

**MICROECONOMICS II**  
**Midterm Exam**  
**Universitat Pompeu Fabra – Winter quarter 2004**  
**Professor: Antonio Cabrales**

1. A mixed-strategy profile  $\sigma^*$  of a strategic-form game is a *strong* equilibrium if there is no other strategy profile  $\sigma'$  such that  $u_j(\sigma') > u_j(\sigma^*)$  for all  $j$  with  $\sigma'_j \neq \sigma_j^*$ .
  - (a) (5) What is the relationship between the set of Nash equilibria and the set of strong equilibria (a subset, a superset, neither)? Why?
  - (b) (10) Give an example of a game with no strong equilibria.
  - (c) (10) Give an example of a game which has at least one strong equilibrium, and where the sets of strong equilibria and Nash equilibria are different.
  
2. There are ten locations, with respective values  $a_1 < a_2 < \dots < a_{10}$ . Player  $i$  ( $i = 1, 2$ ) is endowed with  $n_i$  soldiers ( $n_i < 10$ ) and must allocate them among the locations. To each particular location he can allocate no more than one soldier. The payoff at location  $p$  is  $a_p$  to the player whose soldier is unchallenged, and  $-a_p$  to his opponent, unless both have a soldier at  $p$  or no one has, in which case the payoff is zero to both. The total payoff is obtained by summing up local payoffs.
  - (a) (25) Show that in this game both players have a unique strategy which weakly dominates all others. Does the strategy profile where both agents use their dominating strategy constitute a Nash equilibrium (please justify your answer, positive or negative)?
  
3. Consider the following two-player game. First player 1 can choose either *Stop* or *Continue*. If she chooses *Stop* then the game ends with the pair of payoffs (1, 1). If she chooses *Continue* then the players simultaneously announce nonnegative integers and each player's payoff is the product of the numbers (notice that "infinity" is NOT an integer number).
  - (a) (25) What are the subgame-perfect equilibria of this game (in pure and mixed strategies)?
  
4. The following is the game of the "crazy crab" we analyzed in class. Assume  $v > c$  and write the payoff matrix of the normal form. If in part b) or c) there is a mixed-strategy equilibrium, I only need a proof that such an equilibrium exists, and an indication of the system of equations that need to be solved (and not necessarily the solution for the system of equations).
  - (a) (5) What strategies are strictly dominated?
  - (b) (10) What are the Nash equilibria when  $0.4v - 0.6k < 0$ ?
  - (c) (10) What are the Nash equilibria when  $0.4v - 0.6k > 0$ ?