ECON3014 : GAME THEORY: Oct 2012
Exercise 1

1. Consider the game with payoffs as depicted in the table below. Player 1 is the row player and her payoff is written first in every cell, and player 2 the column player.

	L	C	R
T	2,0	3,7	3,0
M	1,0	2,1	1,8
B	0,4	6,2	7,1

a) Eliminate the strictly dominated strategies for both players. After this elimination, are there any strategies for a player i which are strictly dominated given that the player's opponent j will not use strictly dominated strategies? Answer this question for $i = 1$ and $i = 2$.

b) Show that this game does not have a pure strategy Nash equilibrium.

2. Consider a second price auction for one unit of an indivisible good, with two bidders, where bidder i has valuation $v_i, i \in \{1, 2\}$. That is each bidder has submits a bid, and the object is allocated to the highest bidder, and the price that this bidder pays equals the second highest bid. Show that it is a weakly dominant strategy for a bidder to bid his valuation. That is, show that $b_i = v_i$ weakly dominates any other bid b'_i.

3. Consider a road which is represented by the interval $[0, 1]$. Let a be a number such that $0 < a < 1$. Vendor 1 can locate at any point on the interval $[0, a]$ (that is, he can locate at any point x such that $0 \leq x \leq a$). Vendor 2 can locate at any point on the interval $[a, 1]$. A unit mass of onsumers are uniformly distributed on $[0, 1]$ and each consumer buys one unit of the good from the vendor who is closest to him. If the two vendors locate at the same point a, then each gets one-half of the consumers.

The game is as follows. Vendors choose locations simultaneiously, and a vendor's payoff is given by the number of consumers who purchase from him.

a) Write down the strategy sets and payoff functions in this game.

b) Suppose $a = 0.5$. Show that this game has a unique Nash equilibrium in pure strategies. That is, you need to show (i) there is a Nash equilibrium, and (ii) there is no other Nash equilibrium.

c) Suppose $a < 0.5$. Show that the game does not have a Nash equilibrium in pure strategies.

4. Consider the following inspection game between a boss and a worker, where c is the cost to the boss of carrying out the inspection.

		Boss	
		inspect	sleep
worker	work	$2, 3 - c$	$2, 3$
	shirk	$1, 6 - c$	$3, 2$

a) show that if $c > 0$ and $c < 4$, this game does not have pure strategy equilibrium.

b) Solve for a mixed strategy equilibrum, as function of c (when $0 < c < 4$). What are the numerical mixing probabilities for two specific values, $c = 1$ and $c = 2$.

5. Consider the following game between two players. Each player has to choose a number from the set $\{1, 2, 3\}$. If both players choose the same number n, then player 2 must pay player 1 £n. If they choose different numbers, then neither player pays anything. Suppose that each player's utility, $u_i(x_i)$ only depends upon his own monetary payoff x_i, and is strictly increasing in x_i.

a) Show that the game does not have pure strategy Nash equilibrium.

b) Suppose that players are risk neutral, and that $u_i(x_i) = x_i$ for both players. Solve for a mixed strategy Nash equilibrium.

c) Suppose that player 2 is risk neutral, so that $u_2(x_2) = x_2$. However player 1's utility is given by $u_1(x_1) = \sqrt{x_1}$. That is, player 1 is risk averse. Solve for a mixed strategy Nash equilibrium in this case. How do the mixing probabilities of the two players change as compared to (b)? Explain why this is the case.

d) Optional: modify part (c) so that player 2's utility function is given by $u_2(x_2) = (x_2)^{\alpha}$, where $0 < \alpha < 1$. How does the equilibrium change as α decreases? Interpret your results.