.9 (5.2)	10.4 (6.1)	12.3 (5.4)
School with toilets	0.81 (0.39)	0.84 (0.37)	0.77 (0.42)	0.81 (0.39)	0.85 (0.35)	0.85 (0.36)	0.86 (0.35)	0.78 (0.42)
School with permanent classroom	0.91 (0.28)	0.92 (0.28)	0.92 (0.27)	0.92 (0.27)	0.93 (0.25)	0.88 (0.32)	0.93 (0.26)	0.89 (0.31)
number of extra facilities	2.98 (1.44)	3.22 (1.39)	2.88 (1.60)	3.06 (1.41)	3.04 (1.45)	3.20 (1.27)	3.23 (1.31)	2.83 (1.54)
Percentage of female teachers	0.93 (0.22)	0.93 (0.18)	0.91 (0.25)	0.94 (0.19)	0.94 (0.18)	0.92 (0.21)	0.95 (0.16)	0.90 (0.25)
Percentage of teachers with 3 years of experience	0.77 (0.30)	0.72 (0.32)	0.82 (0.27)	0.77 (0.30)	0.76 (0.31)	0.67 (0.33)	0.73 (0.32)	0.77 (0.29)
Percentage of teachers with university degree	0.31 (0.25)	0.27 (0.24)	0.31 (0.25)	0.31 (0.26)	0.31 (0.26)	0.28 (0.24)	0.30 (0.24)	0.30 (0.26)
Student test score (average)	0.32 (0.11)	0.34 (0.11)	0.32 (0.11)	0.32 (0.11)	0.32 (0.12)	0.34 (0.11)	0.33 (0.11)	0.32 (0.12)
Teacher absenteeism	2.41 (0.86)	2.05 (0.86)	2.13 (3.09)	2.76 (4.92)	2.36 (4.36)	1.94 (3.24)	1.95 (2.85)	2.76 (5.14)
Teacher test score (average)	0.86 (0.07)	0.86 (0.08)	0.86 (0.07)	0.86 (0.07)	0.86 (0.08)	0.87 (0.07)	0.87 (0.07)	0.85 (0.08)
Percentage of Mother with some education (school level)	0.24 (0.21)	0.32 (0.24)	0.24 (0.24)	0.24 (0.21)	0.27 (0.21)	0.30 (0.25)	0.30 (0.24)	0.23 (0.21)
Asset index (school level)	-0.48 (0.79)	-0.27 (0.80)	-0.70 (0.79)	-0.46 (0.82)	-0.31 (0.73)	-0.24 (0.78)	-0.38 (0.75)	-0.45 (0.87)
Distance	0.53 (0.65)	0.46 (0.57)	0.63 (0.69)	0.51 (0.63)	0.44 (0.50)	0.47 (0.67)	0.36 (0.48)	0.71 (0.75)
Attending school	59.5 (50.9)	87.8 (67.2)	53.7 (50.1)	64.9 (52.2)	72.2 (55.1)	77.6 (58.3)	76.6 (57.6)	56.7 (50.4)
Attending private school	22.2 (41.6)	39.2 (48.9)	16.2 (36.9)	24.5 (43.1)	28.0 (45.0)	39.6 (49.0)	30.7 (46.1)	24.4 (43.0)

Notes: Means and standard deviations of school characteristics by household type. The standard deviation is in brackets.

This table is analogous to Table 1, showing characteristics of schools attended by girls, but distinguishing families with different levels of maternal education, and household income, and household distance to village facilities. The first column presents results for the children with illiterate mothers (0 years of education) and column (2) for mother with at least some education (1 or more years of education). Columns (3) to (6) distinguish families by household income: below 25th percentile, between 25th and 50th percentile, and above 50th and 75th percentile, and above 75th percentile. The last two columns present the school characteristics for households leaving near the village facilities (below median) and those leaving more distant from the village facilities (above median), respectively.

Table 4: Summary statistics - school characteristics by type of household (boys)

Variables	Mother Education		Log of income per capita				Household distance to facilities	
	Illiterate	At least some education	≤perc.25	>perc.25 and ≤perc.50	>perc.50 and ≤perc.75	>perc.75 and ≤perc.100	below median	above median
School fees	12.9 (8.7)	13.0 (7.9)	13.2 (4.7)	13.9 (9.2)	12.8 (9.2)	12.1 (8.0)	12.7 (9.0)	13.3 (7.1)
School with toilets	0.62 (0.48)	0.73 (0.44)	0.69 (0.46)	0.62 (0.49)	0.65 (0.48)	0.65 (0.48)	0.65 (0.48)	0.66 (0.48)
School with permanent classroom	0.91 (0.29)	0.93 (0.26)	0.95 (0.22)	0.88 (0.33)	0.94 (0.24)	0.89 (0.31)	0.94 (0.24)	0.89 (0.31)
number of extra facilities	2.52 (1.57)	3.02 (1.63)	2.58 (1.68)	2.60 (1.49)	2.68 (1.63)	2.76 (1.61)	2.86 (1.57)	2.44 (1.61)
Percentage of female teachers	0.20 (0.37)	0.42 (0.45)	0.15 (0.32)	0.30 (0.42)	0.25 (0.40)	0.34 (0.44)	0.30 (0.43)	0.21 (0.37)
Percentage of teachers with 3 years of experience	0.76 (0.29)	0.65 (0.33)	0.80 (0.28)	0.71 (0.30)	0.74 (0.30)	0.67 (0.33)	0.70 (0.32)	0.76 (0.29)
Percentage of teachers with university degree	0.40 (0.28)	0.32 (0.27)	0.45 (0.27)	0.36 (0.28)	0.38 (0.28)	0.33 (0.27)	0.37 (0.27)	0.39 (0.29)
Student test score (average)	0.30 (0.12)	0.35 (0.12)	0.31 (0.11)	0.32 (0.12)	0.32 (0.12)	0.32 (0.12)	0.32 (0.13)	0.31 (0.11)
Teacher absenteeism	1.90 (2.57)	1.43 (1.99)	2.04 (2.65)	1.68 (2.49)	1.77 (2.46)	1.58 (2.06)	1.44 (1.91)	2.11 (2.84)
Teacher test score (average)	0.89 (0.09)	0.87 (0.10)	0.88 (0.10)	0.89 (0.09)	0.88 (0.09)	0.88 (0.08)	0.88 (0.10)	0.89 (0.08)
Percentage of Mother with some education (school level)	0.19 (0.21)	0.28 (0.23)	0.14 (0.18)	0.22 (0.23)	0.22 (0.21)	0.27 (0.24)	0.25 (0.23)	0.17 (0.21)
Asset index (school level)	-0.85 (0.87)	-0.44 (0.87)	-1.19 (0.85)	-0.65 (0.84)	-0.60 (0.80)	-0.48 (0.91)	-0.61 (0.81)	-0.87 (0.95)
Distance	0.74 (0.92)	0.52 (0.71)	0.97 (1.16)	0.63 (0.78)	0.56 (0.69)	0.55 (0.69)	0.39 (0.46)	0.97 (1.08)
Attending school	76.6 (42.2)	90.0 (30.1)	73.5 (44.2)	79.1 (40.7)	83.8 (36.9)	84.2 (36.6)	83.2 (37.4)	76.6 (42.2)
Attending private school	21.5 (41.1)	48.0 (50.0)	16.2 (36.9)	31.8 (46.6)	30.3 (46.0)	37.0 (48.3)	34.0 (47.4)	23.1 (42.2)

Notes: Means and standard deviations of school characteristics by household type. The standard deviation is in brackets.

This table is analogous to Table 1, showing characteristics of schools attended by boys, but distinguishing families with different levels of maternal education, household income, and household distance to village facilities. The first column presents results for the children with illiterate mothers (0 years of education) and column (2) for mother with at least some education (1 or more years of education). Columns (3) to (6) distinguish families by household income: below 25th percentile, between 25th and 50th percentile, and above 50th and 75th percentile, and above 75th percentile. The last two columns present the school characteristics for households leaving near the village facilities (below median) and those leaving more distant from the village facilities (above median), respectively.

school, and the proportion of girls attending private school, vary by maternal education, family income, and household average distance to facilities. These patterns are somewhat similar for boys, with the difference that average private school tuition for those attending private school is negatively related to household income. Again, this is counterbalanced by the fact that both the proportion of boys attending any school and the proportion of boys attending private school greatly increases with household income.

There are some, but not substantial, differences between the infrastructure of schools attended by children with different family backgrounds, namely toilets, boards, classrooms, extra facilities. If anything, some teacher characteristics (such as education and experience) seem to be worse for children in more affluent households, perhaps reflecting the fact that they attend mostly private schools, where teachers are less educated and less experienced on average (notice also that affluent boys are more likely to have female teachers than poorer boys).

Especially striking is the observation that the average test scores of students in the school are not different in schools attended by rich children and in schools attended by poor children. This is true even though the average levels of assets and maternal education in the school differ dramatically across schools attended by children in different income groups. Finally, for both boys and girls, children of high income families attend schools that are much closer to their residence than children of low income families. This is remarkably different from results from other countries where the likelihood of travelling farther is much higher for children from richer households (Neilson (2013)).

There is substantial cross village variation in the proportion of children in school, varying from 49% to 100% for boys (with a mean of 82%, and a standard deviation of 10%), and from 19% to 96% for girls (with a mean of 69%, and a standard deviation of 16%). Similarly, among those in school, the proportion of boys in a private institution can vary from 3% to 72% (with a mean of 29%, and a standard deviation of 16%), while for girls this variation is from 3% to 100% (with a mean of 30%, and a standard deviation of 18%).

Given this geographical variation in patterns of school enrollment, it is natural to first assess the extent to which this is related to geographical variation in the variables we just described, which will be the main ingredients of the school choice model estimated in this paper. We start by calculating, for each village, the average education and income levels for the mothers of the children in our sample. Then we calculate, again for each village and type of school (private and public), the average level of each school characteristic listed in Table 1. In addition, we calculate the average distance between each household and the closest public school, the average distance between each household and the closest private school, and the number of public and private schools in the village. Finally we estimate simple regressions, relating the proportion of children in the village enrolled in any school and the proportion of students in the village in private schools as outcome variables, with school, community and household characteristics as dependent variables, separately for males and females (one observation per village).

The coefficients and R-squared are reported in Table 5 for both girls and boys. We present three specifications for each model, one with school characteristics alone as regressors (including

Table 5: Regression of the proportion of children in the village enrolled in any school and in private schools

	Girls						Boys					
	Total Enrollment			Enrollment in Private Schools			Total Enrollment			Enrollment in Private Schools		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
School fees	0.002 (0.002)	0.002 (0.002)	0.002 (0.003)	-0.001 (0.003)	-0.001 (0.003)	-0.004 (0.003)	0.002 (0.002)	0.002 (0.002)	0.002 (0.002)	0.003 (0.003)	0.003 (0.003)	0.002 (0.002)
School with toilets	-0.123 (0.082)	-0.121 (0.079)	-0.117 (0.081)	0.156 (0.095)	0.138 (0.094)	0.055 (0.078)	-0.052 (0.057)	-0.037 (0.056)	-0.037 (0.056)	0.037 (0.088)	0.038 (0.075)	-0.090 (0.075)
School with permanent classroom	0.073 (0.075)	0.059 (0.073)	0.063 (0.075)	0.012 (0.056)	0.015 (0.056)	-0.030 (0.046)	-0.033 (0.053)	-0.031 (0.052)	-0.035 (0.052)	0.012 (0.051)	-0.002 (0.052)	-0.003 (0.042)
Number of extra facilities	0.027 (0.022)	0.026 (0.022)	0.026 (0.024)	0.025 (0.023)	0.029 (0.022)	0.040 (0.019)**	0.001 (0.015)	0.009 (0.015)	0.009 (0.015)	0.031 (0.020)	0.026 (0.020)	0.023 (0.016)
Percentage of female teachers	0.210 (0.092)**	0.159 (0.092)**	0.175 (0.098)*	-0.041 (0.088)	-0.086 (0.090)	-0.092 (0.074)	0.004 (0.064)	-0.017 (0.065)	-0.118 (0.065)	-0.118 (0.079)	-0.095 (0.082)	-0.168 (0.069)**
Percentage of teachers with 3 years of experience	0.019 (0.104)	0.047 (0.102)	0.043 (0.113)	0.129 (0.090)	0.151 (0.090)*	0.116 (0.073)	0.101 (0.079)	0.106 (0.077)	0.077 (0.078)	0.025 (0.080)	0.024 (0.080)	-0.047 (0.066)
Percentage of teachers with university degree	-0.008 (0.111)	0.018 (0.108)	0.020 (0.115)	-0.139 (0.135)	-0.138 (0.134)	-0.146 (0.111)	-0.122 (0.093)	-0.107 (0.091)	-0.132 (0.078)	-0.040 (0.117)	-0.040 (0.116)	-0.062 (0.095)
Student test score (average)	-0.432 (0.243)*	-0.390 (0.237)	-0.405 (0.255)	-0.100 (0.254)	-0.066 (0.259)	-0.037 (0.212)	0.013 (0.181)	0.055 (0.176)	0.164 (0.185)	-0.156 (0.217)	-0.139 (0.220)	-0.231 (0.181)
Teacher absenteeism	-0.003 (0.007)	-0.004 (0.007)	-0.004 (0.007)	0.001 (0.012)	0.003 (0.013)	0.009 (0.011)	-0.008 (0.008)	-0.007 (0.008)	-0.007 (0.008)	-0.008 (0.011)	-0.007 (0.011)	-0.007 (0.009)
Teacher test score (average)	0.348 (0.272)	0.175 (0.287)	0.155 (0.298)	0.452 (0.303)	0.304 (0.314)	0.436 (0.258)*	0.026 (0.195)	0.129 (0.197)	0.176 (0.196)	0.414 (0.266)	0.372 (0.269)	0.348 (0.220)
Percentage of Mother with some education (school level)	0.177 (0.124)	0.066 (0.124)	0.049 (0.132)	0.106 (0.103)	0.071 (0.107)	-0.002 (0.089)	-0.121 (0.087)	-0.168 (0.085)*	-0.162 (0.085)*	0.155 (0.097)	0.144 (0.098)	0.116 (0.081)
Asset index (school level)	-0.039 (0.029)	-0.038 (0.028)	-0.036 (0.029)	-0.017 (0.026)	-0.024 (0.026)	-0.018 (0.022)	0.001 (0.021)	-0.004 (0.021)	-0.001 (0.022)	-0.008 (0.024)	-0.007 (0.024)	-0.003 (0.020)
Distance to schools (kms)	-0.092 (0.027)**	-0.074 (0.027)**	-0.083 (0.034)**	-0.001 (0.030)	0.020 (0.031)	0.052 (0.031)*	-0.037 (0.017)**	-0.028 (0.017)**	-0.048 (0.022)	-0.048 (0.027)*	-0.044 (0.028)	0.045 (0.031)
Age (years)	-	-0.001 (0.019)	-0.001 (0.020)	-	0.003 (0.026)	-0.014 (0.021)	-	0.012 (0.015)	0.015 (0.015)	-0.015 (0.025)	-0.015 (0.025)	-0.022 (0.021)
Mother Education (years)	-	0.038 (0.015)**	0.036 (0.016)**	-	0.013 (0.019)	0.015 (0.016)	-	0.023 (0.012)**	0.028 (0.012)**	-	0.033 (0.018)*	0.024 (0.015)
Income per capita	-	0.000 (0.000)**	0.000 (0.000)**	-	0.000 (0.000)**	0.000 (0.000)	-	0.000 (0.000)**	0.000 (0.000)**	-	0.000 (0.000)	0.000 (0.000)
Household Distance to facilities (Kms)	-	0.001 (0.005)	0.001 (0.005)	-	-0.002 (0.006)	-0.005 (0.005)	-	-0.003 (0.004)	-0.002 (0.004)	-	0.006 (0.006)	0.003 (0.005)
Number of private schools in the village	-	-	0.002 (0.009)	-	-	0.037 (0.008)**	-	-0.013 (0.006)**	-	-	-	0.040 (0.007)**
Number of public schools in the village	-	-	0.006 (0.016)	-	-	-0.045 (0.014)**	-	0.004 (0.009)	0.004 (0.009)	-	-	-0.054 (0.011)**
Constant	0.332 (0.295)	0.366 (0.317)	0.371 (0.326)	-0.291 (0.316)	-0.315 (0.368)	-0.076 (0.301)	0.892 (0.202)***	0.577 (0.245)**	0.479 (0.247)*	-0.095 (0.284)	0.046 (0.369)	0.377 (0.305)
Observations	106	106	106	106	106	106	106	106	106	106	106	106
R-squared	0.32	0.40	0.41	0.09	0.16	0.37	0.15	0.25	0.28	0.16	0.20	0.49

Notes: This table presents the regression of the proportion of children in the village enrolled in any school and in private schools. We present three specifications for each model, one with school characteristics alone as regressors (including average distance to school), one where we add family background variables, and an additional one where we add the number of private and public schools in each village. The regressions are run separately for girls - columns (1) to (6), and for boys - columns (7) to (12).

average distance to school), one where we add family background variables, and an additional one where we add the number of private and public schools in each village (these variables are obviously correlated with enrollment, but are also possible measures of the availability of schools in each village). In these specifications, for both boys and girls, between 15% and 41% of the variation in school enrollment, and 9% to 49% of the variation in private school enrollment, can be explained by the entire set of dependent variables. The results just shown suggest that we should be able to understand some of what drives parental school choices based on the variables listed above: school characteristics and family background. We turn to this next.

3 Empirical model

The framework we use to model the demand for schools is standard in studies of the demand for differentiated products, and in the recent literature on neighborhood choice. We closely follow that literature, namely, Berry et al. (1995), Berry et al. (2004), and Bayer and Timmins (2007), adapting the procedures proposed in these papers to the particular characteristics of our problem and dataset. To be concrete, we define the village to be the relevant education market for each household. This is consistent with the data in our sample, where students do not attend primary schools outside their village of residence. We estimate different models for boys and girls.

In each village there are several schools with different attributes. A household chooses a single school among the ones present in her market, and derives utility from its attributes. The utility household i obtains from its child (of gender g) attending school j in village/market t is given by

$$u_{ijtg} = \sum_{k=1}^K x_{jktg} \beta_{ikg} + \gamma_{ig} d_{ijtg} + \lambda_{jtg} + \varepsilon_{ijtg} \quad (1)$$

where $j = \{0, \dots, J\}$ indexes each school competing in a market defined by t . The outside option, corresponding to no enrollment in any school, is represented by $j = 0$. Therefore, u_{i0tg} is the utility individual i derives if he does not go to any of the J schools in the village. k indexes observed school characteristics (x_{jktg}) which are valued differently by each individual. λ_{jtg} is an unobserved school attribute valued equally by everyone. d_{ijtg} is the distance from the house of household i to school j (and represents the role of geography, as in Bayer and Timmins (2007)). ε_{ijtg} is an individual-specific preference for school j in market t , which is assumed to have an extreme value type I distribution.

Let r index observed household characteristics, z_{irtg} , and let v_{itg} be an unobservable characteristic of household i . The value of each school characteristic is allowed to vary with the household's own observed and unobserved characteristics. In particular:

$$\beta_{ikg} = \bar{\beta}_{kg} + \sum_{r=1}^R z_{irtg} \beta_{rkg}^o + \beta_{kg}^u v_{itg} \quad (2)$$

and

$$\gamma_{ig} = \bar{\gamma}_g + \sum_{r=1}^R z_{irtg} \gamma_{rg} + \gamma_g^u v_{itg} \quad (3)$$

As we can see in equations (2) and (3), individual preferences can be divided into three parts: $\bar{\beta}_{kg}$, which is constant within gender; β_{rkg}^o and γ_{rg} , which vary with observable student characteristics, z_{irtg} ; and β_{kg}^u and γ_g^u , which vary with unobservable attributes of the individual, v_{itg} .⁴

Integrating (2) and (3) into (1) we get

$$\begin{aligned} u_{ijt} = & \sum_{k=1}^K x_{jktg} \bar{\beta}_{kg} + \lambda_{jtg} + \sum_{k=1}^K \sum_{r=1}^R x_{jktg} z_{irtg} \beta_{rkg}^o + \\ & + \sum_{k=1}^K x_{jktg} v_{itg} \beta_{kg}^u + \bar{\gamma}_g d_{ijt} + \sum_{r=1}^R d_{ijt} z_{irtg} \gamma_{rg} + \gamma_g^u d_{ijt} v_{itg} + \varepsilon_{ijt} \end{aligned} \quad (4)$$

Household i chooses the school for a child of gender g which maximizes (4).⁵ We can further rewrite this equation as:

$$\begin{aligned} u_{ijt} = & \delta_{jtg} + \sum_{k=1}^K \sum_{r=1}^R x_{jktg} z_{irtg} \beta_{rkg}^o + \sum_{k=1}^K x_{jktg} v_{itg} \beta_{kg}^u + \\ & + \bar{\gamma}_g d_{ijt} + \sum_{r=1}^R d_{ijt} z_{irtg} \gamma_{rg} + d_{ijt} v_{itg} \gamma_g^u + \varepsilon_{ijt} \end{aligned} \quad (5)$$

with

$$\delta_{jtg} = \sum_{k=1}^K x_{jktg} \bar{\beta}_{kg} + \lambda_{jtg}. \quad (6)$$

The coefficients of this model can be estimated using the algorithms described in Berry et al. (1995) and Berry et al. (2004) (under standard assumptions on v_{itg} and ε_{ijt} , discussed in the Appendix B) and in Bayer and Timmins (2007), which we adapt slightly to the type of data we have available. As in these papers, we proceed in two steps.

The first step entails estimating δ_{jtg} , β_{rkg}^o , β_{kg}^u , $\bar{\gamma}_g$, γ_{rg} , γ_g^u by maximum likelihood, including a contraction mapping to obtain δ_{jtg} . This is a hybrid of the procedures proposed in Berry

⁴One restriction we impose in our paper relatively to part of the literature is that v_{itg} does not vary with the k^{th} characteristic being considered (although its coefficient, β_{kg}^u , does vary with k). In other words, the unobservable components of the random coefficients in our model are driven by a single factor: v_{itg} . This assumption greatly simplifies our estimation, by reducing the number of unobservables over which we need to integrate. It is also reasonable to think that these random coefficients are driven by a low dimensional set of unobservables, so that considering a single unobservable may not be a poor approximation of reality.

⁵For simplicity, throughout this paper we assume the sample only includes one parent - one child families. We ignore the possibility that more than one child in the sample can come from the same family. In principle one could write a model where the schooling decision for each child in the household is made simultaneously for all children, but this complicates the analysis substantially. Since the main point of this paper is not to understand the choices made between children in the same household, we proceed with the simpler model, and leave this discussion to another paper.

et al. (1995), and in Berry et al. (2004). Although we are using micro data, and in principle we should be able to estimate all the parameters of the model by maximum likelihood, we do not observe enough households per school for a reliable estimation of school fixed effects δ_{jtg} (for most schools we do not observe much more than 10 children in the household survey). However, since we also have a household census of school choice for each village, it is possible to reliably estimate market shares, and recover δ_{jtg} using the contraction mapping procedure proposed in Berry et al. (1995). Apart from this detail, the way we implement these procedures is quite standard and well explained in the literature, so we leave a detailed discussion to the Appendix B.

The second step entails estimating $\bar{\beta}_{kg}$, by running a regression of the school fixed effect (δ_{jtg}) on the observed school characteristics, as in equation (6). δ_{0tg} , which concerns the outside option, is not explicitly included in the model, and it is captured by a village fixed effect (which, among other things, also measures the preferences for enrollment relatively to non-enrollment).

The household and school variables used to estimate the model are described in Appendix - Table A.1. At the school level (x_{jktg}), we use almost every variable available in the dataset.⁶ At the household level (z_{irtg}) to minimize the computational burden of our procedure we focus on three variables: maternal education, household assets, and average household distance to other facilities in the village (capturing the distance to the village center). In addition to the age of the child, we believe these are the most important household characteristics for our problem. Finally, we allow for a single household unobservable, v_{itg} , to affect the coefficients on all observable school characteristics.

In standard consumer market settings the focus is often on a single endogenous product attribute, typically price. However, in principle, other product attributes could also be endogenously chosen, and all observable product attributes could potentially be correlated with unobserved product attributes. This could be true in our setting if there are important unobservable school attributes left out of the regression. The dataset we use includes a rich set of school characteristics which, together with village fixed effects, explain close to 70% of the total variance of school fixed-effects. Nevertheless there is still the possibility that school characteristics are missing from the data. One common approach to this problem is to give up on interpreting the coefficients on these variables as the households' valuation of the corresponding attributes, and consider them instead as coefficients of a projection of all school characteristics on the set of characteristics we observe. In our main set of results we partially depart from this approach by considering carefully the potential endogeneity of three sets of school attributes: prices, measures of peer group "quality" that are likely important determinants of school choice, and distance to school. These are three variables that play an important role in our empirical model. We discuss next how each of these are addressed in the estimation.

⁶Additional detail in the number and type of extra facilities for each schools does not alter the point estimates substantially, but decreases precision.

3.1 The endogeneity of prices

When estimating equation (6) we are sensitive to the concern that price may be correlated with unobservable school attributes λ_{jtg} . In our main results we instrument price with an index of school costs, data that are typically unobserved and therefore not used in models of this sort. In our case, the availability of cost data together with the theoretical justification that in a monopolistic competition model, price should equal marginal cost plus a markup makes the use of cost data particularly attractive. To compute the school costs, we add all reported school costs and divide by the number of students in the school to obtain a measure of per student cost. We exclude rent payments for schools renting their buildings, since there is no available data on user costs for schools which own (instead of rent) their buildings. We include both the total cost per student and its square in the first stage regression of school fees on school costs and other school attributes.

In our regressions we include all other observed school attributes and village fixed-effects (also included in equation (6)). These school attributes include teacher characteristics (gender, experience, education, absenteeism, and test scores), as well as features of the school’s infrastructure (number of toilets, the existence of a permanent classroom, and the number of extra facilities in the school, such as, for example, a library, or a gym). Our assumption is that, conditional on these variables, the remaining variation in school costs within each village is driven by exogenous and unpredictable cost shocks. This is a strong assumption, but probably not unreasonable, given the very detailed characteristics we have for every school. In addition, we can decompose total costs into various components, some which may be thought to be more likely to be unrelated to unobservable school attributes than others, and examine how our estimates change when we use each of these components individually.

In order to assess the robustness of this assumption, we also use an alternative approach, more standard in the literature estimating these type of models, by generating instruments based on variation in mark-ups arising from the market structure. Berry et al. (1995) proposed the use of observed non-price attributes of other competitors as instruments. These variables are meant to capture how crowded a product is in characteristic space, which should affect the price-cost margin and the substitutability across products. The instruments are justified by assuming that they do not affect the choice of unobservable school attributes of the school, conditional on the observable attributes x_{jktg} we include in the model. We incorporate this alternative set of instruments in the model together with cost shocks, although there is no substantive change to our results if we include only cost variables as instruments for fees.⁷

3.1.1 Addressing (zero) fees in public schools

One issue we face which is not standard in the literature using this type of models is that prices for an important sector of the market, public schools, are exogenously set to zero. Therefore,

⁷More generally, Berry and Haile (2009) discuss the non-parametric identification of multinomial choice demand models with heterogeneous individuals. Under standard “large support” and instrumental variables assumptions, they show identifiability of the random utility model. Our model is a special case of the one presented by Berry and Haile (2009).

we can argue that we can generate exogenous variation in fees (uncorrelated with unobservable school attributes) within type of school (public or private), but it is not clear whether this is still true across schools. This problem is common to most other potential studies on this topic, but has not been explicitly addressed in choice models previously. In our main specifications we only instrument private school fees, and set public school fees to zero. Our assumption is that, conditional on the extensive set of school attributes that we observe (and on village fixed effects), the change from zero to (low) positive predicted fees between public and private schools, is unrelated to unobserved school differences between public and private schools.⁸

We have also considered a simple specification where the only explicitly endogenous variable in the model is fees, and the only instrument is costs. Although our main specification is much richer, as described below, the main conclusions of this paper are essentially unchanged if we take only the simplest model. Our estimated preference parameters, estimates of willingness to pay for different school attributes, demand elasticities, and even some of our policy simulations, can be conducted just with the simpler model, with similar results to those that we report.

3.2 The endogeneity of peer characteristics

In our main set of results we also consider explicitly the endogeneity of a second set of school attributes: the average test scores, maternal education, and household assets of other students in the school. These are measures of peer group “quality”, and therefore they are likely to be important determinants of school choice. They are extensively discussed in the literature on school (and neighborhood) choice (e.g. Bayer et al. (2007)).

In principle, one would need to fully specify and solve the equilibrium model governing the sorting of students to schools, taking into account that each household’s decision depends on the decision of every other household in the village. Bayer and Timmins (2007) propose a simpler IV procedure to estimate the individuals’ valuation of peer attributes in a school, which is consistent with an equilibrium model, but does not require the full solution of a model (even in cases where there are likely to be multiple equilibria).⁹

Their paper considers models of sorting of individuals across locations, where a central location attribute is the proportion of individuals choosing that location. Their goal is to estimate the individual’s valuation of this characteristic. They specify a simple equilibrium sorting model which suggests that, as long as individuals only obtain utility from the characteristics of the location they choose, we can instrument the proportion of individuals choosing a particular location using the non-peer (exogenous) attributes of other locations in the same market.

⁸We could introduce a private school indicator as a control, since conditional on such a variable we could argue that all variation in fees would be exogenous. However, this would not solve that problem of how to identify the impact of going from no fees to some fees on school choice, since we would just be conditioning out any such variation. Nevertheless, below we discuss the results of such a model.

⁹Bayer and Timmins (2007) discuss the circumstances under which this procedure is robust to the possibility of multiple sorting equilibria, which arise naturally in settings with social interactions and local spillovers, such as the one we consider. When the number of individuals in each market is large, the probability that each equilibrium is selected conditional on the distribution of preferences and household characteristics in a given market is orthogonal to a particular individual’s preferences and characteristics. Therefore, the choice model can be estimated conditional on the equilibrium selected in each market, regardless of which one was chosen. This simplifies estimation and the assumption on which it is based is reasonable in villages of considerable size, such as the ones studied in this paper.

Starting from one particular location, if the attributes of its close competitors are very attractive, the demand for competitor locations will increase, and the demand for this location will fall. This means that competitor attributes will be good predictors of the proportion of individuals at each location. If, in addition, exogenous attributes of competitors do not directly affect the utility of those choosing this particular location, then the exclusion restriction is likely to be satisfied. One could potentially use any function of competitors' attributes as instruments, and following the literature on optimal instruments, Bayer and Timmins (2007) suggest using the predicted probability that one chooses a particular location, after restricting the coefficient on the (endogenous) peer variable to zero.

Our setting is slightly different than the one in Bayer and Timmins (2007). The peer attributes we care about are not the proportion of students attending a specific school, but the average characteristics of these students. We modify the main ideas of Bayer and Timmins (2007) as follows. Starting from a particular school, the school (non-peer) attributes of its competitors in the same market are likely to affect the composition of the student body in this school. In addition, the attributes of competitor schools will be valid instruments unless they directly affect the utility each household derives from a given school. Therefore, as in Bayer and Timmins (2007), we propose to simulate the equilibrium sorting of households to schools when the valuation of peer attributes is restricted to be equal to zero, and use the predicted average peer characteristics in each school, resulting from this simulation, as an instrument for the actual average peer characteristics in the school.

To be precise, we start by estimating the model of equations (5) and (6), ignoring the endogeneity of peer attributes. We then set equal to zero the coefficients $(\bar{\beta}_{kg}, \beta_{rkg}^o, \beta_{kg}^u)$ on all peer characteristics in each school (average student test scores, average maternal education, average student assets). In addition, we set the school specific unobservable (λ_{jtg}) also equal to zero. We simulate the proportion of students attending each school once these restrictions are imposed, as well as predict their average test scores, the average education of their mothers, and their average assets.

Let $\tilde{\pi}_{ijtg}$ denote the simulated probability that individual i (of gender g in village t) chooses school j , in the absence of peer variables and school unobservables, and given the household's characteristics and the remaining school attributes. Then, for each peer characteristic p_{iptg} ,

we compute $\tilde{p}_{jptg} = \frac{\sum_{i=1}^{N_{tg}} p_{iptg} \tilde{\pi}_{ijtg}}{\sum_{i=1}^{N_{tg}} \tilde{\pi}_{ijtg}}$, which is the simulated value of peer attribute p in school j (in

village t , and considering only gender g), where N_{tg} is the number of families with children of gender g in village t . Finally we can use these predicted values as instruments for the actual peer variables in equation (6), which includes also as regressors the non-peer attributes of each school.

In addition, to increase the power of our estimates, we compute the predicted values of the peer variables for all other schools in the village, giving us additional functions of the instruments which we can use to predict peer characteristics in each school. Then we estimate a weighted average of these values, using as weights the (relative) distance between a school and each of its competitors. We expect the weighted average of predicted peer attributes in

competitor schools to be negatively related to the value of peer variables in a given school. For example, if a village has two schools, as we increase the average value of maternal education in one school, we decrease it in the other school.

Formally, let e_{jlt} be the distance between schools j and l , both in village t . Then, for each peer characteristic p , we compute $\tilde{q}_{jptg} = \frac{\sum_{l=1}^{J_t} e_{jlt} \tilde{p}_{lptg}}{\sum_{l=1}^{J_t} e_{jlt}}$. We use \tilde{q}_{jptg} , in addition to \tilde{p}_{jptg} , as an instrument for the corresponding peer variable in equation (6).

3.3 Distance to School

There is substantial observational and experimental evidence that distance to school is a powerful determinant of school attendance, so we devote particular attention to this variable (e.g. Burde and Linden (2013), Alderman et al. (2001), and Gallego and Hernando (2009)). The main concern in interpreting this coefficient is that households may not locate in a village at random. It is plausible to argue that they do not choose their residence to be close to a particular school, since several of the private schools are small and enter and exit the market with some frequency. However, households living in the center of the village are generally richer and may also be different in unobserved ways to households living elsewhere. Since private schools tend to locate near to the center of villages, these households will also have access to more schools, which would create a correlation between distance to school and unobserved household characteristics.

In order to address this issue we include in the model the average distance between each household and other important facilities in the village, such as for example, hospitals and health clinics. These are often located in the center of the village as well. This allows us to interpret the coefficients on distance to school as the impact of this variable, after controlling for distance to the village center, which captures other household unobservables that could potentially be related to household preferences for schools. This is the strategy developed by Andrabi et al. (2015a) for their causal estimates of the impact of private schooling on test-scores, and is justified with recourse to the historical settlement patterns in these villages.

4 Estimates from the model

We consider a mix of household (z_{irtg}) and school variables (x_{jktg}) in the model. One variable, distance to school (d_{ijtg}), results from the locations of each household and each school, and therefore it is treated in a slightly different way than the other school characteristics (see also Bayer and Timmins (2007)). The valuation of school characteristics is allowed to vary with both observable and unobservable household characteristics (z_{irtg} and v_{itg}), which means that we can entertain a very rich set of substitution patterns in the data. We have estimated a variety of specifications of our model: i) instrumenting only fees; ii) instrumenting both fees and peer attributes; and iii) using alternative sets of instruments, both for fees and for peer variables. In our main results we instrument fees using school costs and BLP type instruments, and instrument peer variables using both \tilde{p}_{jptg} and \tilde{q}_{jptg} . We present the remaining specifications

in the Appendix, and discuss them briefly in the text. Our main results are fairly stable across specifications.¹⁰

4.1 First stage

We estimate equation (5) using maximum likelihood, with an additional step to estimate the school fixed effect (as described above and in Appendix B). The estimated coefficients are shown in tables A.2, A.3 and A.4, in Appendix A. The coefficients in equation (6) can be estimated using instrumental variables, although we also present OLS estimates for comparison. Since distance to school is not a fixed school attribute, but depends on each household’s location, the coefficients related to this variable are estimated in the initial maximum likelihood procedure (see also Bayer and Timmins (2007)).

The results for the first stage regressions are displayed in the Appendix A, and show that per student school costs predict school fees in private schools.¹¹ There we also see that non-fee (and non-peer) attributes of other schools predict peer variables, especially after we use the optimal instrument proposed by adapting the procedure in Bayer and Timmins (2007), explained at the end of section 3: \tilde{p}_{jptg} (the predicted value of the peer variable p in school j) and \tilde{q}_{jptg} (the predicted value of the peer variable p in competitor schools, weighted by distance from school j to each competitor school).¹² For girls, we can predict average maternal education and average wealth of students using these instruments, but not average test scores of other students. For boys, they are good predictors of all three variables. This means that we may have difficulty estimating the valuation of peer test scores for parents of girls.

4.2 Parental willingness to pay for school attributes

Tables 6 (girls) and 7 (boys) show the estimated coefficients for equation (6) using different specifications. The first column shows OLS estimates, the second column shows the main IV estimates, and the remaining columns correspond to IV estimates using alternative sets of instruments: either individual cost components, or total costs. The estimated coefficients vary across columns, but reassuringly their magnitudes (and signs) are in the same range. We will comment on the magnitude and statistical significance of the coefficients below, when we discuss the willingness to pay for school attributes. As mentioned before, we proceed by using the IV estimates in the second column, where total costs (minus rent payments), as well as the non-fee and non-peer attributes of other schools, are used as instruments for school fees (since results are essentially unchanged when we use only costs as instruments).

We combine the estimated coefficients in equations (5) and (6) in tables 8 (girls) and 9

¹⁰In most specifications we find small and statistically insignificant impacts of peer variables on school choices. In Appendix we also present models where the coefficients on peer variables are constrained to be equal to zero.

¹¹The first column of table A.5 reports the specification using only total costs, and the second uses both costs and the BLP instruments. Table A.6 shows what happens when we use individual cost components separately.

¹²Table A.7 shows that the coefficient on \tilde{p}_{jptg} is positive, indicating that the higher the predicted value of the peer variable in the school, based on a model with only exogenous school attributes, the higher the actual value of the peer variable in the school. The coefficient on \tilde{q}_{jptg} is negative, indicating that the value of the peer variables in the school decline with the predicted value of peer variables in competitor schools. Therefore, the signs of the coefficients on these two variables are as expected.

Table 6: OLS vs. IV regressions - girls

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	OLS	IV	IV	IV	IV	IV	IV	IV
School fees	-0.047 [0.011]***	-0.066 [0.013]***	-0.068 [0.014]***	-0.070 [0.014]***	-0.071 [0.014]***	-0.071 [0.014]***	-0.071 [0.014]***	-0.056 [0.020]***
School with toilets	0.085 [0.198]	0.24 [0.260]	0.234 [0.266]	0.31 [0.260]	0.281 [0.259]	0.266 [0.259]	0.27 [0.259]	0.221 [0.274]
School with permanent classroom	0.21 [0.211]	0.202 [0.205]	0.203 [0.208]	0.192 [0.200]	0.204 [0.205]	0.208 [0.207]	0.204 [0.208]	0.101 [0.222]
Number of extra facilities	0.175 [0.049]***	0.255 [0.183]	0.258 [0.183]	0.264 [0.177]	0.265 [0.176]	0.286 [0.186]	0.262 [0.179]	0.243 [0.209]
Percentage of female teachers	0.848 [0.239]***	1.594 [0.426]***	1.581 [0.435]***	1.598 [0.439]***	1.599 [0.429]***	1.601 [0.436]***	1.616 [0.447]***	1.6 [0.450]***
Percentage of teachers with 3 years of experience	1.244 [0.214]***	1.237 [0.808]	1.215 [0.812]	1.179 [0.812]*	1.147 [0.782]	1.058 [0.866]	1.158 [0.802]	1.4 [0.943]*
Percentage of teachers with university degree	0.677 [0.246]***	0.486 [0.301]*	0.48 [0.294]*	0.454 [0.290]	0.439 [0.305]	0.404 [0.316]	0.454 [0.308]	0.572 [0.345]*
Student test score (average)	0.389 [0.500]	9.87 [7.154]	9.813 [7.712]	9.71 [7.443]	9.95 [6.992]	9.263 [7.734]	10.093 [7.303]*	9.091 [8.462]
Teacher absenteeism	0.02 [0.015]	0.008 [0.016]	0.008 [0.016]	0.008 [0.015]	0.008 [0.016]	0.009 [0.016]	0.008 [0.016]	0.008 [0.016]
Teacher test score (average)	0.256 [0.719]	-0.84 [0.902]	-0.801 [0.887]	-0.575 [0.886]	-0.831 [0.890]	-0.798 [0.918]	-0.656 [0.917]	-0.741 [1.050]
Percentage of Mother with some education (school level)	-0.355 [0.258]	-2.809 [1.656]*	-2.721 [1.596]*	-2.695 [1.606]*	-2.855 [1.593]*	-2.97 [1.627]**	-2.731 [1.698]*	-3.378 [1.997]*
Asset index (school level)	-0.072 [0.077]	-0.888 [0.671]	-0.894 [0.653]	-0.92 [0.685]	-0.919 [0.664]	-0.915 [0.677]	-0.944 [0.660]	-0.561 [0.716]

Instruments to:

School fees	Total Cost and BLP	Total Cost	Utilities	Teacher Staff	Non-Teacher Staff	Educ. Material	Other
Peer Variables	Predicted value of the peer variables (student test score, Mother Education and Asset Index) in competitor schools						

Notes: This table shows the estimated coefficients for equation (6) for girls (estimation of $\bar{\beta}_{kg}$ by running a regression of the school fixed effect (δ_{jtg}) on the observed school characteristics) using different specifications. The first column shows OLS estimates, the second column shows our main IV estimates, which includes total costs without rent, BLP instruments, and peer variables as instruments. The remaining columns correspond to IV estimates using alternative sets of instruments to school fees: either total and individual cost components. The BLP instruments are the average of each school characteristic of the competitors except for school fee. For the peer variables (student test score, percentage of mother with some education and asset index) the instruments are the predicted value of the respective peer variable in competitor schools.

Bootstrapped standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 7: OLS vs. IV regressions - boys

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	OLS	IV	IV	IV	IV	IV	IV	IV
School fees	-0.012 [0.008]	-0.031 [0.011]***	-0.034 [0.011]***	-0.033 [0.013]***	-0.033 [0.011]***	-0.033 [0.012]***	-0.035 [0.012]***	-0.030 [0.013]***
School with toilets	0.116 [0.140]	0.118 [0.187]	0.122 [0.191]	0.132 [0.191]	0.121 [0.187]	0.102 [0.191]	0.129 [0.189]	0.044 [0.207]
School with permanent classroom	0.579 [0.150]***	0.676 [0.169]***	0.679 [0.168]***	0.678 [0.165]***	0.68 [0.179]***	0.647 [0.170]***	0.681 [0.170]***	0.606 [0.177]***
Number of extra facilities	0.099 [0.043]***	-0.006 [0.121]	-0.005 [0.119]	-0.006 [0.123]	-0.005 [0.124]	0.003 [0.121]	-0.006 [0.123]	0.013 [0.128]
Percentage of female teachers	-1.153 [0.164]***	-1.529 [0.501]***	-1.524 [0.479]***	-1.531 [0.488]***	-1.526 [0.494]***	-1.543 [0.498]***	-1.528 [0.512]***	-1.610 [0.500]***
Percentage of teachers with 3 years of experience	0.597 [0.187]***	0.995 [0.348]***	0.984 [0.331]***	0.989 [0.343]***	0.986 [0.351]***	0.938 [0.346]***	0.956 [0.356]***	0.884 [0.363]***
Percentage of teachers with university degree	0.721 [0.202]***	0.716 [0.224]***	0.711 [0.220]***	0.716 [0.225]***	0.712 [0.231]***	0.706 [0.238]***	0.708 [0.240]***	0.777 [0.257]***
Student test score (average)	0.059 [0.508]	3.499 [2.132]*	3.490 [2.153]	3.323 [2.118]	3.458 [2.198]	3.385 [2.137]	3.371 [2.219]	3.633 [2.372]
Teacher absenteeism	0.002 [0.016]	-0.005 [0.019]	-0.005 [0.019]	-0.004 [0.019]	-0.005 [0.019]	-0.002 [0.020]	-0.005 [0.019]	0.007 [0.019]
Teacher test score (average)	0.675 [0.663]	-0.132 [0.956]	-0.122 [0.949]	0.014 [0.977]	-0.102 [0.985]	-0.062 [0.969]	-0.008 [0.988]	0.222 [1.068]
Percentage of Mother with some education (school level)	-0.187 [0.237]	-1.296 [2.469]	-1.245 [2.497]	-1.078 [2.476]	-1.225 [2.453]	-1.134 [2.554]	-1.059 [2.525]	-1.080 [2.604]
Asset index (school level)	0.044 [0.065]	0.620 [0.619]	0.618 [0.600]	0.611 [0.610]	0.615 [0.617]	0.604 [0.647]	0.606 [0.623]	0.589 [0.642]

Instruments to:

School fees	Total Cost and BLP	Total Cost	Utilities	Teacher Staff	Non-Teacher Staff	Educ. Material	Other
Peer Variables	Predicted value of the peer variables (student test score, Mother Education and Asset Index) in competitor schools						

Notes: This table shows the estimated coefficients for equation (6) for boys (estimation of $\bar{\beta}_{kg}$ by running a regression of the school fixed effect (δ_{jtg}) on the observed school characteristics) using different specifications. The first column shows OLS estimates, the second column shows our main IV estimates, which includes total costs without rent, BLP instruments, and peer variables as instruments. The remaining columns correspond to IV estimates using alternative sets of instruments to school fees: either total and individual cost components. The BLP instruments are the average of each school characteristic of the competitors except for school fee. For the peer variables (student test score, percentage of mother with some education and asset index) the instruments are the predicted value of the respective peer variable in competitor schools.

Bootstrapped standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

(boys), where we not only display these coefficients, but also show how the effects of the school characteristics on utility (equation (6)), and the willingness to pay for each of them, change with the family background of the child. We evaluate the impacts of the school characteristics at 3 points of the joint distribution of maternal education and household assets. These are the 25th percentiles of both variables, the means of both variables, and the 75th percentiles of both variables. In the tables they are labeled 25th, Mean, and 75th, respectively.

Each table has 3 sets of columns. Columns 1 to 3 show the impact of each school characteristic on parental utility, at 3 different points of the distribution of family background. Columns 5 to 7 report the willingness to pay for changes in each school characteristic at different points of the distribution of family background. The magnitude of the changes considered in the willingness to pay calculations vary across variables, because each variable has a different scale.¹³ The size of the relevant change for each variable is reported in column 4. For example, for the proportion of female teachers we report 0.10 in column 4, indicating that in columns 5-7 we compute the willingness to pay for a 10 percentage point increase in the proportion of female teachers in the school.

The estimates in columns 1-3 in tables 8 and 9 can be broadly divided into three categories: price, distance and other school attributes. Price and distance are very important for both girls and boys. Similarly, point estimates for the coefficients on the average test scores of other students are large for children of both genders, indicating that they place a very high value on peer quality. However, they are generally statistically significant only for boys, and although the magnitude of the IV coefficient is large it is not statistically significant for girls (Table 8).

Other school attributes matter differently for boys and girls. For girls, there are statistically significant valuations (at least at some point in the distribution of family background) of the number of extra facilities, the proportion of other students whose mothers have at least some education, and several teacher attributes (the proportion of female teachers, the proportion of teachers with a university degree and the proportion of teachers with at least 3 years of experience). For boys, the statistically significant coefficients concern the presence of a permanent classroom, and again, several teacher attributes (the proportion of female teachers, the proportion of teachers with at least 3 years of experience and the proportion of teachers with a university degree). At the mean of family background variables, the education and experience of teachers only appears as a statistically important attribute in the case of boys, while the education of the mothers of other students in the school is mainly important for girls.

Notice that we cannot make direct comparisons of the magnitudes of the coefficients across gender groups unless we assume that the variance of ε_{ijtg} in the random utility model does not vary with gender. However, we can still compute demand elasticities, which, in the following sections, we discuss in more detail for three attributes: fees, distance to school, and the proportion of female teachers. These are all attributes with statistically significant coefficients in equation (6) for both gender groups.

¹³We compute willingness to pay for an attribute in the standard way, by dividing the corresponding coefficient by the coefficient on fees, which in this model also measures the marginal utility of income. We then multiply this fraction by the number in the 4th column of the table, generating columns 5, 6 and 7. Notice that all coefficients vary across households, because of household observable and unobservable variables.

Table 8: Willingness to pay for school characteristics - girls

	Willingness to Pay (in U.S. dollars)						
	25th perc.	mean	75th perc.	Variable variation	25th perc.	mean	75th perc.
School fees	-0.092 [0.013]***	-0.066 [0.013]***	-0.051 [0.014]***				
School with toilets	0.194 [0.240]	0.24 [0.260]	0.303 [0.268]	1.00	2.5	4.2	6.9
School with permanent classroom	0.325 [0.201]	0.202 [0.205]	0.112 [0.214]	1.00	4.1	3.6	2.6
Number of extra facilities	0.321 [0.183]*	0.255 [0.183]	0.213 [0.188]	1.00	4.1	4.5	4.9
Percentage of female teachers	1.455 [0.428]***	1.594 [0.426]***	1.675 [0.453]***	0.10	1.8	2.8	3.8
Percentage of teachers with 3 years of experience	0.895 [0.842]	1.237 [0.808]	1.423 [0.809]*	0.10	1.1	2.2	3.3
Percentage of teachers with university degree	0.387 [0.296]	0.486 [0.301]	0.524 [0.309]*	0.10	0.5	0.9	1.2
Student test score (average)	7.77 [7.857]	9.87 [7.154]	11.371 [7.327]	0.13	12.8	22.7	33.9
Teacher absenteeism	0.016 [0.015]	0.008 [0.016]	0.003 [0.016]	1.00	0.2	0.1	0.1
Teacher test score (average)	-0.555 [0.909]	-0.84 [0.902]	-1.048 [0.923]	0.08	-0.6	-1.2	-1.9
Percentage of Mother with some education (school level)	-3.188 [1.572]**	-2.809 [1.656]*	-2.676 [1.682]*	0.10	-4.0	-5.0	-6.1
Asset index (school level)	-0.974 [0.626]	-0.888 [0.671]	-0.859 [0.696]	1.05	-13.0	-16.5	-20.7
Distance	-1.173 [0.099]***	-1.167 [0.095]***	-1.164 [0.100]***	0.50	-7.4	-10.3	-13.3

Notes: This table shows how the effects of the school characteristics in equation (6) on utility, and the willingness to pay for each of them, change with the family background of the girl. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where $m = \{25th \text{ percentile, mean, } 75th \text{ percentile}\}$. We label these: 25th, Mean, and 75th, respectively. Columns 1 to 3 show the impact of each school characteristic on utility at 3 different percentiles of the distribution of family background. Columns 5 to 7 report the willingness to pay for changes in each school characteristic, and the size of the change considered is shown in column 4.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 9: Willingness to pay for school characteristics - boys

	Willingness to Pay (in U.S. dollars)					
	25th perc.	mean	75th perc.	Variable Variation	25th perc.	75th perc.
School fees	-0.040 [0.011]***	-0.031 [0.011]***	-0.028 [0.011]***			
School with toilets	0.135 [0.182]	0.118 [0.187]	0.070 [0.182]	1.00	3.9	2.9
School with permanent classroom	0.633 [0.164]***	0.676 [0.169]***	0.695 [0.172]***	1.00	18.5	29.0
Number of extra facilities	-0.017 [0.114]	-0.006 [0.121]	0.003 [0.124]	1.00	-0.5	0.1
Percentage of female teachers	-1.698 [0.469]***	-1.529 [0.501]***	-1.423 [0.488]***	0.10	-5.0	-5.9
Percentage of teachers with 3 years of experience	1.151 [0.339]***	0.995 [0.348]***	0.832 [0.354]***	0.10	3.4	3.5
Percentage of teachers with university degree	0.818 [0.236]***	0.716 [0.224]***	0.652 [0.226]***	0.10	2.4	2.7
Student test score (average)	3.506 [2.168]	3.499 [2.132]*	3.476 [2.122]*	0.13	13.3	18.9
Teacher absenteeism	-0.009 [0.020]	-0.005 [0.019]	0.001 [0.019]	1.00	-0.3	0.0
Teacher test score (average)	-0.481 [0.949]	-0.132 [0.956]	0.124 [0.955]	0.09	-1.3	0.5
Percentage of Mother with some education (school level)	-1.644 [2.377]	-1.296 [2.469]	-1.206 [2.492]	0.10	-4.8	-5.0
Asset index (school level)	0.629 [0.659]	0.620 [0.619]	0.618 [0.623]	1.14	20.9	29.4
Distance	-0.843 [0.072]***	-0.814 [0.068]***	-0.793 [0.072]***	0.50	-12.3	-16.5

Notes: This table shows how the effects of the school characteristics in equation (6) on utility, and the willingness to pay for each of them, change with the family background of the boy. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where $m = \{25th \text{ percentile, mean, } 75th \text{ percentile}\}$. We label these: 25th, Mean, and 75th, respectively. Columns 1 to 3 show the impact of each school characteristic on utility at 3 different percentiles of the distribution of family background. Columns 5 to 7 report the willingness to pay for changes in each school characteristic, and the size of the change considered is shown in column 4.

Standard errors in brackets.

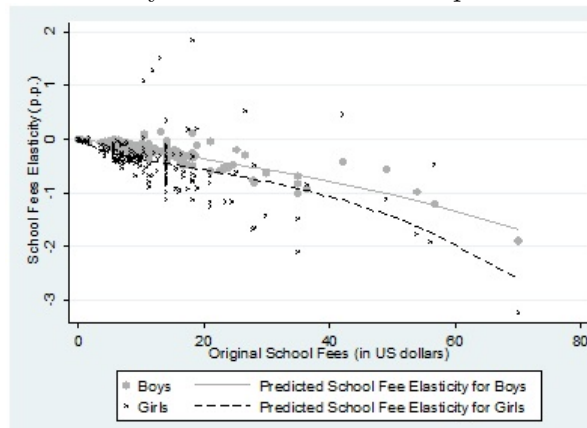
* Significant at 10%; ** significant at 5%; *** significant at 1%.

4.3 School fees

One striking result is that the elasticity of demand with respect to fees is well below 1 for most of the schools. The price elasticity is estimated to be -0.5 for girls and -0.2 for boys. This estimate of the fees elasticity increases (in absolute value) with the level of the fee in the school, suggesting that more expensive schools price in a more elastic section of the demand curve (Figure 1). These small numbers indicate that the demand for private schools is fairly price-inelastic and very large increases in tuition would be required to induce large shifts away from private schools. In other words, parents are willing to pay considerable amounts to keep their children in private schools and avoid any threat to the existence of a private school market (which is what we show in our simulations below).

In addition, this elasticity is higher for girls than for boys, implying that parents are more sensitive to price when it comes to choosing schools for girls than for boys. As we saw above, they may be less sensitive to teacher quality (as measured by teacher education and experience) for girls than for boys. Such a pattern would be consistent with a model where parents perceive boys to be more likely than girls to make use of the skills acquired in school (say in the labor market), leading them to value quality of schooling more for boys than for girls.

Figure 1: Elasticity of enrollment with respect to school fees



Notes: This figure represents the elasticity of demand with respect to fees, as a function of the original school fees, for both girls and boys. The school fee elasticity is a measure of how much the enrollment in each school changes (in percentage points) when the price increases by 1 percent. Schools not charging fees (public) are excluded from the sample.

4.4 Distance

The elasticity of demand with respect to distance to school is of particular interest. Distance is a key determinant of school choice for both boys and girls, but is substantially larger for girls. This could be expected, if parents are more protective of girls than boys and less willing to let them walk longer distances. Our estimates suggest that increasing the distance to school by 500 meters decreases the likelihood of choosing that school by 5.4 p.p for girls and 4.1 p.p for

boys. Tables 8 and 9 show that parents are willing to pay \$15.3 for a 500 meters reduction in distance to school for boys (from an average distance of 680 meters to the current school, and 1250 meters to all schools in the village) and \$10.3 for girls. The magnitudes of the estimates are substantial, especially when compared to the annual fee in a typical private school. Notice also that the willingness to pay for distance is much higher for boys than for girls. This is particularly surprising since the elasticity of demand with respect to distance is higher for girls than for boys. In spite of this, boys are also much less price elastic than girls, and therefore they are willing to pay more than girls for the same decrease in distance.

Another way to highlight the importance of distance relative to other school attributes in the demand for schooling is to express willingness to pay for each school attribute in terms of distance to school, instead of in monetary terms. The question is: how much further travel to school would a parent be willing to tolerate for a given increase in another school attribute (computed by dividing the coefficient of each attribute of equation (6) by the coefficient on distance in the same equation). The results are shown in tables 10 and 11.

We estimate that parents are willing to travel very small additional distances in response to relatively large changes in other school attributes. For example, taking the girls' estimates, parents are only willing to travel 140 meters more on average (190 meters less for boys) for a 10 percentage point change in the proportion of teachers who are females, 110 meters (120 meters for boys) for a 10% increase in the proportion of teachers with at least 3 years of experience, 40 meters (90 meters for boys) for a 10% increase in the proportion of teachers with a university education, or 750 meters (510 meters for boys) for a \$13.3 reduction in school fees, which would make private schools free on average.¹⁴

Finally, note that the elasticity increases in absolute value with distance, so that a given change in distance has a stronger impact on enrollment at long distances than at short distances (Figure 2). One would think that after traveling a certain distance to attend a school, distance no longer becomes a factor in the choice of school, but the opposite seems to be true. However, the range of distances in our data is quite small: The average distance between each household and all schools in the village is 1.09 Km for girls and 1.25 Km for boys, with standard deviations of 1.11 and 1.34 for girls and boys, respectively. Almost no child lives more than 4 Km from any school in their village, and very few children live more than 2 Km away from a school. Therefore, distance is perhaps not very relevant when distance is really short, and becomes

¹⁴Our estimates also allow us to examine the correlation between parental preferences for different school attributes. These correlations are reported in table A.8 in the Appendix for both girls and boys. Recall that two of the attributes in this table have negative coefficients in parental preferences: school fees and distance. We do not show all attributes in this table, but only the ones for which the coefficients were statistically significant in equation (6). Starting with girls, preferences for teachers with experience, female teachers, extra facilities, and teachers with a university degree, are all positively and strongly correlated. Parents who value one of these attributes also value all the others. The patterns for boys are more irregular, and the strength of the correlations is, in general, much weaker for boys than girls. One can also compare this to the bundles of attributes that schools actually offer. Table A.9 in the Appendix compares the correlations between the same list of attributes offered by schools. It shows that the correlation of these attributes are much weaker and not necessarily positive. In particular, and in contrast to the correlation between preferences, the correlation between teachers with experience, female teachers, extra facilities, and teachers with a university degree offered by schools is negative when looking for all schools and for both girls and boys. Regarding private schools the picture is similar, except that schools offering more extra facilities tend to have more teachers with a university degree.

Table 10: Willingness to pay for school characteristics in terms of distance - girls

	25th perc.		mean		75th perc.		Variable variation	Willingness to Pay (in distance terms Kms)		
								25th perc.	mean	75th perc.
School fees	-0.092	[0.013]***	-0.066	[0.013]***	-0.051	[0.014]***	13.3	-1.04	-0.75	-0.58
School with toilets	0.194	[0.240]	0.24	[0.260]	0.303	[0.268]	1.00	0.17	0.21	0.26
School with permanent classroom	0.325	[0.201]	0.202	[0.205]	0.112	[0.214]	1.00	0.28	0.17	0.10
Number of extra facilities	0.321	[0.183]*	0.255	[0.183]	0.213	[0.188]	1.00	0.27	0.22	0.18
Percentage of female teachers	1.455	[0.428]***	1.594	[0.426]***	1.675	[0.453]***	0.10	0.12	0.14	0.14
Percentage of teachers with 3 years of experience	0.895	[0.842]	1.237	[0.808]	1.423	[0.809]*	0.10	0.08	0.11	0.12
Percentage of teachers with university degree	0.387	[0.296]	0.486	[0.301]	0.524	[0.309]*	0.10	0.03	0.04	0.05
Student test score (average)	7.77	[7.857]	9.87	[7.153]	11.371	[7.327]	0.13	0.86	1.10	1.27
Teacher absenteeism	0.016	[0.015]	0.008	[0.016]	0.003	[0.016]	1.00	0.01	0.01	0.00
Teacher test score (average)	-0.555	[0.909]	-0.84	[0.902]	-1.048	[0.923]	0.08	-0.04	-0.06	-0.07
Perc. of Mother with some education (school level)	-3.188	[1.572]**	-2.809	[1.656]*	-2.676	[1.683]*	0.10	-0.27	-0.24	-0.23
Asset index (school level)	-0.974	[0.626]	-0.888	[0.671]	-0.859	[0.696]	1.05	-0.87	-0.80	-0.77
Distance	-1.173	[0.099]***	-1.167	[0.095]***	-1.164	[0.100]***				

Notes: This table shows how the effects of the school characteristics in equation (6) on utility, and the willingness to pay for each of them, change with the family background of the girl. In this table we express willingness to pay for each school attribute in terms of distance to school, instead of expressing it in monetary terms. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where $m = \{25th \text{ percentile, mean, } 75th \text{ percentile}\}$. We label these: 25th, Mean, and 75th, respectively. Columns 1 to 3 show the impact of each school characteristic on utility at 3 different percentiles of the distribution of family background. Columns 5 to 17 report the willingness to pay for changes in each school characteristic, and the size of the change considered is shown in column 4.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 11: Willingness to pay for school characteristics in terms of distance - boys

	Willingness to Pay (in distance terms Kms)						
	25th perc.	mean	75th perc.	Variable variation	25th perc.	mean	75th perc.
School fees	-0.04 [0.011]***	-0.031 [0.01]***	-0.028 [0.011]***	13.3	-0.63	-0.51	-0.47
School with toilets	0.135 [0.182]	0.118 [0.187]	0.07 [0.182]	1.00	0.16	0.15	0.09
School with permanent classroom	0.633 [0.164]***	0.676 [0.169]***	0.695 [0.172]***	1.00	0.75	0.83	0.88
Number of extra facilities	-0.017 [0.114]	-0.006 [0.121]	0.003 [0.124]	1.00	-0.02	-0.01	0.00
Percentage of female teachers	-1.698 [0.469]***	-1.529 [0.501]***	-1.423 [0.488]***	0.10	-0.20	-0.19	-0.18
Percentage of teachers with 3 years of experience	1.151 [0.339]***	0.995 [0.348]***	0.832 [0.354]***	0.10	0.14	0.12	0.10
Percentage of teachers with university degree	0.818 [0.236]***	0.716 [0.224]***	0.652 [0.226]***	0.10	0.10	0.09	0.08
Student test score (average)	3.506 [2.168]	3.499 [2.132]*	3.476 [2.122]*	0.13	0.54	0.56	0.57
Teacher absenteeism	-0.009 [0.020]	-0.005 [0.019]	0.001 [0.019]	1.00	-0.01	-0.01	0.00
Teacher test score (average)	-0.481 [0.949]	-0.132 [0.956]	0.124 [0.955]	0.09	-0.05	-0.01	0.01
Perc. of Mother with some education (school level)	-1.644 [2.377]	-1.296 [2.469]	-1.206 [2.492]	0.10	-0.19	-0.16	-0.15
Asset index (school level)	0.629 [0.659]	0.62 [0.619]	0.618 [0.623]	1.14	0.85	0.87	0.89
Distance	-0.843 [0.072]***	-0.814 [0.068]***	-0.793 [0.072]***				

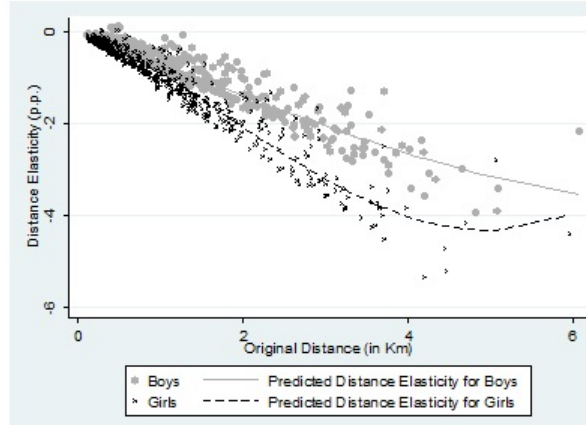
Notes: This table shows how the effects of the school characteristics in equation (6) on utility, and the willingness to pay for each of them, change with the family background of the boy. In this table we express willingness to pay for each school attribute in terms of distance to school, instead of expressing it in monetary terms. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where $m = \{25th \text{ percentile, mean, } 75th \text{ percentile}\}$. We label these: 25th, Mean, and 75th, respectively. Columns 1 to 3 show the impact of each school characteristic on utility at 3 different percentiles of the distribution of family background. Columns 5 to 7 report the willingness to pay for changes in each school characteristic, and the size of the change considered is shown in column 4.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

more relevant as it increases. Finally, virtually no child attends a school outside the village, regardless of whether a potential school is located in a nearby village or a far away village (for very long distances, demand is zero, and unresponsive to further changes in distance, so the elasticity is not well defined).¹⁵

Figure 2: Elasticity of enrollment with respect to distance



Notes: This figure represents the elasticity of demand with respect to distance, as a function of the original distance (in Kms), for girls and boys. The distance elasticity is a measure of how much the enrollment in each school changes (in percentage points) when the distance increases by 1 percent.

4.5 Other school attributes

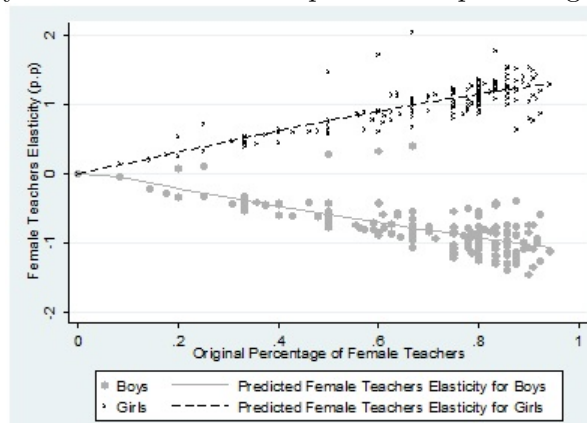
A few more school attributes are of particular interest. First, the valuation of the maternal education of peers is negative, especially for girls (the IV coefficient is negative for boys in Table 7, but not statistically significant). Table 8 shows that girl’s parents are willing to pay \$5 for a 10 percentage point reduction in the proportion of students whose mothers have at least some education. When interpreting this, one should note that the vast majority of mothers in these villages have little or no education. Since the regression already controls for the average test score of peers, one explanation for our results is that conditional on the average test performance of other students, the average mother may prefer to sort into schools with similar mothers, as opposed to schools with very different (and more educated mothers). In fact, we see below that the interaction between this school attribute and the education of each individual mother is positive and statistically significant for girls.

Moreover, the elasticity of demand with respect to the proportion of female teachers is positive for girls and negative for boys (Figure 3), and the willingness to pay is much smaller in absolute value for girls than for boys (tables 8 and 9). On average, girl’s parents are willing to pay an additional \$2.8 per year for an increase of 10 percentage point in the proportion of female teachers in the school, corresponding to about 20% of average annual school fees in a

¹⁵Note that we cannot rule out the hypothesis that this pattern is driven by unobserved heterogeneity. Our assumption is that, conditional on all the observables in the model, household and school location are random.

private school. This is a substantial amount, and is consistent with the fact that the average proportion of female teachers is close to 90% in schools attended by girls. However, parents are willing to pay twice as much (\$5.8) for a 10 percentage point reduction in the proportion of female teachers for boys, (which is equal to 44% in the average school attended by boys). This is consistent with our previous results that parents are more sensitive to teacher attributes for boys rather than girls. It could be a surprising finding, however, if our prior was that parents were more protective of girls than boys, and less willing to let them have any contact with adult males.

Figure 3: Elasticity of enrollment with respect to the percentage of female teachers



Notes: This figure represents the demand elasticity with respect to the proportion of female teachers, as a function of the original percentage of female teachers, for both girls and boys. The female teachers elasticity is a measure of how much the enrollment in each school changes (in percentage points) when the percentage of female teachers increases by 1 percent in each school. The schools with 100 percent of female teachers were excluded from the sample.

Parents of boys are also willing to pay significant amounts for other school attributes: \$25.5 for a school with a permanent classroom (in our sample, 86% of all schools have a permanent classroom), \$3.7 for a 10 percentage point increase in the proportion of teachers with 3 or more years of experience (from an average of 62%), and \$2.7 for a 10 percentage point increase in the proportion of teachers with a university degree (from a sample average of 31%). Since these attributes are not randomly assigned across schools, in order to interpret these numbers as true willingness to pay estimates we need to assume that the variables are exogenously assigned conditional on all the remaining observables in the paper.

4.6 Willingness to pay measures and variation by family background

It is also interesting to examine how the household's valuation of a school attribute varies with the family background of the student, characterized by maternal education and household expenditure. It is possible to do this exercise for every school attribute, but from tables A.2, A.3, and A.4, there are few statistically significant coefficients in the model for the interactions between observable school attributes and observable family characteristics. For girls, the

statistically important interactions are between maternal education and school fees, maternal education and the average maternal education of other students in the school, family expenditure and school fees, and family expenditure and distance to school. For boys, the statistically important interactions are between maternal education and school fees, maternal education and the proportion of female teachers in the school, maternal education and the asset index, and family expenditure and the proportion of teachers with at least 3 years of experience.¹⁶

Columns 1 to 3 of Table 8, shows that the sensitivity of girls' enrollment to fees, average maternal education of peers, and distance to school, declines with family background. As we would expect, the elasticity with respect to fees is significantly lower for girls from a higher family background; the coefficients in the table correspond to an elasticity of -0.8 for girls at the 25th percentile relative to -0.4 for girls from the 75th percentile. Given the steep decline in price elasticity, the willingness to pay for changes in either distance or the family background of peers is estimated to increase with family background.¹⁷ Columns 1 to 3 of Table 9 shows that, like girls, the elasticity with respect to fees declines with family background (-0.4 at 25th percentile relative to -0.2 at the 75th percentile). In addition, the sensitivity of boys' enrollment to whether the school has a permanent classroom rises with background variables, and, with regards to both the proportion of female teachers in the school, and the proportion of teachers with at least 3 years of experience either declines or remains flat with the family background of the student.

4.7 Discussion

Our most critical findings are: (a) that distance matters enormously for both boys and girls, and more educated and wealthier parents of girls are willing to pay the most for reductions in distance to school; and (b) that price matters more for girls than boys, but the price elasticity is higher for children from low family backgrounds.

Given the low levels of consumption in this sample, one could a priori assume that price is the sticking point for enrollment, which would be at odds with our estimates. In fact, this assumption drives much of education policy in low-income countries today, ranging from user-fee reductions to vouchers. It is therefore fair to ask whether our estimates are reasonable. We make three observations.

First, we computed the price elasticity from several available studies and report these in Appendix - Table A.10. Our estimated elasticities of -0.5 for girls and -0.2 for boys are low, but very much within the range of estimated elasticities in the literature. The one study with a much higher elasticity of -1.4 is a recent study of vouchers by Muralidharan and Sundarara-

¹⁶There is also a statistically significant interaction between the number of toilets in the school and maternal education, and between the average wealth of other students in the school and maternal education. However, since the mean impact of these school attributes on enrollment is not statistically different from zero we do not comment on these variables.

¹⁷Restricting all interactions to be linear may lead to puzzling results such as this one. While it is sensible that the negative coefficient on the maternal education of peers becomes less important as one's education increases, it does not make as much sense that (at the same time) the willingness to pay for uneducated mothers is increasing in one's education. This result may be a consequence of our linearity assumptions, and could potentially disappear in a more flexible model.

man (2015).¹⁸ But even in their case, it is likely that the salience and excitement generated by the experiment had effects beyond a simple reduction in price, a point made previously by Dynarski et al. (2009). Furthermore, the Muralidharan and Sundararaman (2015) study provided vouchers only to children already enrolled in public schools, which was possible because this was a one-time program that did not have to worry about children enrolling in public schools only to gain eligibility into the program. Since public school students are poorer, their elasticity may be higher than average. This is in fact what happens in our data, where the price elasticity for children with less favorable family background conditions (25th percentile) is about -0.4 for boys and -0.8 for girls.

Second, the estimates are also consistent with recent experimental evidence among adult women in Pakistan. Cheema et al. (2014) find precisely the same results for a skills training program, also in Punjab, where both distance and price were varied exogenously. They argue that there are strong “border” effects with women unwilling to cross settlement boundaries, an argument that echoes the previous findings of Jacoby and Mansuri (2015) in the case of schooling.

Third, in the classic model of horizontal product differentiation with endogenous price choice, firms trade-off market power (obtained by moving farther apart) with market share (obtained by moving closer to each other), when determining the optimal configuration for their product. When travel costs are very high, the loss in market share from even small movements will be high and thus firms will tend to “cluster” in product space. This is precisely what we see in the data. In each of our villages, the private schools are closely clustered around the center, while government schools, which have different objective functions that emphasize access, are also located in the peripheries. In fact, the average distance between private schools is 770 meters, compared to an average distance of 1690 meters for government schools. Thus, the location patterns that we observe in the cross section are remarkably consistent with the distance elasticities that we estimate.

4.8 Further sensitivity to different specifications

4.8.1 Incorporating a private school indicator and school size into the model

There are two potentially important school attributes that have been excluded from the model so far. One is an indicator for private schools. Private schooling could be an attribute in itself, even after accounting for all other school attributes, if parents intrinsically value the fact that a school is private as opposed to being public.¹⁹ The second attribute left out is a measure of school size. This, again, should affect the value parents put on a given school. Here we describe how our estimates change when we include these variables in the model, and explain why we exclude them from our main specification.

¹⁸Illustrative case from an Experiment where only students from public schools are affected. Illustrative calculation using $\frac{\Delta Share Price}{\Delta Price Share} = \frac{0.15}{0*0.7+0.3*Price_0} \frac{Price_0}{0.35} = \frac{0.15}{0.3} \frac{1}{0.35}$.

¹⁹In addition, if we include a private schooling indicator in the model, then the variation in school fees after instrumenting is even more likely to be exogenous, although we would not be able to identify any of its impacts from the public vs. private fees comparison (which is likely to be somewhat important, given the fairly low levels of fees observed in the private schools in our sample).

Estimates of equation (6) including a private school indicator as an attribute are shown in tables A.11 for girls and A.12 for boys in the Appendix A. For both boys and girls, the private school indicator appears with a strong and negative coefficient. At the same time, the coefficient on school fees falls dramatically, and in the case of boys, becomes statistically insignificant. Given the very rich set of school characteristics available in these data, the private school indicator essentially captures the fact that schools fees are strictly positive in private schools. This explains both the negative coefficient on the private school indicator, and the change in the school fee coefficient.²⁰ For this reason, our view is that it is clearer to remove private school from the specification and keep only the school fees variable.

The inclusion of school size as an attribute is clearly problematic in our model, because schools in high demand will tend to be larger than schools in low demand. The coefficient on school size is therefore likely to be positive, not because parents prefer larger schools, but because high demand is a consequence of good quality. This is precisely what happens in our estimates, shown in tables A.13 and A.14 in the Appendix A. Furthermore, all our remaining coefficients in equation (6) become very imprecise, in particular for boys. Therefore, we prefer to use our main specification instead, omitting school size from the list of school attributes.

4.8.2 Ignoring the endogeneity of peer attributes

In our model the characteristics of the population attending a particular school affect the utility parents achieve from sending their child to that school. Although this is not standard in most applications of BLP, it is a feature of many urban economic models that use this framework (e.g., Bayer et al. (2004), Bayer et al. (2007)), and in models examining the sorting of students into schools (e.g., Nesheim (2002)). In a recent paper, Bayer and Timmins (2007) suggest a simple method to estimate preferences for peer characteristics accounting for the equilibrium sorting of individuals to locations in the presence of spillovers of this type, which does not require fully solving the model, and which we implement in this paper.

In principle it is possible to obtain meaningful estimates of the coefficient on fees in equation (6) ignoring the endogeneity of peer attributes of each school, provided that the instrument which is used for fees is orthogonal to all other school attributes in the model. One could then conduct most of the remaining analysis in the paper using such a model. The main difference relatively to what we showed so far could be that one would not be able to interpret the coefficients on the peer variables as the parental valuation of these variables, as discussed above for all other attributes. We examined how our estimates of equation (6) changed when we either ignored the endogeneity of peer attributes in schools, or simply omitted these variables from the model.²¹ There are some small changes in our estimates, which are shown in tables

²⁰This is a more plausible explanation than one where fees (even after instrumenting) are correlated with unobserved attributes of the school, because private schools both charge fees and have better unobservable attributes. In this case, we would probably expect that the coefficient on fees in a comparison of public vs private to be biased towards zero, since the impact of better unobservable attributes of private schools should cancel the negative impact of school fees. But when we include the private school indicator, the coefficient on fees declines, it does not increase.

²¹One should also look at equation (5). There are hardly any changes in those coefficients relatively to the baseline specification we are considering.

A.15 and A.16 in the Appendix A. This suggests that the main conclusions of our paper are robust to the modeling of peer effects.

5 Simulations

5.1 The value of private schools

The structure of the education system in Pakistan, like in many other low and middle-income countries, has changed substantially in the last 3 decades. In Pakistan, Andrabi et al. (2010) show that the number of primary private schools increased more than 10 times in the last two decades. Given this worldwide evolution, it is important to quantify how much households value the current system against a counterfactual that restricts school choice.

Using estimates from equations (5) and (6), we can simulate the welfare consequences of closing down all private schools or alternatively, leaving one private school open in each village. We can also simulate the welfare impacts of a active schooling policy that provides education vouchers to those attending private schools, implying that effective fees in private schools are reduced to zero.

We use a standard measure of Compensating Variation (CV) to measure changes in welfare from the eradication of private schools, and the introduction of vouchers. It represents the change in a household’s income that equates utility across two states: a benchmark state, which is the status quo, and the alternative state, which is the environment without private schools, or the environment with vouchers. CV corresponds to the amount of income required to compensate a given household for the elimination of private schools.

One non-standard feature of our setting relative to other applications of BLP type models is the existence of potential spillovers, arising through the average peer attributes in each school. Either the closing of private schools, or the provision of school vouchers, is likely to change the peer groups in each school. Therefore, given the parameters of our model, one needs to solve for the new sorting equilibrium after a policy is implemented, accounting for spillover effects.

We begin by ignoring that such spillovers exist. This allows us to perform standard calculations, which assume that product (school) attributes do not change as a result of the policy being simulated.

We then relax this assumption, allowing re-sorting to take place in response to changes in peer attributes (taking seriously the point estimates of the valuation of peer attributes, even when they are imprecisely estimated). Once we do so, the estimated welfare impacts change at most by 1 to 3 percent relative to the more restrictive model, leading us to take the simpler specification as our preferred one in this section.²²

²²For each simulation we estimated the welfare impacts updating \tilde{p}_{jptg} , the simulated value of peer attribute p in school j , with the new simulated probabilities for each individual (without a re-estimation of the model). It should be noted that there are difficult practical obstacles in implementing the full simulation (re-estimation of the model with spillovers). The main problem is that we use the school census to compute the average peer attributes at each school, but we estimate the model in the much smaller household survey. The correlation between the average peer attributes at each school computed using the census and the household survey is about 0.5, which is a high number, but far below 1. This means that if we were to use survey based school attributes for our simulations we are likely to introduce substantial measurement error in the procedure.

We also assume that there are no additional spillover effects of either of these policies through changes in school congestion. Take for example, the first policy, which closes down all private schools. All welfare changes induced by such a policy will be driven by parents of children attending private schools, who are now forced to move to a public school. Those attending public schools in the first place will be indifferent between these two scenarios, under the assumption of no spillover effects due to overcrowding. This is a strong assumption that likely leads to an underestimation of the value of private schooling. One final assumption is that the policy changes do not affect the utility of the outside good (i.e., the utility of not enrolling in any school).

Following Nevo (2000) and shown in McFadden (1980) and Small and Rosen (1981), if the marginal utility of income is fixed for each individual, the compensating variation for individual i is given by

$$CV_i = \frac{\ln \left[\sum_{\tilde{j}=0}^{\tilde{J}} \exp(V_{i\tilde{j}}^{Public}) \right] - \ln \left[\sum_{j=0}^J \exp(V_{ij}^{Private}) \right]}{\frac{\partial V_{ij}^{private}}{\partial school\ fees}} \quad (7)$$

where $V_{ij}^{Private}$ represents the utility in the presence of both private and public schools in the choice set of schools and $V_{i\tilde{j}}^{Public}$ represents the counterfactual scenario where only public schools are available to the students²³. The denominator represents the marginal utility of income.

In order to compute the total change in consumer welfare (TCV), one could average the compensating variation across sample and multiply by the number of students (M):

$$TCV = M \int CV_i dP_v(v) \quad (8)$$

where P is a distribution function. In practice, this average can easily be driven by extreme values both in the upper and the lower tails of the distribution of CV_i , which in our setting are essentially driven by extreme values of $\frac{\partial V_{ij}^{private}}{\partial school\ fees}$. This is a concern because the fact that we allow for very rich observed and unobserved heterogeneity in the valuation of school fees can lead to very extreme values of $\frac{\partial V_{ij}^{private}}{\partial school\ fees}$, which are probably sensitive to modifications in the specification of heterogeneity.

Therefore, a more robust alternative is to present results based on the median value of CV_i in the sample, rather than the average. We use this as our main measure in the calculation of the welfare impacts of different policies. To estimate the total welfare of a policy we multiply this figure by the total number of students in the region we are considering. An alternative, which we also implement (and show in Appendix - Table A.17), is to take the average of CV_i after trimming the bottom and top 1% of the distribution of this variable.

Table 12 presents estimates of the median compensating variation for a policy that forces private schools to shut down, separately for boys and girls. If we close all private schools, the estimated median compensating variation is \$5.4 dollars (about 40 per cent of the average school fee) for boys, and \$1.8 for girls (these numbers should be interpreted as annual compen-

²³ $V_{ij} = \delta_{jtg} + \sum_{k=1}^K \sum_{r=1}^R x_{jktg} z_{irtg} \beta_{rk}^o + \sum_{k=1}^K x_{jktg} v_{itg} \beta_k^u + \bar{\gamma} d_{ijtg} + \sum_{r=1}^R d_{ijtg} z_{irtg} \gamma_r + d_{ijtg} v_{itg} \gamma^u$

sating variation). If we focus only on those affected by the policy, i.e., those attending private schools in the current regime, then the estimated compensating variation is \$12.2 for boys and \$3.5 for girls. This compares to the average value of the fee, of about \$13 and is the amount that would have to be given to households to compensate them fully in money metric utility for the closure of private schools. The net benefit of private schools is therefore positive for both boys and girls attending private schools, and is approximately 25% of the value of fees for girls, and around 90% for boys. Another way to think about the value of private schools is that, for households whose children are in such schools, the benefit is equivalent to around one month’s per-capita income for boys, and about 25% of monthly per-capita income for girls.

Table 12: No private schools - policy that forces private schools to shut down

	Girls	Boys
Median compensating variation (in U.S. dollars)	1.8	5.4
Median compensating variation - affected by the policy	3.5	12.2
Total change in consumer welfare (in thousand U.S. dollars)	68.2	202.7
Changes in total school enrollment rate (in percentage points)	-6.0	-5.3

Notes: In this table we present changes in welfare, and changes in total school enrollment from the eradication of private schools.

We use compensating variation to measure the changes in a household’s income that equates utility across two states: a benchmark state, which is the status quo, and the alternative state, which is the environment without private schools. It corresponds to the amount of income required to compensate a given household for the elimination of private schools.

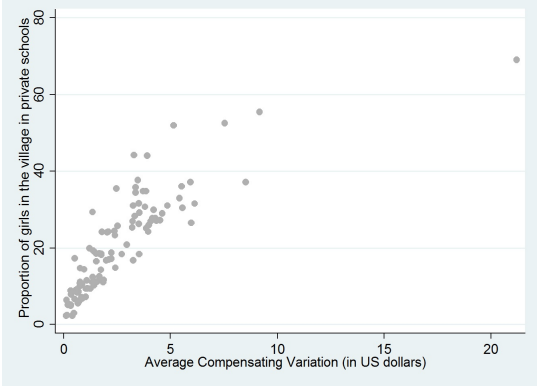
The first two rows present estimates of the median compensating variation (in U.S. dollars) for a policy that forces private schools to shut down, separately for boys and girls. The first row shows the results for everyone, while the second one shows the results for those affected by the policy. In this scenario (no private schools), those not affected by the policy intervention have no change in their consumer surplus. In the third row, we compute a measure of the total change in consumer welfare, in thousand U.S. dollars, taking the median compensating variation across the sample and multiply by the total number of students enrolled in the regions from our sample in rural Punjab, separately for girls and boys. The last row shows how total school enrollment changes (in percentage points) when the “no private schools” policy is implemented, separately for boys and girls. 1 U.S. dollars \approx 85.6 Pakistani Rupees.

One could consider an alternative and less extreme way to restrict access to choice, where instead of forcing the closure of all private schools, we close all but one private school in each village. The private school that is allowed to remain open in this simulation has the average characteristics of all the private schools in the village, and is located at the mean distance of private schools to the village. The amounts required to compensate families for such a change relatively to the status quo (where public and private schools coexist), are much smaller (only about 25% as high) than those reported in the first row of Table 12 (see Table A.18 in the Appendix). This suggests that much of the value of private schools comes from the fact that they make it possible to opt-out from the available public schools. The availability of

variety given by the existence of multiple private schools (as opposed to only one) is valuable, but relatively less important, suggesting a limited role for product differentiation within the market for private schools.

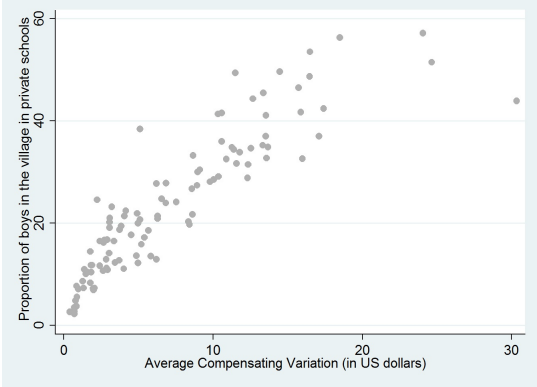
Figure 4 plots the average CV estimates per village against the proportion of female students in the village in private schools. Figure 5 is similar, for male students. Not surprisingly, the correlation between these two variables is very strong for both boys and girls, showing that private school enrollment is high in villages where the valuation of the private school market is also high. The cross-village variation in this valuation is again striking. Our estimates of CV_i for the average student in a village ranges from \$0 to \$30 in the case of boys (with a mean of \$7 and a standard deviation of \$6), and from \$0 to \$21 in the case of girls (with a mean of \$2.5 and a standard deviation of \$2.6).

Figure 4: Compensating variation and proportion of girls in the village in private schools



Notes: This figure represents the average compensating variation per village and the proportion of girls in the village in a private school.

Figure 5: Compensating variation and proportion of boys in the village in private schools



Notes: This figure represents the average compensating variation per village and the proportion of girls in the village in a private school.

In the third row of Table 12 we multiply the numbers in the first row by the total number of students enrolled in the regions of our sample.²⁴ This gives us a measure of the annual welfare benefits of having private schools in these villages, relatively to having no private school, separately for girls and boys. The total value of private schools for parents of children in the regions we are considering in Rural Punjab is \$271000 per year. If we take the whole country, assuming similar valuations in other regions including urban centers (a strong assumption) the value of private schools rises to about \$114 million per year.

The fourth row of Table 12 shows how total school enrollment changes when the ‘no private school’ policy is implemented. Even though girls value private schools much less than boys, the declines in overall school enrollment that we observe as a result of the policy are about 6 percentage points for both gender groups. This is a relatively more important decline for girls, who start from a baseline enrollment rate of about 67%, than for boys, who have an average enrollment rate of 80% in our sample. This means that the differential private school valuation across gender groups does not come from the fact that individuals are less likely to attend any school when private schools cease to exist, but from the fact that they have to switch from a private to a public school which is less desirable.

Table 13 considers a second policy, where school fees are equalized to zero across all schools.²⁵ One way to implement such a policy would be to offer each student a school voucher equal to the fees charged in each private school, which would be \$13 per student if every potential student decided to enrol in private school as a result. Table 13 shows, when we look at the entire population of children in our sample, the median value of such a voucher would be close to \$2 for both girls and boys. Once again, in the second row of this table we multiply these figures by the total number of boys and girls in the region we are considering.²⁶

Note that in our classical welfare analysis, the “value” of a price subsidy to households must be (weakly) lower than its price: Absent any market failures, those who value the product at more than its price are already purchasers. Therefore, it is not a surprise that the median value of the voucher is low. What is of interest is the impact of such a voucher are on total, public and private school enrollment. First, our education market contains both public and private schools and previous research suggests that the per-child cost of private schooling is much lower than that of public schooling. Therefore, if a large number of children move out of public to private schools, the voucher could still be a cost effective policy even absent any market failures (Muralidharan and Sundararaman (2015) and Andrabi et al. (2015a)). Second, it could be that market frictions such as credit constraints or imperfect information lead to erroneous computations of the valuation of private schools and policy makers may therefore be interested in boosting private school enrollment regardless of the household valuation that we can compute in our frictionless model. In that case, our estimates of the deadweight loss show

²⁴This is assuming that the median CV numbers reported above are similar to the mean CV number we would have obtained if we could perfectly correct for outlier CV values that are caused by model misspecification.

²⁵In this simulation, we reduce school fees but retain additional money that parents pay towards textbooks, uniforms and school supplies; in our data these costs can be substantial, reaching around \$12 a year, which is very similar to the cost of private school tuition.

²⁶In Appendix A table A.19 we show that both the value of private schools and the value of school vouchers are considerably higher for children with more educated mothers, and coming from richer families. This is especially true for boys.

how large the shadow value of the market frictions must be for the vouchers to be cost-effective.

Table 13: Voucher program - policy where school fees are equalized to zero

	Girls	Boys
Median compensating variation (in U.S. dollars)	-2.1	-2.0
Total change in consumer welfare (in thousand U.S. dollars)	-76.2	-75.9
Changes in total school enrollment rate (in percentage points)	4.3	1.2
Changes in private school enrollment rate (in percentage points)	7.5	4.2
Changes in public school enrollment rate (in percentage points)	-3.2	-3.0

Notes: In this table we present changes in welfare, and changes in total school enrollment from the introduction of vouchers.

We use compensating variation (CV) to measure the changes in a household’s income that equates utility across two states: a benchmark state, which is the status quo, and the alternative state, which is the environment where school fees are equalized to zero across all schools.

The first row presents estimates of the median compensating variation (in U.S. dollars) for a policy where school fees are equalized to zero across all schools, separately for boys and girls. In the second row, we compute a measure of the total change in consumer welfare, in thousand U.S. dollars, taking the median compensating variation across the sample and multiply by the total number of students enrolled in the regions from our sample in rural Punjab, separately for girls and boys. The last three rows show how total, private, and public school enrollment changes (in percentage points) when the “voucher program” policy is implemented, separately for boys and girls.

1 U.S. dollars \approx 85.6 Pakistani Rupees.

The impacts of the private voucher are not insubstantial. Table 13 shows that total school enrollment increases by 4.3 percentage points for girls and 1.2 percentage points for boys. Private school enrollment rises by 7.5 percentage points for girls (from about 19% to 26%) and 4.2 percentage points for boys (from 23% to 27%).²⁷ Public school enrollments decline by 3.2 and 3.0 percentage points for girls and boys. This means that the cost of the voucher per student is about \$4 for both girls and boys ($= \$13 * 27\%$).²⁸ Further, regarding the cost savings to the government from those who move from public to private schools, Andrabi et al. (2015a) estimate that the cost per student in public schools is around \$26. For girls, the 3% who move from public to private schools will save the government about \$0.83, and for boys the saving is \$0.78. This reduces the deadweight loss, and in fact, it is likely that the shadow value of frictions like credit constraints is higher than the remaining amount. Nevertheless, we also emphasize that the increase in private schooling is not high enough to presume that school fees are the only constraint on higher attendance.

Our approach could be rightly criticized both on the assumption that the voucher is made

²⁷Table 2 shows that 66.8% of all girls are enrolled in a school, and 28.0% of these are in a private school. It is therefore easy to see that the proportion of girls attending a private school is about 19% ($\approx 66.8% * 28.0%$). An analogous calculation can be done for boys.

²⁸Even if we use the median school fee to compute the costs of the policy, which is equal to \$11, we get a total cost per student of around \$3.

uniformly available to all children and villages, and there is no new entry of schools. Incorporating private school entry or changing the targeting design of the voucher would yield different impacts. Nevertheless, the simulation clarifies that there are key differences between providing a voucher to identify test-score differences between public and private schools, and analyzing the welfare consequences of expanding a voucher to an entire schooling system. The specific design and targeting of the voucher will matter. and our simulation provides one methodology that could be used to assess the ex-ante impacts of such policies.

6 Conclusion

In recent years low cost private schools have emerged to expand school choice in very poor areas. There are striking examples from India, Pakistan, Kenya, Nigeria, and Ghana, both in rural and urban settings, where more than half of total school enrollment is taken up by these private institutions. Even though these are all environments where parents are, on average, poor and relatively less educated, they make active decisions about school choice, often opting out of the free public school system. In order to understand the importance of private school markets for education in poor countries, we need to understand the parameters driving demand and supply of private schooling in such settings. This is a central issue in the economics of education, where the roles of choice and competition in the provision of education are increasingly discussed in the context of richer countries (for example Burgess et al. (2015), Bayer et al. (2007), Hastings et al. (2009), and Checchi and Jappelli (2004)). However, even in low-income countries where many excellent researchers have devoted substantial attention to the topic, it remains surrounded with controversy.

In this paper we explored school choice in Punjab, the largest province in Pakistan. In our sample parents can choose among schools (public and private) in a very competitive setting and extensive data on households and schools allows for a rich analysis of the school choice decision. The school level detail in our data allows us to use a long list of school attributes potentially valued by parents. The cost level data provides potentially exogenous sources of variation in tuition fees at the school level, especially after we control for school attributes and village fixed effects. Together these two features of the data permit credible estimation of the parental willingness to pay for a large set of school attributes. The detailed household level data allows us to examine how this willingness to pay for different school attributes varies with household characteristics.

Our demand estimates and policy simulations highlight why such exercises are critical for policy. The high distance elasticities combined with the low price elasticities suggest that parents attach a very high value to private schooling, but not to the product differentiation that occurs when there are multiple private schools in the same village. Further, a voucher program in this setting has some effect on private and public enrollments, but not as large as is usually imagined. These exercises are especially relevant in settings where private schools are abundant, as is increasingly common in a variety of low and middle-income countries. These are also exercises that concern fundamental issues in the economics of school choice that have,

for the most part, not been addressed elsewhere in the literature.

We are also aware of the limitations to this approach. For instance, were we to fully model changes in the schooling system from a counterfactual policy, we would also have to model supply side responses. But to do so, we need to first understand more fundamentally what private schools are maximizing. While clearly they are subject to some market discipline—in that they have to shut down if they cannot cover costs—their pricing decisions may reflect multiple objectives in addition to maximizing profits. As one example, we find that schools price in the inelastic portion of the demand curve with markups below those that would be profit maximizing. These pricing decisions could reflect many different considerations ranging from social concerns to dynamic pricing. Understanding why this is so remains at the frontier of this research, and perhaps is best accomplished with the help actual policy experiments in this context.

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Online appendices to “The Value of Private Schools: Evidence from Pakistan”

A Appendix tables

In this section, we provide additional tables for more details. The household and school variables used to estimate the model are described in Table A.1.

We estimate equation (5) using maximum likelihood, with an additional step to estimate the school fixed effect. The first step estimated coefficients are shown in tables A.2, A.3 and A.4.

The coefficients in equation (6) can be estimated using instrumental variables, although we also present OLS estimates for comparison. The results for the first stage regressions are displayed in tables A.5, A.6, and A.7. Table A.5 reports the specification using only total costs, and using both costs and the BLP instruments. Table A.6 shows what happens when we use individual cost components separately. Table A.7 shows the first stage regressions of peer variables (student test score, mother education, and assets on the predicted value of the peer variables in the school and on the predicted value of the peer variable in competitor schools, weighted by distance to each competitor and other school attributes).

Table A.8 examines the correlation between parental preferences for different school attributes. Table A.9 compares the correlations between the same list of attributes offered by schools. Table A.10 reports the price elasticity from several available studies in the literature.

In Table A.11 for girls and A.12 for boys, we provide estimation of equation (6) including a private school indicator as an attribute of the school. Tables A.13 and A.14 show our estimates when the model includes school size as an attribute. Tables A.15 and A.16 show estimates of equation (6) in a specification where there are no peer variables and where peer variables are taken as exogenous.

Table A.17 present the welfare impacts of the different policies using the average of the compensating variation after trimming the bottom and top 1% of the distribution of this variable. Table A.18 presents changes in welfare from an alternative and less extreme way to restrict access to choice, where we close all but one private school in each village. Finally, table A.19 shows the changes in welfare of the different policies by household type (mother education, income, and household distance to facilities).

Table A.1: Variables definition

Variables	Description
School Variables	
School fees	Tuition annual fees
School with toilets	Dichotomous variable indicating whether schools have toilets
School with permanent classroom	Dichotomous variable indicating whether schools have permanent classroom
Number of extra facilities	Number of extra facilities provided by the school
Student test score (average)	Student test score - Average of Math, Urdu and English
Percentage of female teachers	Percentage of female teachers
Percentage of teachers with 3 years of experience	Percentage of teachers with at least 3 years of experience
Percentage of teachers with university degree	Percentage of teachers with a university degree
Teacher absenteeism	Number of days teacher were absent in a month
Teacher test score (average)	Teacher test score - Average of Math, Urdu and English
Percentage of Mother with some education (school level)	Percentage of mothers with at least 1 year of education
Asset index (school level)	Average of the asset index at school level
Total costs without rent	Monthly expenditure on Utilities, Pay and Allowance of Teaching and Non-Teaching staff, purchase of educational material such as textbooks and other current disbursements.
BLP instruments	Average school characteristics of competitors except for school fee (School with toilets, School with permanent classroom, School with board, number of extra facilities, Student test score, Percentage of female teachers, Percentage of teachers with 3 years of experience, Percentage of teachers with university degree, Teacher absenteeism, Teacher test score, Percentage of Mother with some education and Asset index.
Individual/Household variables	
Distance	Reports the distance in Kms from the house to any school available in the village
Age	Reports the children's age in years
Mother Education	Reports the students' mother education in years
Income	Total monthly expenditure divided by household size
Household distance to facilities	Reports the average distance in Kms. from the house to the main facilities in the village

Table A.2: Estimates of interaction terms - observables

Individual/household characteristic	School Characteristic	Girls	Boys
Age	school fees	-0.005 (0.003)*	-0.003 (0.002)
	number of extra facilities	0.012 (0.009)***	0.025 (0.008)***
	Percentage of female teachers	0.040 (0.053)	-0.076 (0.041)*
	Percentage of teachers with 3 years of experience	0.001 (0.041)	0.025 (0.047)
	Percentage of teachers with university degree	0.057 (0.046)	0.080 (0.045)*
	Student test score (average)	-0.081 (0.110)	-0.070 (0.114)
	Teacher absenteeism	-0.002 (0.003)	0.003 (0.004)
	Teacher test score	0.005 (0.134)	-0.047 (0.128)
	Mother education (school level)	0.049 (0.057)	-0.088 (0.058)*
	Asset index (school level)	0.011 (0.015)	0.014 (0.014)
	outside option - not enrolled	0.253 (0.139)*	0.155 (0.129)
	School with toilets	0.006 (0.037)	-0.003 (0.027)
	School with permanent classroom	0.056 (0.039)	0.060 (0.039)*
	distance	0.024 (0.015)*	0.014 (0.012)
	mother education	school fees	0.011 (0.003)***
number of extra facilities		0.007 (0.010)	0.009 (0.009)
Percentage of female teachers		0.061 (0.069)	0.093 (0.043)**
Percentage of teachers with 3 years of experience		0.057 (0.047)	0.080 (0.053)*
Percentage of teachers with university degree		0.036 (0.054)	-0.049 (0.053)
Student test score (average)		-0.075 (0.123)	0.095 (0.131)
Teacher absenteeism		0.000 (0.004)	0.001 (0.006)
Teacher test score		-0.043 (0.165)	-0.061 (0.124)
Mother education (school level)		0.178 (0.065)***	-0.003 (0.059)
Asset index (school level)		0.009 (0.018)	0.035 (0.016)***
outside option - not enrolled		-0.130 (0.181)	0.035 (0.132)
School with toilets		-0.018 (0.045)	0.084 (0.033)***
School with permanent classroom		-0.029 (0.041)	0.058 (0.046)
distance		0.004 (0.018)	-0.001 (0.015)

Notes: This table reports estimates of the interaction terms (β_{rkg}^o , and γ_{rg}) for students' age and mother education in equation (5) for both girls and boys. This step entails estimating $\delta_{jtg}, \beta_{rkg}^o, \beta_{kg}^u, \bar{\gamma}_g, \gamma_{rg}, \gamma_g^u$ by maximum likelihood, including a contraction mapping to obtain δ_{jtg} .

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.3: Estimates of interaction terms - observables

Individual/household characteristic	School Characteristic	Girls	Boys	
log of income	school fees	0.033 (0.014)***	0.003 (0.013)	
	number of extra facilities	-0.020 (0.049)	0.011 (0.038)	
	Percentage of female teachers	0.020 (0.276)	0.149 (0.193)	
	Percentage of teachers with 3 years of experience	-0.049 (0.210)	-0.617 (0.239)***	
	Percentage of teachers with university degree	-0.071 (0.231)	-0.101 (0.213)	
	Student test score (average)	-0.625 (0.596)	-0.513 (0.550)	
	Teacher absenteeism	-0.012 (0.015)	0.012 (0.024)	
	Teacher test score	-0.381 (0.819)	0.791 (0.523)*	
	Mother education (school level)	-0.080 (0.264)	-0.058 (0.274)	
	Asset index (school level)	0.021 (0.075)	-0.016 (0.066)	
	outside option - not enrolled	-1.045 (0.763)	-0.389 (0.554)	
	School with toilets	0.246 (0.187)	-0.284 (0.131)**	
	School with permanent classroom	-0.259 (0.195)	-0.053 (0.178)	
	distance	0.178 (0.072)***	0.057 (0.050)	
	household distance to facilities	school fees	0.008 (0.022)	0.002 (0.015)
		number of extra facilities	0.102 (0.085)	-0.026 (0.048)
Percentage of female teachers		0.058 (0.476)	0.082 (0.245)	
Percentage of teachers with 3 years of experience		0.429 (0.376)	0.149 (0.325)	
Percentage of teachers with university degree		0.194 (0.380)	-0.180 (0.266)	
Student test score (average)		0.695 (1.032)	-0.130 (0.810)	
Teacher absenteeism		0.016 (0.025)	0.016 (0.026)	
Teacher test score		-0.542 (1.446)	-0.019 (0.858)	
Mother education (school level)		0.186 (0.471)	-0.068 (0.369)	
Asset index (school level)		-0.019 (0.122)	0.012 (0.084)	
outside option - not enrolled		0.346 (1.427)	-0.060 (0.844)	
School with toilets		-0.047 (0.305)	-0.073 (0.172)	
School with permanent classroom		-0.267 (0.335)	0.045 (0.225)	
distance		0.048 (0.044)	0.040 (0.028)	

Notes: This table reports estimates of the interaction terms (β_{rkg}^o , and γ_{rg}) for log of income and household distance to facilities in equation (5) for both girls and boys. The first step entails estimating δ_{jtg} , β_{rkg}^o , β_{kg}^u , $\bar{\gamma}_g$, γ_{rg} , γ_g^u by maximum likelihood, including a contraction mapping to obtain δ_{jtg} . Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.4: Estimates of interaction terms - unobservables

School Characteristics	Girls	Boys
school fees	-0.0001 (0.0521)	0.0006 (0.0498)
number of extra facilities	-0.0004 (0.051)	0.0006 (0.039)
Percentage of female teachers	-0.0005 (0.239)	0.0002 (0.278)
Percentage of teachers with 3 years of experience	0.0002 (0.208)	0.0004 (0.286)
Percentage of teachers with university degree	0.0003 (0.234)	-0.0002 (0.225)
Student test score (average)	0.00003 (0.493)	-0.00003 (0.496)
Teacher absenteeism	0.0004 (0.010)	-0.0001 (0.022)
Teacher test score	-0.0003 (0.339)	0.0001 (0.327)
Mother education (school level)	0.0004 (0.340)	-0.0002 (0.264)
Asset index (school level)	0.00002 (0.075)	0.00001 (0.055)
outside option - not enrolled	-0.0002 (0.002)	-0.0001 (0.031)
School with toilets	-0.0001 (0.181)	-0.0001 (0.130)
School with permanent classroom	-0.0002 (0.158)	0.0003 (0.159)
distance	-0.0005 (0.049)	-0.001 (0.024)

Notes: This table reports estimates of the interaction terms for the individual unobservable characteristics in equation (5) for both girls and boys ($\beta_{rk_g}^u$, and γ_g^u). The first step entails estimating $\delta_{jt_g}, \beta_{rk_g}^o, \beta_{k_g}^u, \bar{\gamma}_g, \gamma_{r_g}, \gamma_g^u$ by maximum likelihood, including a contraction mapping to obtain δ_{jt_g} .

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.5: First stage - school fee equation - total cost and BLP instruments

	(1)	(2)
School with toilets	194.0 [259.7]	-37.6 [550.6]
School with permanent classroom	174.0 [147.2]	495 [323.6]
Number of extra facilities	4.6 [43.7]	-19.5 [90.1]
Percentage of female teachers	4.3 [205.6]	-139.1 [469.0]
Percentage of teachers with 3 years of experience	-24.6 [186.1]	-311.4 [468.2]
Percentage of teachers with university degree	366.3 [298.2]	960.9 [670.1]
Teacher absenteeism	-9.9 [23.1]	-25.5 [53.1]
Teacher test score (average)	929.1 [555.3]*	1,360.5 [961.1]
total cost normalised by student without rent	542.4 [139.6]***	509.8 [143.4]***
total cost normalised by student without rent (squared)	-32.8 [8.6]***	-31.0 [8.8]***
School with toilets (BLP)		-1,450.5 [3,371.7]
School with permanent classroom (BLP)		2,145.4 [2,006.7]
number of extra facilities (BLP)		-180.9 [643.4]
Percentage of female teachers (BLP)		-1,127.5 [3,693.5]
Percentage of teachers with 3 years of experience (BLP)		-2,069.5 [2,928.7]
Percentage of teachers with university degree (BLP)		4,145.2 [4,553.8]
Teacher absenteeism (BLP)		-162.7 [444.6]
Teacher test score (average) (BLP)		1,646.0 [4,077.4]
Constant	-932.4 [843.4]	3,057.8 [10,264.9]
Observations	280	280
R-squared	0.67	0.68
Other controls	Village FE	Village FE
F-Test (instrument)	7.62	1.92
p-value	0.0007	0.0468

Notes: This table reports estimates of the first stage regression of school fees on school costs, BLP instruments and other school attributes. We can only run this first stage regression for private schools, since public schools are free, and therefore do not contribute any information. The first column we use total costs without rent, and the second has the main specification using total costs without rent and the BLP instruments.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.6: First stage - school fee equation - individual cost components

	(1)	(2)	(3)	(4)	(5)
School with toilets	285.7 [291.3]	213.6 [264.6]	206.9 [296.9]	177.3 [276.4]	195.4 [298.6]
School with permanent classroom	176.1 [153.9]	183.4 [149.8]	141.4 [161.7]	198.6 [154.5]	213.3 [149.0]
number of extra facilities	47.3 [44.2]	14.3 [44.5]	27.4 [47.3]	58.8 [44.5]	103.5 [42.2]**
Percentage of female teachers	25.9 [214.3]	-23.1 [209.8]	26.3 [218.8]	36.3 [216.9]	56 [200.9]
Percentage of teachers with 3 years of experience	-135.5 [195.2]	-60.4 [188.8]	-149.1 [193.8]	-154 [200.8]	-265 [182.3]
Percentage of teachers with university degree	891.6 [286.3]***	497 [300.8]	756.7 [295.2]**	770.9 [287.0]***	665.2 [271.4]**
Teacher absenteeism	-7.8 [24.0]	-8.2 [23.5]	-10.6 [24.4]	-9.7 [24.3]	1.2 [21.8]
Teacher test score (average)	901.80 [595.1]	927.5 [566.0]	827.6 [590.2]	625.9 [596.0]	592 [615.0]
Cost with utilities	1,497.6 [1,133.7]				
Cost with utilities (squared)	-2,276.5 [1,479.0]				
Cost with teacher staff		528.8 [178.0]***			
Cost with teacher staff (squared)		-33.0 [11.0]***			
Cost with non-teacher staff			2,450.6 [1,830.6]		
Cost with non-teacher staff (squared)			-3,494.8 [4,137.9]		
Cost with Educational Material				3,952.3 [5,540.1]	
Cost with Educational Material (squared)				-10,270.9 [44,280.7]	
Cost with other costs					-1,116.3 [533.3]**
Cost with other costs (squared)					1,141.4 [279.7]***
Other Controls	Village FE	Village FE	Village FE	Village FE	Village FE
F-test (instrument)	1.20	4.55	1.26	1.02	15.21
p-value	0.305	0.012	0.286	0.364	0.000

Notes: This table reports estimates of the first stage regression of school fees on individual cost components separately and other school attributes. We can only run this first stage regression for private schools, since public schools are free, and therefore do not contribute any information. Columns (1) to (5) report the specifications using cost with utilities, teacher staff, non-teacher staff, educational material and other costs, respectively.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.7: First stage - peer variables equations

	GIRLS				BOYS				
	Student Test	Mother Education	Asset Index	Student Test	Mother Education	Asset Index	Student Test	Mother Education	Asset Index
School with toilets	0.008 [0.018]	0.068 [0.040]*	0.05 [0.145]	0.03 [0.014]**	-0.01 [0.029]	-0.13 [0.112]			
School with permanent classroom	-0.006 [0.017]	0.003 [0.038]	-0.009 [0.137]	-0.014 [0.016]	-0.03 [0.034]	-0.166 [0.131]			
number of extra facilities	0.019 [0.004]***	0.036 [0.009]***	0.161 [0.034]***	0.016 [0.004]***	0.031 [0.008]***	0.169 [0.032]***			
Perc. of female teachers	-0.033 [0.020]*	0.072 [0.043]*	0.412 [0.155]***	0.056 [0.017]***	0.134 [0.035]***	0.637 [0.138]***			
Perc. of teachers with 3 years of exp.	-0.099 [0.017]***	-0.083 [0.038]**	-0.646 [0.135]***	-0.063 [0.019]***	-0.023 [0.040]	-0.462 [0.155]***			
Perc. of teachers with univ. degree	-0.016 [0.024]	-0.037 [0.051]	-0.176 [0.183]	-0.006 [0.021]	0.021 [0.044]	0.037 [0.171]			
Teacher absenteeism	0.001 [0.002]	-0.001 [0.003]	0.006 [0.012]	-0.001 [0.002]	0.003 [0.004]	0.007 [0.015]			
Teacher test score (average)	0.063 [0.073]	-0.237 [0.159]	0.004 [0.574]	0.059 [0.061]	-0.27 [0.125]**	0.221 [0.486]			
Predicted Value Student Test	0.158 [0.199]			0.84 [0.197]***					
Predicted Value St-Test competitors weighted by distance	-0.336 [0.242]			-0.468 [0.279]*					
Predicted Value Mother Education		0.3 [0.103]***			0.151 [0.096]				
Predicted Value Mother Educ competitors weighted by distance		-0.062 [0.207]			-0.142 [0.188]				
Predicted Value Asset Index			0.211 [0.096]**			0.095 [0.123]			
Predicted Value Asset Index competitors weighted by distance			-0.045 [0.168]			-0.447 [0.261]*			
Constant	0.544 [0.219]**	0.082 [0.331]	-0.725 [0.925]	0.087 [0.159]	0.096 [0.195]	-0.043 [0.889]			
other controls	village FE	village FE	village FE	village FE	village FE	village FE			
F-test (instruments)	1.17	4.72	2.45	9.95	1.84	2.57			
p-value	0.31	0.009	0.088	0.000	0.161	0.078			

Notes: This table reports estimates of the first stage regression of peer variables (student test score, mother education and assets on the predicted value of the peer variable in the school and on the predicted value of the peer variable in competitor schools, weighted by distance to each competitor and other school attributes. Columns (1) to (3) report the results for Girls and columns (4) to (6) the results for boys. Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.8: Correlation of the individual preferences regarding school characteristics

GIRLS							
	School Fee	Distance	Teachers with experience	Female Teachers	Number of extra facilities	Teachers with Univ. Degree	Having a Perm. Classroom
School Fee	1						
Distance	0.5508	1					
Teachers with Experience	0.4638	0.6646	1				
Female Teachers	0.5913	0.7488	0.69	1			
Number of Extra Facilities	0.3638	0.6868	0.9898	0.6995	1		
Teachers with University Degree	0.3412	0.7179	0.9562	0.7978	0.9815	1	
Having a Permanent Classroom	-0.6504	-0.7029	-0.9477	-0.6059	-0.9045	-0.8352	1

BOYS							
	School Fee	Distance	Teachers with experience	Female Teachers	Number of extra facilities	Teachers with Univ. Degree	Having a Perm. Classroom
School Fee	1						
Distance	0.0940	1					
Teachers with Experience	0.1752	0.4344	1				
Female Teachers	0.9529	0.3271	0.2344	1			
Number of Extra Facilities	0.3096	-0.3486	-0.4118	-0.5202	1		
Teachers with University Degree	-0.6601	-0.6538	-0.5147	-0.8432	0.8045	1	
Having a Permanent Classroom	0.2210	0.6130	0.6101	0.2138	0.2375	-0.2680	1

Notes: This table reports the correlation between individual preferences for different school attributes for both girls and boys. Recall that two of the attributes in this table have negative coefficients in parental preferences: school fees and distance. We do not show all attributes in this table, but only the ones for which the coefficients were statistically significant in equation (6).

Table A.9: Correlation of the attributes offered by schools

GIRLS							
All Schools							
	School Fee	Distance	Teachers with experience	Female Teachers	Number of extra facilities	Teachers with Univ. Degree	Having a Perm. Classroom
School Fee	1.00						
Distance	-0.14	1.00					
Teachers with experience	-0.49	0.18	1.00				
Female Teachers	-0.17	-0.15	0.10	1.00			
Number of extra facilities	0.49	-0.26	-0.32	-0.02	1.00		
Teachers with University Degree	-0.03	0.04	-0.02	-0.01	-0.01	1.00	
Having a Permanent Classroom	0.02	0.06	0.05	0.08	0.12	0.13	1.00
Private schools							
	School Fee	Distance	Teachers with experience	Female Teachers	Number of extra facilities	Teachers with Univ. Degree	Having a Perm. Classroom
School Fee	1.00						
Distance	0.10	1.00					
Teachers with experience	-0.06	0.03	1.00				
Female Teachers	-0.17	-0.31	-0.08	1.00			
Number of extra facilities	0.31	-0.04	0.04	-0.01	1.00		
Teachers with University Degree	0.30	0.05	0.01	-0.14	0.23	1.00	
Having a Permanent Classroom	0.14	0.09	0.01	0.04	0.21	0.09	1.00
BOYS							
All Schools							
	School Fee	Distance	Teachers with experience	Female Teachers	Number of extra facilities	Teachers with Univ. Degree	Having a Perm. Classroom
School Fee	1.00						
Distance	-0.16	1.00					
Teachers with experience	-0.49	0.18	1.00				
Female Teachers	0.47	-0.25	-0.54	1.00			
Number of extra facilities	0.53	-0.22	-0.35	0.42	1.00		
Teachers with University Degree	-0.16	0.13	0.13	-0.33	-0.12	1.00	
Having a Permanent Classroom	0.05	0.02	0.01	-0.01	0.14	0.16	1.00
Private schools							
	School Fee	Distance	Teachers with experience	Female Teachers	Number of extra facilities	Teachers with Univ. Degree	Having a Perm. Classroom
School Fee	1.00						
Distance	0.13	1.00					
Teachers with experience	-0.07	0.02	1.00				
Female Teachers	-0.17	-0.32	-0.07	1.00			
Number of extra facilities	0.30	-0.03	0.04	-0.01	1.00		
Teachers with University Degree	0.30	0.05	0.00	-0.14	0.22	1.00	
Having a Permanent Classroom	0.14	0.08	0.02	0.04	0.21	0.09	1.00

Notes: This table reports the correlation between the list of attributes offered by schools (all and private) for both girls and boys. We do not show all attributes in this table, but only the ones for which the coefficients were statistically significant in equation (6).

Table A.10: School fee elasticity

	Fee Elasticity	Country
Alderman et al. (2001)	-0.2	Pakistan
Dynarski et al. (2009)	-0.2	U.S.
Within “BLP” framework *		
Bau (2015)	-0.6	Pakistan
Gallego and Hernando (2009)	-0.8	Chile
Illustrative case from an Experiment where only students from public schools are affected **		
Muralidharan and Sundararaman (2015)	-1.4	India

Notes:

* Based on the coefficients of the BLP model using the formula:

$elasticity = \alpha * price * (1 - share)$.

** Illustrative calculation using $\frac{\Delta Share}{\Delta Price} \frac{Price}{Share} = \frac{0.15}{0*0.7+0.3*Price_0} \frac{Price_0}{0.35} = \frac{0.15}{0.3} \frac{1}{0.35}$. In this paper only students from public schools were affected by the policy intervention.

Table A.11: Robustness check - private school as a school characteristic - girls

	25th perc			mean			75th perc		
	IV	IV	IV	IV	IV	IV	IV	IV	IV
School fees	-0.092 [0.013]***	-0.048 [0.015]***	-0.066 [0.013]***	-0.023 [0.015]	-0.051 [0.014]***	-0.008 [0.016]			
Private		-1.027 [0.235]***		-0.992 [0.238]***		-0.987 [0.244]***			
School with toilets	0.194 [0.240]	0.26 [0.199]	0.24 [0.260]	0.304 [0.202]	0.303 [0.269]	0.367 [0.207]*			
School with permanent classroom	0.325 [0.201]	0.185 [0.167]	0.202 [0.205]	0.067 [0.169]	0.112 [0.214]	-0.023 [0.173]			
Number of extra facilities	0.321 [0.183]*	0.367 [0.151]**	0.255 [0.183]	0.3 [0.153]*	0.213 [0.188]	0.257 [0.157]			
Percentage of female teachers	1.455 [0.428]***	1.391 [0.351]***	1.594 [0.426]***	1.533 [0.355]***	1.675 [0.453]***	1.614 [0.364]***			
Percentage of teachers with 3 years of experience	0.895 [0.842]	0.367 [0.701]	1.237 [0.808]	0.727 [0.709]	1.423 [0.809]*	0.915 [0.727]			
Percentage of teachers with university degree	0.387 [0.296]	-0.002 [0.271]	0.486 [0.301]	0.111 [0.274]	0.524 [0.309]*	0.15 [0.281]			
Student test score (average)	7.77 [7.857]	7.026 [5.879]	9.87 [7.154]	9.152 [5.952]	11.371 [7.327]	10.657 [6.102]*			
Teacher absenteeism	0.016 [0.015]	0.007 [0.015]	0.008 [0.016]	0.000 [0.015]	0.003 [0.016]	-0.005 [0.016]			
Teacher test score (average)	-0.555 [0.909]	-0.756 [0.829]	-0.84 [0.902]	-1.033 [0.839]	-1.048 [0.923]	-1.24 [0.860]			
Perc. of Mother with some education (school level)	-3.188 [1.572]**	-3.158 [1.415]**	-2.809 [1.656]*	-2.78 [1.432]*	-2.676 [1.683]*	-2.648 [1.468]*			
Asset index (school level)	-0.974 [0.626]	-1.009 [0.530]*	-0.888 [0.671]	-0.922 [0.536]*	-0.859 [0.696]	-0.893 [0.550]			

Notes: This table shows estimates of equation (6) in a specification where an indicator for whether a school is private is included in the model as another attribute. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where m = 25th percentile, mean, 75th percentile. We label these: 25th, Mean, and 75th, respectively. Columns 1, 3, and 5 show the impact of our preferred specification. Remaining columns (2, 4, and 6) report the results where an indicator for whether a school is private is included in the model as another attribute. Standard errors in brackets.
* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.12: Robustness check - private school as a school characteristic - boys

	25th perc			mean			75th perc		
	IV	IV	IV	IV	IV	IV	IV	IV	IV
School fees	-0.040 [0.011]***	-0.014 [0.016]	-0.031 [0.011]***	-0.006 [0.015]	-0.028 [0.011]***	-0.002 [0.015]			
Private		-0.691 [0.278]**		-0.672 [0.275]**		-0.675 [0.276]**			
School with toilets	0.135 [0.188]	0.167 [0.164]	0.118 [0.187]	0.149 [0.162]	0.07 [0.191]	0.101 [0.163]			
School with permanent classroom	0.633 [0.173]***	0.56 [0.171]**	0.676 [0.169]**	0.606 [0.169]**	0.695 [0.167]**	0.624 [0.170]**			
Number of extra facilities	-0.017 [0.122]	0.011 [0.105]	-0.006 [0.121]	0.021 [0.104]	0.003 [0.129]	0.031 [0.104]			
Percentage of female teachers	-1.698 [0.503]***	-1.448 [0.444]**	-1.529 [0.501]**	-1.286 [0.439]**	-1.423 [0.523]**	-1.179 [0.440]**			
Percentage of teachers with 3 years of experience	1.151 [0.349]***	0.979 [0.325]**	0.995 [0.348]**	0.828 [0.321]**	0.832 [0.355]**	0.664 [0.322]**			
Percentage of teachers with university degree	0.818 [0.226]**	0.63 [0.211]**	0.716 [0.224]**	0.533 [0.209]**	0.652 [0.231]**	0.469 [0.209]**			
Student test score (average)	3.506 [2.151]	3.077 [2.157]	3.499 [2.132]*	3.082 [2.133]	3.476 [2.159]	3.057 [2.138]			
Teacher absenteeism	-0.009 [0.019]	-0.015 [0.020]	-0.005 [0.019]	-0.01 [0.019]	0.001 [0.019]	-0.004 [0.019]			
Teacher test score (average)	-0.481 [0.971]	-0.585 [0.984]	-0.132 [0.956]	-0.233 [0.973]	0.124 [0.945]	0.023 [0.975]			
Perc. of Mother with some education (school level)	-1.644 [2.470]	-1.648 [2.539]	-1.296 [2.469]	-1.3 [2.511]	-1.206 [2.461]	-1.21 [2.517]			
Asset index (school level)	0.629 [0.617]	0.641 [0.518]	0.620 [0.619]	0.631 [0.512]	0.618 [0.643]	0.63 [0.514]			

Notes: This table shows estimates of equation (6) in a specification where an indicator for whether a school is private is included in the model as another attribute. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where m = 25th percentile, mean, 75th percentile. We label these: 25th, Mean, and 75th, respectively. Columns 1, 3, and 5 show the impact of our preferred specification. Remaining columns (2, 4, and 6) report the results where an indicator for whether a school is private is included in the model as another attribute. Standard errors in brackets. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.13: Robustness check - school size as a school characteristic - girls

	25th perc			mean			75th perc		
	IV	IV	IV	IV	IV	IV	IV	IV	IV
School fees	-0.092 [0.013]***	-0.077 [0.011]***	-0.066 [0.013]***	-0.051 [0.011]***	-0.051 [0.014]***	-0.036 [0.011]***			
School Size		0.004 [0.000]***		0.004 [0.000]***		0.004 [0.000]***			
School with toilets	0.194 [0.240]	0.16 [0.194]	0.24 [0.260]	0.216 [0.195]	0.303 [0.269]	0.285 [0.200]			
School with permanent classroom	0.325 [0.201]	0.371 [0.156]**	0.202 [0.205]	0.248 [0.157]	0.112 [0.214]	0.158 [0.161]			
Number of extra facilities	0.321 [0.183]*	0.188 [0.146]	0.255 [0.183]	0.126 [0.147]	0.213 [0.188]	0.086 [0.151]			
Percentage of female teachers	1.455 [0.428]***	1.575 [0.351]***	1.594 [0.426]***	1.729 [0.353]***	1.675 [0.453]***	1.819 [0.362]***			
Percentage of teachers with 3 years of experience	0.895 [0.842]	0.778 [0.664]	1.237 [0.808]	1.102 [0.668]	1.423 [0.809]*	1.276 [0.685]*			
Percentage of teachers with university degree	0.387 [0.296]	0.039 [0.249]	0.486 [0.301]	0.127 [0.251]	0.524 [0.309]*	0.157 [0.257]			
Student test score (average)	7.77 [7.857]	10.546 [5.659]*	9.87 [7.154]	12.701 [5.695]**	11.371 [7.327]	14.246 [5.838]**			
Teacher absenteeism	0.016 [0.015]	0.021 [0.015]	0.008 [0.016]	0.014 [0.015]	0.003 [0.016]	0.009 [0.015]			
Teacher test score (average)	-0.555 [0.909]	-1.371 [0.839]	-0.84 [0.902]	-1.705 [0.844]**	-1.048 [0.923]	-1.944 [0.865]**			
Perc. of Mother with some education (school level)	-3.188 [1.572]**	-4.242 [1.374]***	-2.809 [1.656]*	-3.915 [1.382]***	-2.676 [1.683]*	-3.815 [1.417]***			
Asset index (school level)	-0.974 [0.626]	-0.988 [0.522]*	-0.888 [0.671]	-0.931 [0.525]*	-0.859 [0.696]	-0.92 [0.539]*			

Notes: This table shows estimates of equation (6) in a specification where an indicator of school size is included in the model as another attribute. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where m = 25th percentile, mean, 75th percentile. We label these: 25th, Mean, and 75th, respectively. Columns 1, 3, and 5 show the impact of our preferred specification. Remaining columns (2, 4, and 6) report the results where an indicator of school size is included in the model as another attribute.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.14: Robustness check - school size as a school characteristic - boys

	25th perc			mean			75th perc		
	IV	IV	IV	IV	IV	IV	IV	IV	IV
School fees	-0.040 [0.011]***	-0.013 [0.010]	-0.031 [0.011]***	-0.005 [0.010]	-0.028 [0.011]***	-0.002 [0.010]			
School Size		0.004 [0.000]***		0.004 [0.000]***		0.004 [0.000]***			
School with toilets	0.135 [0.188]	0.024 [0.148]	0.118 [0.187]	0.008 [0.146]	0.07 [0.191]	-0.04 [0.146]			
School with permanent classroom	0.633 [0.173]***	0.457 [0.151]***	0.676 [0.169]***	0.501 [0.149]***	0.695 [0.167]***	0.519 [0.150]***			
Number of extra facilities	-0.017 [0.122]	-0.13 [0.095]	-0.006 [0.121]	-0.118 [0.094]	0.003 [0.129]	-0.109 [0.094]			
Percentage of female teachers	-1.698 [0.503]***	-1.277 [0.393]***	-1.529 [0.501]***	-1.106 [0.388]***	-1.423 [0.523]***	-0.999 [0.389]**			
Percentage of teachers with 3 years of experience	1.151 [0.349]***	0.731 [0.288]**	0.995 [0.348]***	0.578 [0.284]**	0.832 [0.355]***	0.414 [0.285]			
Percentage of teachers with university degree	0.818 [0.226]***	0.323 [0.182]*	0.716 [0.224]***	0.222 [0.179]	0.652 [0.231]***	0.158 [0.180]			
Student test score (average)	3.506 [2.151]	1.953 [1.929]	3.499 [2.132]*	1.959 [1.902]	3.476 [2.159]	1.938 [1.907]			
Teacher absenteeism	-0.009 [0.019]	-0.028 [0.018]	-0.005 [0.019]	-0.024 [0.018]	0.001 [0.019]	-0.018 [0.018]			
Teacher test score (average)	-0.481 [0.971]	-1.478 [0.888]*	-0.132 [0.956]	-1.126 [0.876]	0.124 [0.945]	-0.872 [0.878]			
Perc. of Mother with some education (school level)	-1.644 [2.470]	-4.022 [2.277]*	-1.296 [2.469]	-3.673 [2.246]	-1.206 [2.461]	-3.586 [2.252]			
Asset index (school level)	0.629 [0.617]	0.835 [0.467]*	0.620 [0.619]	0.823 [0.460]*	0.618 [0.643]	0.821 [0.462]*			

Notes: This table shows estimates of equation (6) in a specification where an indicator of school size is included in the model as another attribute. We compute the 25th, and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where m = 25th percentile, mean, 75th percentile. We label these: 25th, Mean, and 75th, respectively. Columns 1, 3, and 5 show the impact of our preferred specification. Remaining columns (2, 4, and 6) report the results where an indicator of school size is included in the model as another attribute.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.15: Robustness check - specifications without peer variables and without endogeneity of peer variables - girls

	25th perc.		mean		75th perc.	
	No endogeneity	No peers	No endogeneity	No peers	No endogeneity	No peers
School fees	-0.095 [0.012]***	-0.097 [0.011]***	-0.069 [0.012]***	-0.069 [0.011]***	-0.053 [0.013]***	-0.054 [0.012]***
School with toilets	-0.006 [0.168]	-0.042 [0.170]	0.077 [0.171]	0.056 [0.171]	0.16 [0.176]	0.143 [0.175]
School with permanent classroom	0.346 [0.163]**	0.337 [0.165]**	0.203 [0.166]	0.198 [0.166]	0.102 [0.170]	0.098 [0.170]
Number of extra facilities	0.22 [0.044]***	0.197 [0.044]***	0.215 [0.045]***	0.2 [0.045]***	0.208 [0.047]***	0.197 [0.046]***
Percentage of female teachers	0.664 [0.186]***	0.559 [0.186]***	0.763 [0.190]***	0.694 [0.187]***	0.801 [0.195]***	0.748 [0.192]***
Percentage of teachers with 3 years of experience	1.00 [0.187]***	1.052 [0.187]***	1.065 [0.190]***	1.098 [0.189]***	1.079 [0.195]***	1.105 [0.194]***
Percentage of teachers with university degree	0.541 [0.218]**	0.555 [0.221]**	0.581 [0.223]***	0.59 [0.223]***	0.586 [0.229]**	0.593 [0.228]***
Student test score (average)	0.645 [0.506]		0.477 [0.516]		0.346 [0.530]	
Teacher absenteeism	0.025 [0.015]*	0.026 [0.015]*	0.019 [0.015]	0.02 [0.015]	0.014 [0.015]	0.015 [0.015]
Teacher test score (average)	0.473 [0.663]	0.665 [0.669]	0.305 [0.676]	0.424 [0.675]	0.188 [0.694]	0.278 [0.691]
Perc. of Mother with some education (school level)	-0.623 [0.222]***		-0.368 [0.226]		-0.283 [0.232]	
Asset index (school level)	-0.093 [0.062]		-0.067 [0.063]		-0.052 [0.065]	

Notes: This table shows estimates of equation (6) in a specification where there are no peer variables and where peer variables are taken as exogenous. We compute the 25th and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where m = 25th percentile, mean, 75th percentile. We label these: 25th, Mean, and 75th, respectively. Columns 1, 3, and 5 show the impact of the specification without endogeneity. Remaining columns (2, 4, 6) report the results where peer school variables were excluded from the model.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.16: Robustness check - specifications without peer variables and without endogeneity of peer variables - boys

	25th perc.			mean			75th perc.		
	No endogeneity	No peers	No endogeneity	No endogeneity	No peers	No endogeneity	No endogeneity	No peers	No endogeneity
School fees	-0.039 [0.012]***	-0.038 [0.011]***	-0.032 [0.012]***	-0.032 [0.012]***	-0.03 [0.011]***	-0.028 [0.012]**	-0.026 [0.011]**		
School with toilets	0.149 [0.128]	0.155 [0.127]	0.143 [0.127]	0.143 [0.127]	0.138 [0.125]	0.1 [0.127]	0.089 [0.125]		
School with permanent classroom	0.54 [0.146]***	0.542 [0.145]***	0.578 [0.144]***	0.578 [0.144]***	0.575 [0.144]***	0.595 [0.144]***	0.59 [0.144]***		
Number of extra facilities	0.099 [0.038]***	0.099 [0.037]***	0.114 [0.037]***	0.114 [0.037]***	0.119 [0.036]***	0.124 [0.037]***	0.13 [0.037]***		
Percentage of female teachers	-1.282 [0.168]***	-1.28 [0.163]***	-1.09 [0.166]***	-1.09 [0.166]***	-1.069 [0.161]***	-0.978 [0.166]***	-0.953 [0.162]***		
Percentage of teachers with 3 years of experience	0.663 [0.188]***	0.657 [0.187]***	0.51 [0.185]***	0.51 [0.185]***	0.496 [0.185]***	0.348 [0.186]*	0.332 [0.185]*		
Percentage of teachers with university degree	0.776 [0.190]***	0.774 [0.189]***	0.677 [0.188]***	0.677 [0.188]***	0.68 [0.187]***	0.615 [0.188]***	0.619 [0.188]***		
Student test score (average)	0.25 [0.467]		0.187 [0.461]	0.187 [0.461]		0.094 [0.462]			
Teacher absenteeism	-0.004 [0.017]	-0.004 [0.017]	-0.002 [0.017]	-0.002 [0.017]	-0.001 [0.017]	0.004 [0.017]	0.005 [0.017]		
Teacher test score (average)	0.378 [0.580]	0.440 [0.575]	0.637 [0.574]	0.637 [0.574]	0.697 [0.568]	0.875 [0.575]	0.928 [0.570]		
Perc. of Mother with some education (school level)	-0.154 [0.223]		-0.163 [0.220]	-0.163 [0.220]		-0.177 [0.221]			
Asset index (school level)	0.013 [0.058]		0.054 [0.057]	0.054 [0.057]		0.071 [0.057]			

Notes: This table shows estimates of equation (6) in a specification where there are no peer variables and where peer variables are taken as exogenous. We compute the 25th and 75th of maternal education and household assets (our two family background variables), as well as their mean. Then we evaluate the impacts of the school characteristics at 3 points: (m of the distribution of maternal education, m of the distribution of household assets), where m = 25th percentile, mean, 75th percentile. We label these: 25th, Mean, and 75th, respectively. Columns 1, 3, and 5 show the impact of the specification without endogeneity. Remaining columns (2, 4, 6) report the results where peer school variables were excluded from the model.

Standard errors in brackets.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table A.17: Compensating variation - after trimming the bottom and top 1% of the distribution

Panel A - Average Compensating Variation (in U.S. dollars)				
	All		Affected by the Policy	
	GIRLS	BOYS	GIRLS	BOYS
No Private schools	2.3	8.5	2.8	10.2
Voucher Program	-3.0	-3.2	-	-

Panel B - Total Compensating Variation (in thousand U.S. dollars)				
	GIRLS	BOYS	TOTAL	DIF
No Private schools	86.1	318.1	404.2	-232.0
Voucher Program	-113.5	-118.2	-231.7	-4.7

Notes: In this table we present changes in welfare using the average of the compensating variation after trimming the bottom and top 1% of the distribution of this variable. Panel A shows the estimates of the median compensating variation (in U.S. dollars) for a policy that forces all private schools to shut down and from the introduction of vouchers. Columns (1) and (2) show the results for everyone, and columns (3) and (4) display the estimates for those affected by the policy intervention. In the “no private schools” scenario those not affected by the policy intervention have no change in their consumer surplus. In Panel B we obtain the total welfare change, in U.S. thousand dollars, taking the median compensating variation across the sample and multiply by the total number of students enrolled in the regions from our sample in rural Punjab. As before, Columns (1) and (2) show the results for everyone. Column (3) presents the sum of welfare change for girls and boys, and column (4) displays the difference between boys and girls. 1 U.S. dollars \approx 85.6 Pakistani Rupees.

Table A.18: Compensating variation - one private school
 Panel A - Median Compensating Variation (in U.S. dollars)

	All		Affected by the Policy	
	GIRLS	BOYS	GIRLS	BOYS
Only one Private school	0.4	1.2	1.0	3.4

Panel B - Total Compensating Variation (in thousand U.S. dollars)				
	GIRLS	BOYS	TOTAL	DIF
Only one Private school	13.3	46.1	59.3	-32.8

Notes: In this simulation we present the changes in welfare for a policy where we close all but one private school in each village. The private school that is allowed to be open in this simulation has the average characteristics of all private schools in the village. Panel A shows estimates of the median compensating variation, in U.S. dollars, separately for boys and girls. Columns (1) and (2) show the results for everyone, and columns (3) and (4) display the estimates for those affected by the policy intervention. In Panel B we obtain the total welfare change, in U.S. thousand dollars, taking the median compensating variation across the sample and multiply by the total number of students enrolled in the regions from our sample in rural Punjab. As before, Columns (1) and (2) show the results for everyone, and columns (3) and (4) display the estimates for those affected by the policy intervention. 1 U.S. dollars \approx 85.6 Pakistani Rupees.

Table A.19: Compensating variation - by household type

GIRLS							
		Mother Education		Income		Household distance to facilities	
		At least some education	below mean	above mean	below mean	above mean	
		Median (in U.S. dollars)					
No Private schools	No education	1.7	3.9	1.6	3.1	2.5	0.5
Voucher program	No education	-1.8	-2.8	-1.8	-2.5	-2.3	-1.0
BOYS							
		Mother Education		Income		Household distance to facilities	
		At least some education	below mean	above mean	below mean	above mean	
		Median (in U.S. dollars)					

Notes: This table shows the changes in welfare by household type (mother education, income, and household distance to facilities) for a policy that forces all private schools to shut down (“no private schools”) and from the introduction of vouchers. For both, girls and boys, the table shows the median compensating variation (in U.S. dollars).

We use compensating variation to measure changes in a household’s income that equates utility across two states: a benchmark state, which is the status quo, and the alternative state, which is the environment without private schools and the scenario where school fees are equal to zero. For example, it corresponds to the amount of income required to compensate a given household for the elimination of private schools.

1 U.S. dollars \approx 85.6 Pakistani Rupees.

B Appendix - BLP (First step)

In this part of the Appendix, we discuss the estimation procedure of the first step. The coefficients of this model can be estimated using the algorithms described in Berry et al. (1995) and Berry et al. (2004), which we adapt slightly to the type of data we have available.

The first step entails estimating $\delta_{jtg}, \beta_{rkg}^o, \beta_{kg}^u, \bar{\gamma}_g, \gamma_{rg}, \gamma_g^u$ by maximum likelihood, including a contraction mapping to obtain δ_{jtg} .

Under the assumption that ε_{ijtg} has an extreme value Type I distribution, the probability of household i choose school j for children of gender g (i.e. the probability of $u_{ijtg} > u_{iqtg}, \forall j \neq q$) is

$$\begin{aligned} P_{ijtg} &= \Pr(y_i = j | z_{itg}, x_{jtg}, d_{ijtg}, v_{itg}, \beta_g, \gamma_g) \\ &= \frac{\exp(\delta_{jtg} + \sum_{k=1}^K \sum_{r=1}^R x_{jktg} z_{irtg} \beta_{rk}^o + \bar{\gamma}_g d_{ijtg} + \sum_{r=1}^R d_{ijtg} z_{irtg} \gamma_{rg} + \sum_{k=1}^K x_{jktg} v_{itg} \beta_{kg}^u + d_{ijtg} v_{itg} \gamma_g^u)}{\sum_{q=0}^J \exp(\delta_{qtg} + \sum_{k=1}^K \sum_{r=1}^R x_{qktg} z_{irtg} \beta_{rk}^o + \bar{\gamma}_g d_{iqtg} + \sum_{r=1}^R d_{iqtg} z_{irtg} \gamma_{rg} + \sum_{k=1}^K x_{qktg} v_{itg} \beta_{kg}^u + d_{iqtg} v_{itg} \gamma_g^u)} \end{aligned} \quad (9)$$

and the likelihood function is given by:

$$L(\beta_g, \gamma_g) = \prod_{j=0}^J \prod_{i \in A_j} P_{ijtg}$$

and the log-likelihood by:

$$LL(\beta_g, \gamma_g) = \sum_{j=0}^J \sum_{i \in A_j} \ln(P_{ijtg})$$

where, the set of households that choose school j is given by

$$A_{jtg}(x_{jtg}, d_{ijtg}; \delta_{jtg}, \beta_{rkg}^o, \bar{\gamma}_g, \gamma_{rg}) = \{(\varepsilon_{i0tg}, \dots, \varepsilon_{iJtg}) | u_{ijtg} > u_{iltg}, \forall j \neq l\}$$

As v_{itg} is unobserved, the expected value of the probability unconditional on v_{itg} is given by:

$$\hat{P}_{ijtg}(z_{itg}, x_{jtg}, d_{ijtg}, \beta_g, \gamma_g) = \int P_{ijtg} f(v) d(v)$$

To calculate the log-likelihood function we approximate this integral using simulation and then sum the log of this probability over students i of gender g .

Let \tilde{P}_{iqtg} be a simulated approximation to P_{iqtg} . The simulated choice probability is given by

$$\tilde{P}_{ijtg} = \sum_{n=1}^{ND} \frac{\exp(\delta_{jtg} + \sum_{k=1}^K \sum_{r=1}^R x_{jktg} z_{irtg} \beta_{rk}^o + \bar{\gamma}_g d_{ijtg} + \sum_{r=1}^R d_{ijtg} z_{irtg} \gamma_{rg} + \sum_{k=1}^K x_{jktg} v_{itgn} \beta_{kg}^u + d_{ijtg} v_{itgn} \gamma_g^u)}{\sum_{q=0}^J \exp(\delta_{qtg} + \sum_{k=1}^K \sum_{r=1}^R x_{qktg} z_{irtg} \beta_{rk}^o + \bar{\gamma}_g d_{iqtg} + \sum_{r=1}^R d_{iqtg} z_{irtg} \gamma_{rg} + \sum_{k=1}^K x_{qktg} v_{itgn} \beta_{kg}^u + d_{iqtg} v_{itgn} \gamma_g^u)} \quad (10)$$

for random draws $v_{itgn}, n = 1, \dots, ND$.

The Simulated log-likelihood function is given by

$$SLL(\beta, \gamma) = \sum_{j=0}^J \sum_{i \in A_j} \ln(\tilde{P}_{ijt})$$

This procedure is the same as Maximum Likelihood except that simulated probabilities are used instead of the exact probabilities²⁹.

Partially differentiating (10) with respect to δ_{qtg} we get

$$\frac{\partial SLL}{\partial \delta_{qtg}} = \sum_{j=0}^J \sum_{\substack{i \in A_j \\ j \neq q}} \frac{1}{\tilde{P}_{ijt}} \frac{\partial \tilde{P}_{ijt}}{\partial \delta_{qtg}} + \sum_{i \in A_q} \frac{1}{\tilde{P}_{iqt}} \frac{\partial \tilde{P}_{iqt}}{\partial \delta_{qtg}} \quad (11)$$

Given that

$$\frac{\partial \tilde{P}_{iqt}}{\partial \delta_{qtg}} = \tilde{P}_{iqt}(1 - \tilde{P}_{iqt}) \quad (12)$$

$$\frac{\partial \tilde{P}_{ijt}}{\partial \delta_{qtg}} = -\tilde{P}_{iqt} \tilde{P}_{ijt}, j \neq q \quad (13)$$

the FOC with respect to δ_{qtg} of the MSL problem becomes:

$$\begin{aligned} \frac{\partial SLL}{\partial \delta_{qtg}} &= \sum_{i \in A_q} 1 - \sum_{j=0}^J \sum_{i \in A_j} \tilde{P}_{iqt} \\ &= N_q - \sum_{i=1}^N \tilde{P}_{iqt} = 0 \end{aligned}$$

Dividing by N we get:

$$sh_{qg} - \frac{1}{N} \sum_{i=1}^N \tilde{P}_{iqt} = 0 \quad (14)$$

where sh_{qg} is the share of students that attend school q and N is the total number of students³⁰.

This condition implies that the estimated δ_{jt} has to guarantee that the empirical share of students attending school j has to be equal to the average probability that a student attends this school.

²⁹See Train (2009) for further details.

³⁰The procedure is done for each gender. The market is the combination of village t and gender g .

In order to find estimates for the parameters of interest we need to iterate over

$$\delta_{qtg}^{t+1} = \delta_{qtg}^t - \left[\log(sh_{qg}) - \log\left(\frac{1}{N} \sum_{i=1}^N \tilde{P}_{iqtg}\right) \right] \quad (15)$$

Each iteration over (15) requires a new calculation of the probabilities in (10)