ECON3014 : GAME THEORY, Practical 1 (**Oct 2012)**

1. Players 1 and 2 each choose a positive integer from the set $\{1, 2, .., K\}$. If the two players choose the same integer, then player 2 pays £1 to player 1; otherwise, no payment is made. Each player seeks to maximize her expected monetary payoff. Show that the game has a mixed strategy Nash equilibrium, in which each player chooses every integer upto with equal probability, $1/K$.

2. General A is defending a territory which is accessible by two mountain passes against general B. A has 3 divisions at her disposal and B has two divisions. Each general allocates her divisions between the two passes. A wins the battle at a pass if and only if she assigns at least as many divisions to the pass as does B. A successfully defends her territory if and only if she wins the battle at *both* passes. Assume that each general only cares whether or not A defends her territory (i.e. they do not care additionally about the number of battles that they win).

a) Formulate this situation as a strategic game.

b) What are the weakly dominated strategies for each player?

c) Find a mixed strategy Nash equilibrium. (Hint: explain why a player cannot assign positive probability to a weakly dominated strategy.)

3. *An all pay auction.* There are two bidders for an object, which is of value 1 to both the bidders. Each bidder must simultaneously bid a real number $b_i \geq 0$. The object goes to the person making the higher bid, but both the bidders must pay their bid (if the bids are tied, then each bidder gets the object with probability one-half).

a) Show that this game does not have a pure strategy Nash equilibrium.

b) Solve for a mixed strategy Nash equilibrium.