Structural Econometrics:
Equilibrium search models

Jean-Marc Robin

Plan

1. Facts on wage distributions
2. On-the-job search and wage posting – Burdett-Mortensen
3. Sequential auctions – Postel-Vinay Robin
Facts on wage distributions

In the past ten years, after Abowd, Kramarz and Margolis’s (Econometrica, 1999) initial push, many matched employer-employee datasets (MEED) have been constructed in Denmark, Italy, Sweden, Austria, etc, to estimate wage equations.

MEED have the following basic structure:

- One panel of workers with individual index $i \in \{1, ..., I\}$ and time index $t \in \{1, ..., T\}$,
- One panel of firms with individual index $j \in \{1, ..., J\}$ and same time index $t \in \{1, ..., T\}$,
- A matching function $j(i,t) \in \{1, ..., J\}$ that defines worker $i$’s employer at time $t$.

They usually are register data (employer payroll reports collected for tax purposes).

Matching wage register data with firm data (accounting data like EBIT, book value, etc.), feasible by availability of firm ID, is not always done. Matching with other individual social security or health insurance data is realised in some countries (Denmark, Sweden).
Basic model used by AKM is a standard error-component model with firm fixed effects:

\[
\ln w_{it} = x_{it}\beta + \psi_{j(i,t)} + \alpha_i + u_{it} \\
= x_{it}\beta + \sum_{j=1}^{J} \psi_j d_{i,t}^j + \alpha_i + u_{it}
\]

where

- \( w_{it} \) is individual wage,
- \( x_{it} \) is a vector of time-varying individual characteristics (experience, tenure), and
- \( d_{i,t}^j, j \in \{1, \ldots, J\} \), are indicator variables:

\[
d_{i,t}^j = \begin{cases} 
1 & \text{if } j(i,t) = j \\
0 & \text{otherwise} 
\end{cases}
\]

- AKM propose to estimate \( \beta \), person effects \( \alpha = (\alpha_1, \ldots, \alpha_N) \) and firm effects \( \psi = (\psi_1, \ldots, \psi_J) \) by OLS.
Identification

Requires VERY large datasets

• Only mobile workers, for whom \( d_{it}^j \neq d_{i*}^j \), contribute to identification, as the OLS estimator of \( \psi \) is the within estimator:

\[
\begin{align*}
w_{it} - w_{i*} &= (x_{it} - x_{i*}) \beta + \sum_{j=1}^{J} \psi_j \left( d_{it}^j - d_{i*}^j \right) + u_{it} - u_{i*}
\end{align*}
\]

• One needs to observe mobile workers in every firm \( j \) for the identification of \( \psi_j \).
  
  – E.g., if only one worker with no job mobility is observed in firm \( j \), then \( \psi_j \) is not identified.
  
  – More generally, partition of the firm sample into disconnected “clusters” with no observed cross-cluster worker mobility creates multicollinearity and requires normalisation of some \( \psi_j \)’s.
• OLS estimator of \( \psi \) consistent for **fixed** \( J \), **large** \( I \), if firm-worker assignment is strictly exogenous:

\[
\left( d^j_{it} \right)_{t \in \{1, ..., T\}, j \in \{1, ..., J\}} \perp (u_{it})_{t \in \{1, ..., T\}, i \in \{1, ..., N\}}, \forall i \in \{1, ..., N\}.
\]

[Acceptable if \( u_{it} \) is transitory.]

• OLS estimator of \( \alpha \):

\[
\widehat{\alpha}_i = w_{i} - x_{i} \widehat{\beta} - \sum_{j=1}^{J} \widehat{\psi}_j d^j_{i}.
\]

consistent for **large** \( T \).

**Conclusion:** If too few individuals, too few time periods and too little mobility:

• Imprecise fixed effect estimates;

• Spurious negative correlation between \( \widehat{\alpha}_i \) and \( \widehat{\psi}_{j(i,t)} \) (across individuals).
### Correlations of Components of Real Annual Wage Rates


#### France 1976-1996

<table>
<thead>
<tr>
<th>Component</th>
<th>StD.</th>
<th>ln w</th>
<th>$x \beta$</th>
<th>$\alpha = z \gamma + \theta$</th>
<th>$z \gamma$</th>
<th>$\theta$</th>
<th>$\psi$</th>
<th>u</th>
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<tbody>
<tr>
<td>Log real annual wage rate</td>
<td>0.9772</td>
<td>1.0000</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>Experience</td>
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<td></td>
<td></td>
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<tr>
<td>Person effect</td>
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<td>0.4569</td>
<td>0.0698</td>
<td>1.0000</td>
<td></td>
<td></td>
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<tr>
<td>Schooling</td>
<td>0.1522</td>
<td>0.1510</td>
<td>-0.0469</td>
<td>0.2917</td>
<td>1.0000</td>
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<tr>
<td>Unobservable</td>
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<td>0.4316</td>
<td>0.0872</td>
<td>0.9565</td>
<td>0.0000</td>
<td>1.0000</td>
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<tr>
<td>Firm effect</td>
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<td>0.4287</td>
<td>0.1670</td>
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<td>0.0293</td>
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<tr>
<td>Residual</td>
<td>0.5545</td>
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<td>0.0000</td>
<td>0.0000</td>
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<td>1.0000</td>
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#### US 1990-1999

<table>
<thead>
<tr>
<th>Component</th>
<th>StD.</th>
<th>ln w</th>
<th>$x \beta$</th>
<th>$\alpha = z \gamma + \theta$</th>
<th>$z \gamma$</th>
<th>$\theta$</th>
<th>$\psi$</th>
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<tr>
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<td>Experience</td>
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<td>Person effect</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.0000</td>
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</table>

- A,K,L,R’s estimate of spurious correlation around $-0.10$ for France and $-0.02$ for the US.
Alternative wage equation

- We use French wage register data matched with firm accounting data.

- Regress log wages $\ln w_{it}$ on employer’s mean log productivity (measured by mean log value-added per worker):

$$
\ln w_{it} = x_{it}\beta + \alpha_i + \gamma \bar{y}_{j(i,t)} + u_{it},
$$

where

$$
\bar{y}_j = \frac{1}{T} \sum_{t=1}^{T} \ln y_{jt}.
$$

<table>
<thead>
<tr>
<th></th>
<th>France, 1990-2000</th>
<th>St.D.</th>
<th>$\ln w_{it}$</th>
<th>$x_{it}\beta$</th>
<th>$\alpha_i$</th>
<th>$\gamma \bar{y}_{j(i,t)}$</th>
<th>$u_{it}$</th>
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<tr>
<td>Log real annual labor cost ($\ln w_{it}$)</td>
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<td>0.477</td>
<td>1.000</td>
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<tr>
<td>Quartic in age ($x_{it}\beta$)</td>
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<td>0.077</td>
<td>0.139</td>
<td>1.000</td>
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<tr>
<td>Person effect ($\alpha_i$)</td>
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<td>0.420</td>
<td>0.888</td>
<td>-0.029</td>
<td>1.000</td>
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<tr>
<td>Firm effect ($\gamma \bar{y}_{j(i,t)}$)</td>
<td></td>
<td>0.022</td>
<td>0.290</td>
<td>0.047</td>
<td>0.269</td>
<td>1.000</td>
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<tr>
<td>Residual ($u_{it}$)</td>
<td></td>
<td>0.204</td>
<td>0.428</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Sample: all private sector employees, 20-50 in initial year.

- **Competitive models** can generate interindustry wage differentials (see for example Heckman and Honoré (Ecta, 1990): worker sort themselves by industry according to particular characteristics). Difficult to extend this idea to interfirm differentials.

- **Equilibrium search models** offer a natural framework in which to analyse this multiform wage dispersion.
  - Labor market competition between employers is the fundamental determinant of wages.
  - Competition is limited by search frictions (information imperfection on job offer locations).

- This framework encompasses two extreme cases:
  - **Competitive wages.** When workers can freely force employers into competition, workers get paid their marginal productivity.
  - **Monopsony wage.** When the cost of finding alternative employers is infinite, firms offer unemployed workers their reservation wages (Diamond, *JET*, 1971).
Equilibrium wage dispersion

Diamond’s (JPE, 1971) critique.

Question: Can there exist wage dispersion at the equilibrium in a sequential search equilibrium model?

When firm behavior is made endogenous, $F(\cdot)$ is shown to degenerate to a mass point at $b$. The search model thus collapses into a simple monopsony model of the labor market.

Why? Suppose firms seek to maximize profit flow $(p - w)\ell(w)$ where $\ell(w)$ is the labor force that they can expect if they offer a wage $w$. (Firms may differ in labor productivity $p$.)

A firm offering less than $\phi$ in equilibrium attracts no worker and all firms offering more than $\phi$ attract the same number of workers. Hence, firms’ best interest is to offer the smallest acceptable wage, that is $\phi$.

Now, if $F$ is a mass point at $\phi$, then $\phi = b + \frac{\lambda_0}{\rho + \delta} \int_{\phi}^{w} F(w)dw = b.$
Reservation wages heterogeneity

Albrecht and Axell (JPE, 1984).

Workers are heterogeneous by nature in this approach: They differ in their opportunity costs of employment, $b$, which are distributed according to some $H$.

Heterogeneity in $b$ is an exogenous source of heterogeneity in $\phi$.

This is enough to generate a non degenerate equilibrium wage distribution $G$, such that $\text{Supp}(G) \subseteq \text{Supp}(H)$.

Low-wage firms make higher per capita profits, but they only attract low-$b$ workers. High-wage firms make less profit per worker, but they make it up on volume since they are able to attract workers with higher values of $b$.

Predicts negative duration-dependence of the exit rate of unemployment.
On-the-job search and wage posting


Equilibrium wage dispersion can be generated among *ex ante* identical workers by allowing workers to search for a better job while employed.

Firms play a wage posting game: Each firm randomly chooses a particular wage in some wage offer distribution $F$, and offers this wage to any worker—employed or not—that it meets.

The unique non cooperative equilibrium of the game is characterized by a non degenerate distribution of wage offers $F$, which leads to a non degenerate earnings distribution $G$.

The Burdett and Mortensen model predicts:

1. Equilibrium wage dispersion among *ex ante* identical workers;
2. “Larger firms pay higher wages”;
3. Workers with more tenure tend to be better paid and less mobile than “junior” workers;
4. Allows for worker and/or firm heterogeneity...
Framework

$L$ identical workers and $N$ heterogenous firms.

Mobility shocks:

- Matches exogenously destroyed at rate $\delta$.
- Job offer arrival rate to unemployed = $\lambda_0$.
- Job offer arrival rate to employed workers = $\lambda_1$.

Job values:

- $\exists$ a job value index, $y$, such that job $y'$ is preferred to job $y$ iff $y' > y$ (wage, marginal productivity of labor, amenities, etc.).
- Sampling distribution of job values = $F \neq \Gamma = \text{distribution of } y \text{ across firms.}$

$$f(y) = \sqrt{v(y) \cdot \gamma(y)}$$

Flow value of non labor time = $b$. 

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Steady-state characterized by a triple \((u, G, \ell)\) where \(u\) is unemployment rate, \((1 - u) G(y)\) is the number of employees in firms with job value less than \(y\), and \(\ell(y)\) is the steady-state size of a firm offering \(y\).

- \(u\) is the steady-state unemployment rate:

\[
\lambda_0 u \overline{F}(\phi) = \delta (1 - u) \iff u = \frac{\delta}{\delta + \lambda_0}.
\]

Note that \(w \geq \phi \Rightarrow \overline{F}(\phi) = 1\).
Unemployment Rates From Stocks and Flows
(French LFS and US CPS)

From stocks: Fraction of unemployed in March of year \(t\)

From flows: \(t/t+1\) job destruction rate divided by \(t/t+1\) job destruction rate plus \(t/t+1\) re-employment rate

Rates are computed by comparing the state in March of year \(t+1\) to the state in March of year \(t\).
• $G$ is the steady-state distribution of job values across employees:

$$
\lambda_0 u F(y) = [\delta + \lambda_1 (1 - F(y))] G(y) (1 - u) \iff G(y) = \frac{F(y)}{1 + \kappa_1 \overline{F}(y)}
$$

for $\kappa_1 = \lambda_1 / \delta$ is the average number of offers an employed worker can expect before the next layoff.

• $\ell(y)$ is the steady-state size of firm $y$:

$$
\ell(y) = \frac{L \cdot g(y)}{N \cdot \gamma(y)} = \frac{\# \text{ workers in firms } y}{\# \text{ firms } y} = \frac{L}{N} \frac{1 + \kappa_1}{\left[ 1 + \kappa_1 \overline{F}(y) \right]^2} \frac{f(y)}{\gamma(y)} = \frac{L}{N} \frac{1 + \kappa_1}{\left[ 1 + \kappa_1 \overline{F}(y) \right]^2} v(y).
$$

Note that $\ell(y)$ needs not be increasing if $v(y)$ is not monotonic.

Equivalent flow equation:

$$
\left[ \lambda_0 u + \lambda_1 G(y) (1 - u) \right] L \times f(y) = \left[ \delta + \lambda_1 \overline{F}(y) \right] \times \ell(y) \gamma(y) N
$$

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The Burdett-Mortensen model:

- Job value $y = \text{wage } w$.
- Uniform hiring effort: $v(w) = 1$.
- Firms differ in marginal labor productivity with $p \sim \Gamma$.

A **market equilibrium** is a non cooperative, maybe dissymmetric, Nash equilibrium where firms seek to maximize profit flows

$$\pi(p) = \max_{w \geq \phi} (p - w) \ell(w)$$

subject to the steady-state employment condition:

$$\ell(w) = \frac{L}{N} \frac{1 + \kappa_1}{[1 + \kappa_1 \overline{F}(w)]^2},$$

and workers apply the reservation strategy

$$\phi = b + (\lambda_1 - \lambda_0) \int_{\phi}^{\overline{w}} \frac{\overline{F}(w) dw}{\rho + \delta + \lambda_1 \overline{F}(w)}.$$
Main equilibrium property

There does not exist a mass point in the equilibrium wage offer distribution.

If there existed a mass of firms offering the same wage $w$, then any one of them would be better off by offering slightly more. The additional wage cost would more than compensated by the additional labor force that this deviating firm would be able to attract.

Technically: the function $\ell(w)$ solving the (now exact) flow condition

$$\left[\lambda_0 u + \lambda_1 G(w^-) (1-u)\right] Lf(w) = \left[\delta + \lambda_1 F(w)\right] \ell(w) \gamma(w) N$$

$$\Leftrightarrow \left[\delta + \lambda_1 G(w^-)\right] (1-u) L = \left[\delta + \lambda_1 F(w)\right] \ell(w) N$$

is discontinuous at any point of discontinuity of $F$ or $G$.

We write $G(w^-) = \lim_{x \to w^-} G(x)$ instead of $G(w)$ because only firms posting a wage strictly less than $w$ lose their employees when they are contacted by a firm posting wage $w$. 
Homogeneous equilibrium

Even if firms are ex ante identical (degenerate distribution of labor productivities), they must offer different wages for an equilibrium to exist. The firms offering higher wages compensating higher wage costs by a bigger size.

For the non degenerate equilibrium to exist, firms must be indifferent in the wage they offer:

$$\forall w \in \text{Supp}(F), \ (p - w) \ell(w) = (p - w) \ell(w)$$

where $w = \inf [\text{Supp}(F)]$.

As $\ell(w) = \frac{L}{N} \frac{1+\kappa_1}{[1+\kappa_1 F(w)]^2} = \frac{L}{N} \frac{1}{1+\kappa_1}$, the equal profit condition writes

$$\forall w \in \text{Supp}(F), \ (p - w) \frac{(1 + \kappa_1)^2}{[1 + \kappa_1 F(w)]^2}(w) = (p - w)$$

$$\Leftrightarrow 1 + \kappa_1 F(w) = (1 + \kappa_1) \sqrt{\frac{p - w}{p - w}}.$$ 

The optimal minimum wage offer is $w = \phi$. The firm offering the lowest wage cannot raise its profit by offering a lower wage than $\phi$. 

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The equilibrium wage offer density is increasing!

\[ f(w) = \frac{1}{2} \frac{1 + \kappa_1}{\kappa_1} \frac{1}{\sqrt{(p - w)(p - w)}}. \]

The only reason that incites firms to offer a wage higher than the monopsony wage \( \phi \) is the Bertrand competition induced by on-the-job search. Offering a higher wage is a way to reduce turnover. But the higher the wage offered the lower the competition pressure. Wage concentration hence also increases.
Now, suppose that $\Gamma$ is non degenerate and continuous.

Each firm offers a wage $w(p)$ so as to maximize profit flows $(p - w) \ell(w)$ given $F$ and $w \geq \phi$.

One first shows that the wage offer function $w(p)$ is continuous and increasing.

For all $p_1 > p_2$, let $w_1 = w(p_1)$ and $w_2 = w(p_2)$, then,

$$
\frac{(p_1 - w_1) \ell(w_1)}{= A_1} \geq \frac{(p_1 - w_2) \ell(w_2)}{= A_2} > \frac{(p_2 - w_2) \ell(w_2)}{= A_3} \geq \frac{(p_2 - w_1) \ell(w_1)}{= A_4}
\Rightarrow A_1 - A_4 \geq A_2 - A_3 \Leftrightarrow \ell(w_1) \geq \ell(w_2) \Rightarrow w_1 \geq w_2.
$$

Hence $w(p)$ is non decreasing.

As there cannot exist mass points in $F$ and as $\Gamma$ is continuous, then $w(p)$ must be almost surely strictly increasing.
Equilibrium wage function

The FOC of the profit maximization programme are:

\[-\ell(w) + (p - w) \ell'(w) = 0\]
\[\Leftrightarrow -1 + \frac{2\kappa_1 (p - w)}{1 + \kappa_1 F'(w)} f(w) = 0\]

The equilibrium \(w(p)\) is increasing. Hence, \(F(w(p)) = \Gamma(p)\) and \(f(w)w'(p) = \gamma(p)\). The FOC can thus be transformed into the following first-order differential equation:

\[-w'(p) + \frac{2\kappa_1 (p - w)}{1 + \kappa_1 \Gamma(p)} \gamma(p) = 0.\]

One can solve it using the initial condition \(w(p) = \phi\). Indeed, nothing forces the least productive firm to post a wage higher than the smallest acceptable one by unemployed workers.
Alternatively, use the Envelope Theorem to show that

\[
\pi'(p) = \frac{d}{dp} \left[ (p - w(p)) \ell(w(p)) \right] = \ell(w(p))
\]

\[
= \frac{L}{N} \frac{1 + \kappa_1}{[1 + \kappa_1 \bar{F}(w(p))]^2} = \frac{L}{N} \frac{1 + \kappa_1}{[1 + \kappa_1 \bar{F}(p)]^2}.
\]

Integrating \(\pi'(p)\), it thus follows that

\[
\pi(p) = [p - w(p)] \frac{L}{N} \frac{1 + \kappa_1}{[1 + \kappa_1 \bar{F}(p)]^2} = \int_p^p \pi'(x) dx + \pi(p)
\]

\[
= \frac{L}{N} \left( \int_p^p \frac{1 + \kappa_1}{[1 + \kappa_1 \bar{F}(x)]^2} dx + \frac{p - w(p)}{1 + \kappa_1} \right).
\]

Hence, since \(w(p) = \phi\),

\[
w(p) = p - \left[ 1 + \kappa_1 \bar{F}(p) \right]^2 \left( \int_p^p \frac{1}{[1 + \kappa_1 \bar{F}(x)]^2} dx + \frac{p - \phi}{(1 + \kappa_1)^2} \right).
\]
No point in estimating the homogeneous model as it predicts an increasing wage offer density $f$ as well as an increasing earnings density $g$ (the distribution of currently employed workers' earnings $G$ as opposed to the wage offer distribution $F$).

Dependent variables:

- Employment state at the time the survey starts: $x = 1$ if employment, $= 0$ if unemployed.

- Unemployed at first observation time:
  
  $t_{0b} =$ elapsed, $t_{0f} =$ residual unemployment duration
  
  $d_{0b} = 1$ or $0$ whether unemployment duration left-censored
  
  $d_{0f} = 1$ or $0$ whether unemployment duration right-censored
  
  $w_0 =$ wage accepted by unemployed individuals

- Employed at first observation time:

  $t_{1b} =$ elapsed, $t_{1f} =$ residual employment duration

  $d_{1b} = 1$ if job duration left-censored, otherwise $= 0$

  $d_{1f} = 1$ if job duration right-censored, otherwise $= 0$

  $w_1 =$ wage of employees at time of first interview

  $v = 1$ if job-to-unemployment transition, $= 0$ if j-t-j transition
• Worker unemployed at the time of the first interview:

\[
\underbrace{\frac{\delta}{\delta + \lambda_0}}_{\text{proba of being unemployed}} \times \underbrace{\lambda_0^{2-d_{0b}-d_{0f}} \exp \left[ -\lambda_0 (t_{0b} + t_{0f}) \right]}_{\text{density of } \left(t_{0b},t_{0f},d_{0b},d_{0f}\right)} \times \underbrace{f(w_0)^{1-d_{0f}}}_{\text{density of accepted wage}}.
\]

• Individual who is employed at the time of the first interview:

\[
\underbrace{\frac{\lambda_0}{\delta + \lambda_0}}_{\text{proba of being employed}} \times \underbrace{g(w_1)}_{\text{density of current wage}} \times \left(\delta + \lambda_1 \overline{F}(w_1)\right)^{1-d_{1b}} \underbrace{\exp \left[ -\left(\delta + \lambda_1 \overline{F}(w_1)\right) (t_{1b} + t_{1f}) \right]}_{\text{density of } \left(t_{1b},t_{1f},d_{1b},d_{1f}\right)} \times \underbrace{\left[\delta^v \cdot \left(\lambda_1 \overline{F}(w_1)\right)^{1-v}\right]^{1-d_{1f}}}_{\text{proba of exit destination}}.
\]
Two-stage estimation procedure

1. First, estimate $G$ and $g$ using a non-parametric estimator (a kernel estimator for example). For any $\kappa_1$, let

$$\hat{F}(w; \kappa_1) = \frac{1 - \hat{G}(w)}{1 + \kappa_1 \hat{G}(w)} \quad \text{and} \quad \hat{f}(w; \kappa_1) = \hat{F}'(w; \kappa_1).$$

2. Second, replace $\bar{F}$ and $f$ by these expressions in the expression for the likelihood and maximize it with respect to $\lambda_0$, $\kappa_1$ and $\delta$.

3. Use the FOC of the profit maximization programme to estimate a productivity value for each observed wage:

$$-1 + \frac{2\kappa_1 (p - w)}{1 + \kappa_1 \bar{F}(w)} f(w) = 0 \Leftrightarrow p - w = \frac{1 + \kappa_1 \bar{F}(w)}{2\kappa_1 f(w)}$$

$$\Rightarrow \hat{p}_i = w_i + \frac{1 + \kappa_1 \hat{G}(w_i)}{2\kappa_1 \hat{g}(w_i)}.$$
Results

Estimations results for the frictional parameters

<table>
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### Earnings distribution

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<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$P_{90}$</th>
<th>$\frac{P_{90}}{P_{10}}$</th>
<th>$Q_3$</th>
<th>$\frac{Q_3}{Q_1}$</th>
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<td>5000</td>
<td>5850</td>
<td>7200</td>
<td>9694</td>
<td>13650</td>
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<td>1.65</td>
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<td>6700</td>
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Properties of the estimated productivity distribution

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<th>$Q_2$</th>
<th>$Q_3$</th>
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</table>
Main empirical lesson of the BM model

- BM models fit turnover and wage distributions well.
- But estimated distribution of firm productivity implausible: exceedingly long right tail.

Why?

- High-productivity firms have a lot of market power in the BM model.
- This tends to concentrate wages towards the upper part of the distribution.
- In order to generate the very long, thin tails of observed wage distributions, productivity distributions with much longer and thinner tails are thus necessary.
- Irreconcilable with actual productivity distributions.
Sequential Auctions

Postel-Vinay and Robin (Econometrica, 2002).

- Match productivity:
  - Workers are perfect substitutes and differ in ability $\varepsilon$.
  - Marginal productivity of efficient labor $p$ is constant and firm-specific.
  - Marginal productivity of a match $(\varepsilon, p)$ is $p\varepsilon$.

- Matches break up exogenously at rate $\delta$ and workers meet employers at rate $\lambda_1$.

- Workers draw type $p$ firms according to the same sampling distribution $F$, whatever their type.

- Wage contracts are negotiated between employers and employees under complete information about each other’s type and can be renegotiated by mutual consent only.

- Lifetime value of unemployment for a worker of type $\varepsilon$ is $V_0(\varepsilon)$.

- Lifetime value of current wage $w$ for a worker $\varepsilon$ in firm $p$ is $V(\varepsilon, w, p)$. 
Employers have full market power. [Assumption relaxed in Cahuc, Postel-Vinay and Robin, 2005.]

When an unemployed worker meets a potential employer, worker is paid a wage $\phi_0 (\varepsilon, p)$ such that

$$V (\varepsilon, \phi_0 (\varepsilon, p), p) = V_0 (\varepsilon).$$

When an employed worker paid $w$ in firm $p$ receives an offer from a firm $p'$:

- If $p < p'$, moves to $p'$ for wage $\phi(\varepsilon, p, p')$ (possibly lower than $w$).
- If $p > p'$ and $w < \phi(\varepsilon, p', p)$, then worker stays at firm $p$ but wage rises to $\phi(\varepsilon, p', p)$.
- If $\phi(\varepsilon, p', p) < w$ nothing happens.

Wage value $\phi(\varepsilon, p, p')$, $p < p'$, solves the equation:

$$V (\varepsilon, \phi, p') = V (\varepsilon, \varepsilon p, p).$$

[Similar to a second-price auction.]
All wages have the form \( \phi(\varepsilon, p, p'), p < p' \), where:

\[
\phi(\varepsilon, p, p') = \varepsilon \cdot \phi(1, p, p') \\
= \varepsilon \cdot \left( p - \frac{\lambda_1}{\rho + \delta} \int_p^{p'} F(x) \, dx \right).
\]

Within-firm wage distribution can be determined in closed-form.

- In particular:

\[
\mathbb{E}(w|p) = \mathbb{E}(\varepsilon) \cdot \left( p - [1 + \kappa_1F(p)]^2 \int_{p_{\min}}^p \frac{1 + (1 - \sigma)\kappa_1f(q)}{[1 + \kappa_1F(q)]^2} \, dq \right),
\]

where \( \kappa_1 = \lambda_1/\delta \) and \( \sigma = \rho/ (\rho + \delta) \).
Log-wage variance decomposition

- Use French wage register data (DADS).

- *Not matched* with firm accounting data (comparable to AKM).

- Decompose the log-wage variance as:

\[
\text{Var} \left( \ln w \right) = \text{Var} \left( \ln \phi \left( \varepsilon, q, p \right) \right) = \begin{cases} 
\text{Var} \left( \ln \varepsilon \right) & \text{(person effect)} \\
+ \text{Var} \mathbb{E} \left[ \ln \phi \left( 1, q, p \right) | p \right] & \text{(firm effect)} \\
+ \mathbb{E} \text{Var} \left[ \ln \phi \left( 1, q, p \right) | p \right] & \text{(effect of frictions)}.
\end{cases}
\]

**Main results:**

- Person effect explains 40% of \( \text{Var} \left( \ln w \right) \) for managers and quickly drops to 0 for unskilled categories.

- Contribution of market imperfections \( \approx 50\% \).
Firm productivity

Cahuc, Postel-Vinay and Robin (Econometrica, >2005).

- Match administrative data on wages with accounting firm data to obtain independent productivity estimates.

- Estimate production function with firm-level data:

\[ Q_{jt} = \sum p_j (\alpha_1 L_1 t + \alpha_2 L_2 t + \ldots)^\zeta, \quad \sum_j p_j = 0, \]

for unskilled (1) and skilled (2) workers.

- \(\alpha_1\) and \(\alpha_2\) are mean ability \(\varepsilon\) within each skill category;
- \(p_j\) is firm \(j\)'s labor productivity;
- A measure of \(p_j\) is thus obtained, independent of wage data.

- Predict firm-level mean wages \(w_{kj}\) using these estimates \(\hat{p}_j\) and the theoretical \(E(w_{kj} | \hat{p}_j)\). Then assess the fit.
Figure 1: The wage–productivity relationship
Figure 2: Productivity densities