Nonlinear Persistence and Partial Insurance: 
Income and Consumption Dynamics in the PSID

By Manuel Arellano, Richard Blundell, and Stephane Bonhomme

In this paper we highlight the important role the PSID has played in our understanding of income dynamics and consumption and income shocks, see for example, Krueger and Perri (2006); Blundell, Pistaferri, and Preston (2008), henceforth, (BPP); and Guvenen and Smith (2014). In the partial insurance approach, transmission parameters are specified that link “shocks” to income with consumption growth. These transmission parameters can change across time and may differ across individuals reflecting the degree of “insurance” available. They encompass self-insurance through simple credit markets as well as other mechanisms used to smooth consumption.

We explore the nonlinear nature of income shocks and describe a new quantile-based panel data framework for income dynamics, developed in Arellano, Blundell, and Bonhomme (ABB 2017). In this approach the persistence of past income shocks is allowed to vary according to the size and sign of the current shock. We find that the model provides a good match with data on family earnings and on individual wages from the PSID. We confirm the results on income dynamics using the extensive population register data from Norway.

Exploiting the enhanced consumption and asset data in recent waves of the PSID, we show that nonlinear persistence has key implications for consumption insurance. The approach is used to provide new empirical measures of partial insurance in which the transmission of income shocks to consumption varies systematically with assets, the level of the shock, and the history of past shocks.

I. Earnings and Consumption Dynamics

A prototypical “canonical” panel data model of (log) family (earned) income $y_{it}$ is

$$y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T,$$

where $y_{it}$ is net of a systematic component, $\eta_{it}$ is a random walk with innovation $v_{it}$,

$$\eta_{it} = \eta_{it-1} + v_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T,$$

and $\varepsilon_{it}$ is a transitory shock.

There is good economic reasoning behind this decomposition: persistent shocks to earned income are more difficult to insure, especially for young families with low assets. How families cope with persistent shocks is the main focus of this research. Short-run fluctuations will matter too, of course, especially for households with low assets (or low access to liquid assets).

In the partial insurance framework, consumption growth is related to income shocks,

$$\Delta c_{it} = \phi_{t} v_{it} + \psi_{t} \varepsilon_{it} + \nu_{it},$$

where $\Delta c_{it} = c_{it-1} - c_{i,t-1}$, $c_{it}$ is log consumption net of a systematic component, $\phi_{t}$ is the
transmission of persistent shocks $\nu_t$, and $\psi_t$ the transmission of transitory shocks; the $\nu_t$ are taste shocks, assumed to be independent across periods. BPP show identification and efficient estimation using GMM. They also allow for measurement error and extend to MA(1) transitory shocks.

The parameters $\phi_t$ and $\psi_t$ link the evolution of consumption inequality to income inequality. They indicate the degree of partial insurance, and will differ by age, assets, and human capital. For example, using a linearized approximation to a simple benchmark intertemporal consumption model, Blundell, Low, and Preston (2013) show

$$\phi_t = (1 - \pi_t) \quad \text{and} \quad \psi_t = (1 - \pi_t) \gamma_{Lt},$$

where $\pi_t \approx \frac{\text{Assets}_t}{\text{Assets}_t + \text{Human Wealth}_t}$ and $\gamma_{Lt}$ is the annuity value of a temporary shock to income for an individual aged $t$ retiring at age $L$.

For the PSID, estimates of $(1 - \pi_t)$ typically average at around 0.82. BPP estimate a partial insurance coefficient $\phi$ of 0.642 (0.09). This represents excess insurance relative to $(1 - \pi_t)$. They document higher values for samples without college education, for older cohorts, and for low wealth samples.

This linearized partial insurance framework provides key insights on the distributional dynamics of income and consumption. However, it rules out the nonlinear transmission of shocks and restricts interactions in consumption responses.

II. Nonlinear Persistence

The aim in the new work on nonlinear persistence is to step back from the standard panel data model of income dynamics and take a different track: develop an alternative approach in which the impact of past shocks can be altered by the size and sign of new shocks. The framework allows “unusual” shocks to wipe out the memory of past shocks. Additionally the future persistence of a current shock will depend on future shocks. We will see that the presence of “unusual” shocks matches the data well and has a key impact on consumption and saving decisions over the life cycle.

In this framework we maintain the factor structure for log income but allow $\eta_{it}$ to follow a general first-order Markov process.\footnote{The first-order Markov assumption can be generalized to Markov ($p$), with any fixed $p$ (although this requires larger $T$).} Denoting the $\tau$th conditional quantile of $\eta_{it}$ given $\eta_{i,t-1}$ as $Q_\tau(\eta_{i,t-1}, \tau)$, we specify

$$\eta_{it} = Q_\tau(\eta_{i,t-1}, T_{it}),$$

where $(u_{it}, \eta_{i,t-1}, \eta_{i,t-2}, \ldots) \sim \text{Uniform}(0, 1)$, and $\varepsilon_{it}$ has zero mean, independent over time. The conditional quantile functions $Q_\tau(\eta_{i,t-1}, u_{it})$ and the marginal distributions $F_{\eta_{it}}$ can all be age specific.

This framework allows for quite general nonlinear dynamics of income, allowing a general form of conditional heteroscedasticity, skewness, and kurtosis. To see this, consider the following measure of persistence:

$$\rho_t(\eta_{i,t-1}, \tau) = \left. \frac{\partial Q_\tau(\eta_{i,t-1}, \tau)}{\partial \eta} \right|_{\eta = \eta_{i,t-1}},$$

which measures the persistence of $\eta_{i,t-1}$ when, at age $t$, it is hit by a shock $u_{it}$ that has rank $\tau$. This measures the persistence of histories, which may depend on both the level of earnings $(\eta_{i,t-1})$ and the percentile of the shock $(\tau)$. Below we show strong evidence for such nonlinearities in persistency.

III. An Empirical Model for Consumption

To motivate the specification of consumption we use a standard life-cycle incomplete markets model. Let $c_{it}$ and $a_{it}$ denote log-consumption and assets (beginning of period) net of age dummies. We model consumption in levels and leave the nonlinear rule flexible. Our empirical specification is based on

$$c_{it} = g_t(a_{it}, \eta_{i,t-1}, \varepsilon_{it}, \nu_{it}), \quad t = 1, \ldots, T,$$

where $\nu_{it}$ are independent across periods, and $g_t$ is a nonlinear, age-dependent function, monotone in $\nu_{it}$; $\nu_{it}$ may be interpreted a taste shifter that increases marginal utility. This consumption rule is consistent, in particular, with the standard life-cycle model. ABB derive conditions under which $g$ is nonparametrically identified.
The consumption responses to $\eta$ and $\varepsilon$ are

$$
\phi_t(a, \eta, \varepsilon) = E \left[ \frac{\partial g_t(a, \eta, \varepsilon, \nu)}{\partial \eta} \right],
$$

$$
\psi_t(a, \eta, \varepsilon) = E \left[ \frac{\partial g_t(a, \eta, \varepsilon, \nu)}{\partial \varepsilon} \right],
$$

where $\phi_t(a, \eta, \varepsilon)$ and $\psi_t(a, \eta, \varepsilon)$ reflect the transmission of the persistent and transitory earnings components, respectively. They generalize the partial insurance coefficients of BPP.

Similar techniques can be used in the presence of advance information, consumption habits, and in cases where the consumption rule depends on lagged $\eta_i$, or when $\eta$ follows a second-order Markov process, see Section 3 in ABB. The framework allows for additional, unobserved heterogeneity in earnings and consumption. Households can also differ in their initial productivity $\eta_i$ and initial assets.

IV. Data and Estimation

The PSID went through a redesign in the late 1990s, introducing new consumption and asset modules. Since 1999 it collects some 70 percent of non-durable consumption expenditures, and more than 90 percent since 2005. We use the sum of food at home, food away from home, gasoline, health, transportation, utilities, etc. We also make use of the more detailed asset data, see Blundell, Pistaferri, and Saporta-Eksten (2016). For comparison we use family earnings data from administrative registers in the Norwegian population registers, see Blundell, Graber, and Mogstad (2015).

The results we present on the PSID use data from the 1999–2009 surveys. Assets holdings are the sum of financial assets, real estate value, pension funds, and car value, net of mortgages and other debt. Income $y_{it}$ are residuals of log total pretax household labor earnings on a set of demographics—cohort and calendar time dummies, family size and composition, education, race, and state dummies. Log consumption $c_{it}$ is also a residual, using the same set of demographics as for earnings. Following BPS, we select married male heads aged between 25 and 59. We focus on a subsample of 792 households.

The conditional quantile function for the permanent income factor $\eta_{i,t}$, given $\eta_{i,t-1}$, is specified as

$$
Q_t(\eta_{t-1}, \tau) = Q(\eta_{t-1}, age_t, \tau)
$$

$$
= \sum_{k=0}^{K} a_k(\tau) \varphi_k(\eta_{t-1}, age_t),
$$

where $\varphi_k$, $k = 0, 1, \ldots, K$, are polynomials (Hermite). We proceed similarly for $\varepsilon_{it}$, etc. The consumption (log) function, $g(a, \eta_i, \varepsilon_t, age_t)$, is specified as a flexible polynomial in assets, permanent income factor, the transitory shock and age, $g_t$, is additive in $\nu_t$.

Estimation takes place in two steps, see ABB for details. The first step recovers estimates of the income parameters. The second step recovers estimates of the consumption parameters, given an estimate of the income parameters. The estimation algorithm alternates between draws of latent variables from candidate posteriors and quantile regressions using those draws, see also Arellano and Bonhomme (2016).

V. Empirical Results

Figure 1 provides our initial evidence for non-linear income dynamics. It presents estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$ for both the PSID in panel A, and the Norwegian register data in panel B. These are evaluated at percentiles of the shock $\tau_{\text{shock}}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the distribution of $y_{i,t-1}$.

The estimates in Figure 1 display distinct and systematic nonlinearity. The persistence of income shocks is much lower for large negative (positive) shocks for high (low) initial incomes. The results for the PSID are confirmed in panel B for the Norwegian data.

Turning to the income model, Figure 2 in panel A provides estimates of the average derivative of the conditional quantile function of the persistent income factor $\eta_{i,t}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $\eta_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the distribution of $\eta_{i,t-1}$. The estimates

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2The Norwegian results are part of the project on “Labor Income Dynamics and the Insurance from Taxes, Transfers and the Family.” See ABB Appendix C.
are evaluated at mean age in the sample. Panel B in Figure 2 is based on data simulated according to our nonlinear earnings model with parameters set to their estimated values. It shows a close accordance with the persistence in the PSID income data. Moving to the estimated consumption model, Figure 3 displays the average derivative of the conditional mean of $c_{it}$ given $y_{it}$, $a_{it}$, and $age_{it}$ with respect to $y_{it}$, evaluated at values of $a_{it}$ and $age_{it}$ corresponding to their $\tau_{assets}$ and $\tau_{age}$ percentiles, and averaged over the values of $y_{it}$. It shows consumption responses vary systematically with age and assets and in a way that accords with standard life-cycle theory. It also shows an agreement between the consumption model and data.

Finally, we provide preliminary evidence that the nonlinear persistence we have uncovered in family earnings data is also evident in hourly

**Figure 1. Quantile Autoregressions**

*Notes:* Residuals $y_{it}$ of log pretax household labor earnings, Age 25–60 1999–2009 (United States), Age 25–60 2005–2006 (Norway). Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$. Quantile functions are specified as third-order Hermite polynomials.

*Source:* Arellano, Blundell, and Bonhomme (2017)

**Figure 2. Nonlinear Persistence**

*Notes:* PSID data. Panel A shows estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$. Quantile functions are specified as third-order Hermite polynomials. Panel B is based on data simulated according to our nonlinear earnings model with parameters set to their estimated values.

*Source:* Arellano, Blundell, and Bonhomme (2017)
wage data. Figure 4, panel A, presents estimates of nonlinear persistence in PSID male hourly earnings. Panel B provides the estimates of the average derivative of the conditional quantile function of the component $\eta$ from the nonlinear model. These results show an important role for unusual shocks and nonlinear persistence in hourly wage data, suggesting nonlinear persistence may be a key feature for life-cycle models of family labor supply.

**VI. Summary and Conclusions**

We have outlined a new framework for income dynamics and the nonlinear transmission of income shocks to consumption. We have exploited important new measurements for consumption and assets in the PSID. We have also shown the complementarities between the use of “big” administrative data, e.g., Norwegian registers, and purpose-designed panel surveys, like the PSID.
A Markovian permanent-transitory model of household income, which reveals asymmetric persistence of unusual shocks, is shown to accord well with the persistence of income in the PSID and in Norwegian register data. An age-dependent nonlinear consumption rule as a function of assets, permanent income, and transitory income is also applied to the PSID and shown to generate new empirical measures of the degree of partial insurance.

We also find nonlinearities in the dynamics of individual male wages in the PSID, suggesting a role for nonlinear persistence in dynamic family labor supply, e.g., Blundell, Pistaferri, and Saporta-Eksten (2016) and Heathcote, Storesletten, and Violante (2014). In future work it would be useful to examine firm-to-firm transitions, the role of housing equity, and the importance of health and other (partially insured) shocks.

A final word to the PSID: congratulations at 50 and looking forward to the next 50 years!

REFERENCES


