Nash and game theory

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I am asked to give my view on the contribution of John Nash to the development of game theory. Since I have received most of my early influence through textbooks, let me take look at the subject indices of two important textbooks in game theory: Fudenberg and Tirole (1991) and Osborne and Rubinstein (1994). John Nash is the only person whose contributions to game theory appear indexed by his name more than once in both books. All the other pioneers appear at most once, if at all. To me this is a quite telling indicator of intellectual influence. And it is probably representative of how the profession views his work.

So what has he done that is so important? He may not have *invented* game theory, but he has shown the way to issues that are of essential importance in game theory. Namely, *how to solve all games*, and *how to use game theory to solve social problems*. I will now explain these grandiose italics in turn.

Other people had proposed solutions for games before Nash. John von Neumann (1928) suggested the “maximin” criterion, a solution concept with a long tradition in decision theory. Each player assumes that for every strategy she chooses, the opponent will choose the one that hurts her the most. She then chooses a strategy that is optimal for her, given this expected reaction of the opponent. As Von Neumann already makes clear, this makes most sense for strictly competitive games, where what is good for one player is necessarily bad for the other. With this solution concept it does not matter very much whether you are playing against Nature or another player, or what this player is like. Each player can find her preferred strategy by only looking at her own payoffs.

The revolutionary aspect of Nash's (1950a) equilibrium concept is that it uses heavily the aspect that differentiates games from pure decision theoretic contexts. Games are interactive situations, where players are optimizers who are confronted with other optimizers. If players used the maximin criterion, outside zero-sum games, they could be making a terrible mistake. By thinking

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that the opponents are evil beings who are "out to get them", they would be forgoing possible gains from cooperation. In most applications of game theory these gains are ubiquitous. In fact, one could argue that what makes game theory interesting from an applied point of view is the mixture of cooperation and competition, which is pervasive in real life interactions. So the applicability of a solution concept that depended on strictly competitive situations would be very reduced. The Nash equilibrium concept did not have that problem.

But if players are not pessimists who think that the opponents are going to damage them as much as they can, what should they think? After all, one can defend the maximin criterion because it is well defined and easy to calculate, since a player only needs to know her own payoffs. The answer given by the Nash equilibrium concept sounds radical the first time you hear it. Agents choose strategies that are optimal, given the strategies that are *actually* played by the opponents. A macroeconomist would say that game players have "rational expectations", they forecast accurately the reactions of the opponents.

The immediate question is how can a player know what the other players will actually do. That seems to demand almost supernatural predictive powers from the agents. One possible answer was given by other scholars who had proposed the Nash equilibrium "avant la lettre" as the solution to some particular games that they were studying: Cournot (1838), when he studied the oligopoly problem in economics; or Fisher (1930) when studying the problem of the sex-ratio in biology. Both of them answered that agents over time "learned"' or "adapted to" the actual actions of the opponents.

While this is a sensible answer, I think that focusing on the particular process through which equilibrium can be reached was not a good idea. It may have prevented these scholars from reaching a general answer to the question of what can be a solution for all games. Different equilibrating processes will lead to different solutions, and sometimes to no solution at all. What made Nash unique is that, in typical mathematical fashion, he would not be bothered by examples when starting to analyze the problem. He proposed a solution concept that applied to *all* games, and then he showed that an equilibrium existed in *all* games. And then it turns out that it is very hard to find dynamic equilibrating processes whose steady states are not Nash equilibria.

In the case of the "Nash program" for cooperative games, he went also one giant step further than the competition. He proposed a solution to a "fair" division problem that seemed appealing in an abstract sense and then showed that this solution could in fact be *decentralized*, that is, the solution could be the equilibrium outcome of a game.

Nash (1950b) proposes an axiomatic solution for a bargaining problem in which the parties involved have available a set of (pairs of) utilities and they have to agree to one element in the set. The axioms are quite reasonable: (Pareto) efficiency, symmetry, independence of irrelevant alternatives. The solution is still used by applied economists the world over. But this by itself would not deserve such a prominent place in the history of science. There are other axiomatic solutions that also make sense; Kalai-Smorodinski (1975) and others (see Roth 1979 for a survey) have also proposed systems of axioms to solve bargaining problems.

The really fundamental contribution of Nash in this area was to propose (and partially solve) the following question. Suppose that a third party is interested in obtaining the outcome proposed by Nash (1951) and that she does not know the actual preferences of the players, but just the set of possible agreements. Could she attain the desired solution by proposing the parties to play a game, where the third party only has to make sure that the final outcomes are what she dictates for each pair of strategies? Just posing the question is a phenomenal achievement, as this defines a whole area of work in economics: the theory of implementation and mechanism design.

Nash actually gave an answer to the question. Let each player simultaneously announce a "demand" (how much of the pie she wants): if both demands are compatible, each player obtains what she announced, otherwise both get nothing. As one can easily see, this game has many equilibria. If one player expects the other to "demand" an (arbitrary) *x* percent of the pie, she will "demand" *100-x* percent; and vice versa. But Nash showed that, with a "small" degree of uncertainty, the outcome would be "close" to the desired solution. This is not entirely satisfactory, and other people have given better answers to this particular problem. Rubinstein's (1982) bargaining game with alternating offers (this was first pointed out by Binmore 1987) is perhaps the better known one, although Howard (1992) is probably more exact. But this does not detract from the greatness of a proposal that set the agenda for a vast research program that is still very active and has known a variety of real-life applications. One can say that the much discussed (see McMillan 1994) auctions of the spectrum, which paved the way for a lot of work in "socially optimal privatizations", are the intellectual offspring of Nash's (1951) paper.

There is one further (unintended) contribution of the work of John Nash. It is my feeling that academics nowadays write too much, and think too little. As a result, a lot of our work is of no real importance. This outcome is probably an equilibrium of the game we play, but it seems to me that there are better possible outcomes. Nash showed that one only needs a few pages of extremely good work to make a lasting impact in the world of science. Perhaps we should think about changing the rules of the academic game, so that the (hopefully unique) Nash equilibrium produces more high quality work.

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