## PLIN0056 Semantics Research Seminar

Lecture 2: Functional readings and reconstruction

## 1 Karttunen-style compositional semantics with intensional pronouns

### 1.1 Compositional semantics with intensional pronouns

We'll deal with intensionality with intensional pronouns (see von Fintel \& Heim 2021: Part II, Percus 2000, Schwarz \& Keshet 2019).

- Variables
- Individual variables (type $e$ ): $\mathbf{x}, \mathbf{x}^{\prime}, \ldots, \mathbf{y}, \mathbf{y}^{\prime}, \ldots, \mathbf{z}, \mathbf{z}^{\prime}, \ldots$
- Intensional variables (type $s$ ): $\mathbf{w}, \mathbf{w}^{\prime}, \mathbf{w}^{\prime \prime}, \ldots$
- Assignments map individual variables to entities and intensional variables to possible worlds. We will also be dealing with variables of functional types.
- Propositional variables (type $\langle s, t\rangle$ ): $\mathbf{p}, \mathbf{p}^{\prime}, \ldots, \mathbf{q}, \mathbf{q}^{\prime}, \ldots$
- Functional variables (various types, e.g., $\langle e, e\rangle): \mathbf{f}, \mathbf{f}^{\prime}, \ldots, \mathbf{g}, \mathbf{g}^{\prime}, \ldots, \mathbf{h}, \mathbf{h}^{\prime}, \ldots$
- Every atomic expression takes an intensional pronoun as its first argument.
a. $\quad \llbracket \mathbf{s a w}]^{g}=\lambda w_{s} \cdot \lambda x_{e} \cdot \lambda y_{e} . y$ saw $x$ in $w$
b. $\quad[\mathbf{c a t}]^{g}=\lambda w_{s} \cdot \lambda x_{e} \cdot x$ is a cat in $w$

Some expressions are intensionally rigid.
a. $\quad \llbracket$ Chomsky $\rrbracket^{g}=\lambda w_{s}$. Chomsky
b. $\llbracket$ every $]^{g}=\lambda w_{s} . \lambda P_{\langle e, t\rangle} \cdot \lambda Q_{\langle\langle, t\rangle} .\left\{x \in D_{e} \mid P(x)=1\right\} \subseteq\left\{x \in D_{e} \mid P(x)=Q(x)=1\right\}$
c. $\llbracket$ something $\rrbracket^{g}=\lambda w_{s} \cdot \lambda P_{\langle e, t\rangle} \cdot\left\{x \in D_{e} \mid P(x)=1\right\} \neq \varnothing$

- Every intensional pronoun is bound by some $\lambda$-operator.
- Every clause has a $\lambda$-operator on top.

- Intensional pronouns will be indicated as subscripts below, instead of independent nodes. Those on intensionally rigid expressions will be all omitted. We'll ignore tense.

- Benefit 1: The compositional rules will be similar to extensional systems. Intensionality only appears on the leaves of the tree!
(3) Functional Application

If $\llbracket \alpha \rrbracket^{g} \in D_{\langle\sigma, \tau\rangle}$ and $\llbracket \beta \rrbracket^{g} \in D_{\sigma}$, then $\llbracket \alpha \beta \rrbracket^{g}=\llbracket \beta \alpha \rrbracket^{g}=\llbracket \alpha \rrbracket^{g}\left(\llbracket \beta \rrbracket^{g}\right)$.
(4) Predicate Abstraction

If $\xi$ is a variable and $\tau$ is a type, then $\llbracket \lambda \xi_{\tau} \alpha \rrbracket^{g}=\lambda v_{\tau} \cdot \llbracket \alpha \rrbracket^{g[\xi \mapsto v]}$.
(5) Variable Rule

If $\alpha$ is a (null or overt) pronoun or trace bearing a variable $\xi$, then $\llbracket \alpha \rrbracket^{g}=\lambda w_{s} . g(\xi)$.
(6) $\quad$ a. $\quad \llbracket$ someone $\rrbracket^{g}=\lambda w_{s} . \lambda P_{\langle e, t\rangle} .\left\{x \in D_{e} \mid x\right.$ is a person $w$ and $\left.P(x)=1\right\} \neq \varnothing$
b. $\quad \llbracket \mathbf{s a w} \rrbracket^{g}=\lambda w_{s} \cdot \lambda x_{e} \cdot \lambda y_{e} . y$ saw $x$ in $w$
c. $\quad \llbracket$ Chomsky $]^{g}=\lambda w_{s}$. Chomsky

$$
\begin{align*}
& \left.\llbracket \lambda \mathbf{w}_{s} \text { someone }_{\mathbf{w}} \mathbf{s a w}_{\mathbf{w}} \text { Chomsky }\right]^{g} \\
= & \lambda w_{s} \cdot \llbracket \text { someone }_{\mathbf{w}} \mathbf{s a w}_{\mathbf{w}} \text { Chomsky } \rrbracket^{g[\mathbf{w} \mapsto w]}  \tag{PA}\\
= & \lambda w_{s} \cdot \llbracket \text { someone }_{\mathbf{w}} \rrbracket^{g[\mathbf{w} \rightarrow w]}\left(\llbracket \mathbf{s a w}_{\mathbf{w}} \text { Chomsky } \rrbracket^{g[\mathbf{w} \mapsto w]}\right)  \tag{FA}\\
= & \lambda w_{s} \cdot \llbracket \text { someone }_{\mathbf{w}} \rrbracket^{g[\mathbf{w} \rightarrow w]}\left(\llbracket \mathbf{s a w}_{\mathbf{w}} \rrbracket^{g[\mathbf{w} \mapsto w]}\left(\llbracket \text { Chomsky }^{g[\mathbf{w} \rightarrow w]}\right)\right) \tag{FA}
\end{align*}
$$

$\vdots$
$=\lambda w_{s} . \llbracket$ someone $_{\mathbf{w}} \rrbracket^{g[\mathbf{w} \rightarrow w]}\left(\left[\lambda x_{e} \cdot \lambda y_{e} . y\right.\right.$ saw $x$ in $\left.w\right]$ (Chomsky) $)$
$=\lambda w_{s} .\left[\lambda P_{\langle e, t\rangle} \cdot\left\{x \in D_{e} \mid x\right.\right.$ is a person $w$ and $\left.\left.P(x)=1\right\} \neq \varnothing\right]\left(\left[\lambda y_{e} . y\right.\right.$ saw Chomsky in $\left.\left.w\right]\right)$
$\vdots$
$=\lambda w_{s}$. at least one person in $w$ saw Chomsky in $w$

- Benefit 2: More flexible analysis of de re/de dicto (in fact, it's a bit too powerful; see the works cited above).


### 1.2 Karttunen-style semantics for questions

$W h$-phrases are existential quantifiers that take scope between the $w h$-question operator and abstraction over its propositional argument.
(7) a. $\quad \llbracket$ what $]^{g}=\llbracket$ something $\rrbracket^{g}=\lambda w_{s} . \lambda P_{\langle e, t\rangle}$. for some $x \in D_{e}, P(x)=1$
b. $\quad \llbracket \mathbf{w h o} \rrbracket^{g}=\llbracket$ someone $\rrbracket^{g}=\lambda w_{s} . \lambda P_{\langle e, t\rangle}$. for some $x \in D_{e}, x$ is a person in $w$ and $P(x)=1$
(8) $\quad$ a. $\quad \llbracket \mathbf{C}_{\mathbf{w h}} \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} \cdot \lambda q_{\langle s, t\rangle} \cdot q=p$
b. $\quad \llbracket \mathbf{C}_{\mathbf{y n}} \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} \cdot \lambda q_{\langle s, t\rangle} \cdot p=q$ or $p=\lambda w_{s}^{\prime} \cdot p(w)=0$
(9)

(10) $\llbracket(9) \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle}$. for some person $x$ in $w, p=\lambda w_{s}^{\prime}$. every child in $w^{\prime}$ saw $x$ in $w^{\prime}$ For any $w, \llbracket(9) \rrbracket^{g}(w)$ characterises

$$
\left\{\lambda w_{s}^{\prime} \text {. every child in } w^{\prime} \text { saw } x \text { in } w^{\prime} \mid x \text { is a person in } w\right\}
$$

- Here we chose $\mathbf{w}^{\prime}$ as the intensional pronoun for everything in TP (de dicto).
- Child could take the other intensional pronoun $w$ (de re). Call this LF (9)'. Then, the denotation and the set it characterises in $w$ will be:
(11) $\quad$ a. $\quad \llbracket(9)^{\prime} \rrbracket^{g}=\lambda w_{s} . \lambda p_{\langle s, t\rangle}$. for some person $x$ in $w, p=\lambda w_{s}^{\prime}$. every child in $w$ saw $x$ in $w^{\prime}$ b. $\quad\left\{\lambda w_{s}^{\prime}\right.$. every child in $w$ saw $x$ in $w^{\prime} \mid x$ is a person in $\left.w\right\}$
- Not easy to demonstrate the difference between these two readings at this point. We will come back to it when we discuss pragmatics next week.
- Below, we'll ignore de re readings of things in the TP (except for the traces of which-phrases).


## 2 Which and reconstruction

Which is analysed as an existential quantifier.
$\llbracket \mathbf{w h i c h} \rrbracket^{g}=\lambda w_{s} . \lambda P_{\langle e, t\rangle} \cdot \lambda Q_{\langle e, t\rangle}$. for some $x \in D_{e}, P(x)=Q(x)=1$
(13) Which cat did every child see?
(14)


$$
\begin{equation*}
\llbracket(14) \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} . \text { for some cat } x \text { in } w, p=\lambda w_{s}^{\prime} . \text { every child in } w^{\prime} \operatorname{saw} x \text { in } w^{\prime} \tag{15}
\end{equation*}
$$

For any $w, \llbracket(14) \rrbracket^{g}(w)$ characterises

$$
\left\{\lambda w_{s}^{\prime} . \text { every child in } w^{\prime} \text { saw } x \text { in } w^{\prime} \mid x \text { is a cat in } w\right\}
$$

- According to this analysis, who and which person have the same meaning.
- But which + singular NP gives rise to a uniqueness presupposition, e.g., (14) presupposes that each child saw exactly one cat. We will come back to this next week.


### 2.1 Intensional reconstruction?

Note that the restrictor of which is interpreted relative to $w$.


Would we want to derive a reading where cat is relative to some other possible world (i.e., a de dicto reading)?
(16) Q: Which relative of yours does John want you to introduce to him?

A: My younger sister. I don't have one, but he thinks I do.
But maybe the following is not completely out. It might be a kind of (mixed) quotation.

Q: Which relative of yours did John meet?
A: My younger sister. I don't have one, but he thinks I do.
The problem becomes more apparent in embedded questions.
(18) The student is wondering which relatives of mine live in Japan.

But let's not talk about embedded questions (yet).

### 2.2 Bound pronouns in which-phrases

Which-phrases containing bound pronouns pose a similar, more obvious issue.
E.g., in (19), his can be bound by every man.
(19) Which relative of his does every man hate?
(20)


- In (20), his $_{\mathbf{y}}$ is not bound. $\lambda \mathbf{y}_{e}$ is too low!
- We don't want to reconstruct the entire which-phrase, because if we do, it'll function as an existential quantifier, similarly to a relative of his $\mathbf{y}_{\mathbf{y}}$.


## 3 Engdahl 1986

To account for data like (19), Engdahl 1986 postulates a phonologically null binder, $E$, within the which-phrase. It's basically a type shifter.

(22)
a. $\quad \llbracket$ relative $\rrbracket^{g}=\lambda w_{s} \cdot \lambda x_{e} \cdot \lambda y_{e} \cdot y$ is a relative of $x$ 's in $w$
b. $\left.\quad \llbracket \mathbf{h i s}_{\mathbf{z}}\right]^{g}=\lambda w_{s} . g(\mathbf{z})$
a. $\quad \llbracket$ relative $_{\mathbf{w}} \mathbf{o f} \mathbf{h i s}_{\mathbf{z}} \rrbracket^{g}=\lambda y_{e} . y$ is a relative of $g(\mathbf{z})$ in $g(\mathbf{w})$
b. $\quad \llbracket \lambda \mathbf{z}_{e}$ relative $_{\mathbf{w}} \mathbf{o f} \mathbf{h i s}_{\mathbf{z}} \rrbracket^{g}=\lambda z_{e} . \lambda y_{e} . y$ is a relative of $z$ in $g(\mathbf{w})$
a. $\quad \llbracket E \rrbracket^{g}=\lambda w_{s} \cdot \lambda R_{\langle e,\langle e, t\rangle\rangle} \cdot \lambda f_{\langle e, e\rangle}$. for each $x \in D_{e}, R(x)(f(x))=1$
b. $\quad \llbracket E \lambda \mathbf{z}_{e}$ relative $_{\mathbf{w}}$ of $\mathbf{h i s}_{\mathbf{z}} \rrbracket^{g}=\lambda f_{\langle e, e\rangle}$. for each $x \in D_{e}, f(x)$ is a relative of $x$ 's in $g(\mathbf{w})$

It's not always true that every individual has a relative, so for such a world, there won't be an $f$ that (24b) maps to 1 . To deal with this, we can revise $E$ as follows:
a. $\quad \llbracket E \rrbracket^{g}=\lambda w_{s} \cdot \lambda R_{\langle e,\langle e, t\rangle\rangle} \cdot \lambda f_{\langle e, e\rangle}$.
for each $x \in D_{e}$, if $\left\{y \in D_{e} \mid R(x)(y)=1\right\} \neq \varnothing$, then $R(x)(f(x))=1$
b. $\llbracket E \lambda \mathbf{z}_{e}$ relative $_{\mathbf{w}}$ of his $_{\mathbf{z}} \rrbracket^{g}$
$=\lambda f_{\langle e, e\rangle}$. for each $x \in D_{e}, f(x)$ is a relative of $x$ 's in $g(\mathbf{w})$, if there's one
Engdahl 1986 treats which as a type-flexible existential quantifier.

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a. \(\quad \llbracket\) which \(_{\langle e, e\rangle} \rrbracket^{g}=\lambda w_{s} . P_{\langle\langle e, e\rangle, t\rangle} \cdot \lambda Q_{\langle\langle e, e\rangle, t\rangle}\). for some \(f \in D_{\langle e, e\rangle}, P(f)=Q(f)=1\)
b. \(\quad \llbracket\) which \(_{\langle e, e\rangle} E \lambda \mathbf{z}_{e}\) relative \(_{\mathbf{w}}\) of his \(\mathbf{z}_{\mathbf{z}} \rrbracket^{g}\)
\(=\lambda Q_{\langle\langle e, e\rangle, t\rangle}\). for some \(f \in D_{\langle e, e\rangle}\), for each \(x \in D_{e}\),
\(f(x)\) is a relative of \(x\) 's in \(g(\mathbf{w})\), if there's one, and \(Q(f)=1\)
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$$
\begin{equation*}
\llbracket(27) \rrbracket^{g}=\lambda w_{s} \cdot \lambda q_{\langle s, t\rangle} \text {. for some } f \in D_{\langle e, e\rangle}, \tag{28}
\end{equation*}
$$

for each $x \in D_{e}, f(x)$ is a relative of $x$ 's in $w$, if there's one, and $q=\lambda w_{s}^{\prime}$. for every man $x$ in $w^{\prime}, x$ hates $f(x)$ in $w^{\prime}$

For any possible world $w, \llbracket(27) \rrbracket^{g}(w)$ characterises:

$$
\left\{\begin{array}{l|l}
\lambda w_{s}^{\prime} . \text { for every man } x \text { in } w^{\prime}, x \text { hates } f(x) \text { in } w^{\prime} & \begin{array}{l}
f \in D_{\langle e, e\rangle} \text { and for each } x \in D_{e}, \\
f(x) \text { is a relative of } x \text { 's in } w, \text { if there's one }
\end{array}
\end{array}\right\}
$$

Each of these propositions is a (partial) answer to the question (in w), e.g., the proposition that Andy hates his sister Ann, Benjamin hates his mother Becky, Chris hates his father Charles, Daniel hates his brother David, etc.

## 4 Heim 2012

Heim 2012 accounts for reconstruction via NP-reconstruction + Trace Conversion à la Fox 2000.

### 4.1 Trace Conversion

Copy theory of movement (We'll ignore head movement)
(29)


Trace conversion

(30)
a. $\quad[\text { IDENT }]^{g}=\lambda w_{s} \cdot \lambda x_{e} \cdot \lambda P_{\langle e, t\rangle} \cdot \lambda y_{e} \cdot y=x$ and $P(y)=1$
b. $\quad \llbracket \mathbf{T H E}]^{g}=\lambda w_{s} \cdot \lambda P_{\langle e, t\rangle}:\left|\left\{x \in D_{e} \mid P(x)=1\right\}\right|=1 . \iota x[P(x)=1]$

### 4.2 Presupposition

Partial function theory of presupposition (Heim 1982, 1983, Heim \& Kratzer 1998)

- Presupposition triggers denote partial functions. For example, for any $w$,
$\operatorname{dom}\left(\llbracket \mathbf{T H E} \rrbracket^{g}(w)\right)=\left\{P \in D_{\langle e, t\rangle} \mid\right.$ there is exactly one $x \in D_{e}$ such that $\left.P(x)=1\right\}$
- $P(x)=1$ entails $x \in \operatorname{dom}(P)$, so we don't write the latter here.
- With partial functions, sets and their characteristic functions are no longer isomorphic, e.g., $\lambda x_{e} . A(x)=B(x)=1$ and $\lambda x_{e}: A(x)=1 . B(x)=1$ are different functions but characterise the same set. We therefore need to be explicit about the distinction between sets and functions.
- Compositional rules specify how presuppositions project. Functional Application partialises $\llbracket \cdot \rrbracket^{g}$.
(31) Functional Application

Let $\alpha$ be a branching node with two daughter constituents $\beta$ and $\gamma$.
a. $\left.\quad{ }^{'} \alpha^{\prime} \in \operatorname{dom}(\llbracket \cdot]^{g}\right)$ iff all of the following is the case.
(i) $\quad{ }^{\ulcorner } \beta^{\top} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$
(ii) $\quad{ }^{`} \gamma \quad \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$
(iii) $\llbracket \gamma \rrbracket^{g} \in \operatorname{dom}\left(\llbracket \beta \rrbracket^{g}\right)$
b. If ${ }^{\ulcorner } \alpha^{\top} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$, then $\llbracket \alpha \rrbracket^{g}=\llbracket \beta \rrbracket^{g}\left(\llbracket \gamma \rrbracket^{g}\right)$

## Predicate Abstraction

Let $\xi$ be a variable and $\tau$ a type.
a. $\quad{ }^{「} \lambda \xi_{\tau} \alpha^{\top} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$.
b. $\quad \llbracket \lambda \xi_{\tau} \alpha \rrbracket^{g}=\lambda v_{\tau}:{ }^{\ulcorner } \alpha^{\top} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g[\xi \mapsto v]}\right) \cdot \llbracket \alpha \rrbracket^{g[\xi \mapsto v]}$

## Variable rule

a. $\quad \llbracket t_{\xi} \rrbracket^{g}=\lambda w_{s} . g(\xi)$
b. $\quad \llbracket \boldsymbol{p r} \boldsymbol{o}_{\xi} \rrbracket^{g}=\lambda w_{s} . g(\xi)$
c. $\quad\left[\quad \operatorname{him}_{\xi} \rrbracket^{g}=\lambda w_{s}: g(\xi)\right.$ is male in $w . g(\xi)$
d. $\quad \llbracket \operatorname{her}_{\xi} \rrbracket^{g}=\lambda w_{s}: g(\xi)$ is female in $w . g(\xi)$
etc.

- Note that Functional Application only projects the presuppositions triggered in the function $\beta$. If $\gamma$ is itself a function and triggers a presupposition, the meaning of $\beta$ needs to specify how to project it, e.g.

$$
\begin{equation*}
\llbracket \text { everybody } \rrbracket^{g}=\lambda w_{s} \cdot \lambda P_{\langle e, t\rangle}: \text { for each person } x \text { in } w, x \in \operatorname{dom}(P) \text {. for each person } x \text { in } w, P(x)=1 \tag{34}
\end{equation*}
$$

- Textbooks like Heim \& Kratzer 1998 hide this complication.
- Eventually we want to have a more general, predictive theory of presupposition projection, so that we won't have to lexically specify projection rules like (34) (cf. Heim 1983, Schlenker 2008, 2009, etc.).

For the wh-complementiser, let us for now assume that it is a 'presupposition plug', i.e., it doesn't project a presupposition of the TP denotation $q$.

$$
\begin{equation*}
\llbracket \mathbf{C}_{\mathbf{w h}} \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} \cdot \lambda q_{\langle s, t\rangle} \cdot q=p \tag{35}
\end{equation*}
$$

But alternatively, we could project it like (36). This could make differences when we talk about pragmatics.

$$
\begin{equation*}
\llbracket \mathbf{C}_{\mathbf{w h}} \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} \cdot \lambda q_{\langle s, t\rangle}: w \in \operatorname{dom}(q) \cdot q=p \tag{36}
\end{equation*}
$$

### 4.3 Total reconstruction

Heim 2012 proposes that the NP-restrictor of which reconstructs, and which is a unary existential quantifier.
(37) Which cat did Chonsky see?

a. $\quad{ }^{\text {r THE }} \mathbf{c a t} \mathbf{w}_{\mathbf{w}}$ IDENT $\mathbf{x}^{\top} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$
(i) iff there is exactly one cat in $g(\mathbf{w})$ that is identical to $g(\mathbf{x})$
(ii) iff $g(\mathbf{x})$ is a cat in $g(\mathbf{w})$
b. If ${ }^{\text {r THE }} \mathbf{~ c a t ~} \mathbf{w}_{\mathbf{w}}$ IDENT $\mathbf{x}^{`} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right), \llbracket$ THE cat $\mathbf{t}_{\mathbf{w}}$ IDENT $\mathbf{x} \rrbracket^{g}=g(\mathbf{x})$
a. $\llbracket \lambda \mathbf{w}_{s}^{\prime}$ Chomsky $^{\text {saw }_{\mathbf{w}^{\prime}}}$ THE cat IDENT $_{\mathbf{w}} \rrbracket^{g}$
$=\lambda w_{s}^{\prime}: g(\mathbf{x})$ is a cat in $g(\mathbf{w})$. Chomsky saw $g(\mathbf{x})$ in $w^{\prime}$
b. $\llbracket \lambda \mathbf{x}_{e} \mathbf{C}_{\mathbf{w h}} \mathbf{p} \lambda \mathbf{w}_{s}^{\prime}$ Chomsky saw $\mathbf{w}^{\prime}$ THE cat $\mathbf{w}_{\mathbf{w}}$ IDENT $\mathbf{x} \rrbracket^{g}$
$=\lambda x_{e} . g(\mathbf{p})=\lambda w_{s}^{\prime}: x$ is a cat in $g(\mathbf{w})$. Chomsky saw $x$ in $w^{\prime}$

$$
\begin{equation*}
\llbracket \mathbf{w h i c h} \rrbracket^{g}=\lambda P_{\langle e, t\rangle}:|\operatorname{dom}(P)|>0 \text {. for some } x \in D_{e}, P(x)=1 \tag{40}
\end{equation*}
$$

NB: This entry existentially projects the presupposition of $P$, whereby giving rise to the Binding Problem: (If the presupposition is independent from the assertion) the two dimensions of meaning will be both existentially quantified and independent from each other. We can't solve this problem in the framework we are assuming here.

$$
\begin{equation*}
\llbracket(37) \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} . \text { for some } x \in D_{e}, p=\lambda w_{s}^{\prime}: x \text { is a cat in } w \text {. Chomsky saw } x \text { in } w^{\prime} \tag{41}
\end{equation*}
$$

For any possible world $w, \llbracket(37) \rrbracket^{g}(w)$ characterises:

$$
\begin{equation*}
\left\{\lambda w_{s}^{\prime}: x \text { is a cat in } w . \text { Chomsky saw } x \text { in } w^{\prime} \mid x \in D_{e}\right\} \tag{42}
\end{equation*}
$$

- These partial propositions are almost the same as Rullmann \& Beck 1998.
- One difference is that the set in (42) contains a pathological proposition $p \in D_{\langle s, t\rangle}$ such that $\operatorname{dom}(p)=\varnothing$, unless everything in the model, including Chomsky, is a cat in $w$ (But if a cat has a tail, that's enough to give rise to a pathological proposition!). Let's denote the pathological proposition by $\perp$.
- For those entities $x$ that are cats in $w$, the proposition will be total (because the presupposition is not about $w^{\prime}$ ). Assuming that the model contains something that is not a cat in $w$, then the set is equivalent to (43).

$$
\begin{equation*}
\left\{\lambda w_{s}^{\prime} . \text { Chomsky saw } x \text { in } w^{\prime} \mid x \text { is a cat in } w\right\} \cup\{\perp\} \tag{43}
\end{equation*}
$$

- We could also have $\mathbf{w}^{\prime}$ on cat, in which case the set of propositions will be independent of $w$ :

$$
\left\{\lambda w_{s}^{\prime}: x \text { is a cat in } w^{\prime} \text {. Chomsky saw } x \text { in } w^{\prime} \mid x \in D_{e}\right\}
$$

### 4.4 Functional readings

Heim 2012 accounts for bound pronouns in which-phrases as follows. Following Engdahl 1986, she uses a type-flexible denotation for which, and a functional trace.
(44) Which relative of his does every man hate?
(45)

(46) a. $\quad$ 'THE relative $\mathbf{w}_{\mathbf{w}}$ of his $_{\mathbf{y}}$ IDENT $\mathbf{f} \mathbf{y}^{\text {' }} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$
iff $g(\mathbf{y})$ is male in $g(\mathbf{w})$ and there is exactly one relative of $g(\mathbf{y})$ 's in $g(\mathbf{w})$ that is identical to $g(\mathbf{f})(g(\mathbf{y}))$
iff $g(\mathbf{y})$ is male in $g(\mathbf{w})$ and $g(\mathbf{f})(g(\mathbf{y}))$ is a relative of $g(\mathbf{y})$ 's in $g(\mathbf{w})$
b. If 'THE relative $\mathbf{w}_{\mathbf{w}}$ of his $\mathbf{y}_{\mathbf{y}} \operatorname{IDENT} \mathbf{f} \mathbf{y}^{\prime} \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{g}\right)$, then $\llbracket$ THE relative $\mathbf{w}_{\mathbf{w}}$ of his hid $_{\mathbf{y}}$ IDENT $\mathbf{f} \mathbf{y} \rrbracket^{g}=$ $g(\mathbf{f})(g(\mathbf{y}))$
(47) $\llbracket \lambda \mathbf{y}_{e} \mathbf{T}^{\text {hate }} \mathbf{w}_{\mathbf{w}^{\prime}}$ THE relative $\mathbf{w}_{\mathbf{w}}$ of his $\mathbf{y}_{\mathbf{y}}$ IDENT $\mathbf{f} \mathbf{y} \rrbracket^{g}$
$=\lambda y_{e}: y$ is male in $g(\mathbf{w})$ and $g(\mathbf{f})(y)$ is a relative of $y$ 's in $g(\mathbf{w}) . y$ hates $g(\mathbf{f})(y)$ in $g\left(\mathbf{w}^{\prime}\right)$
(48) a. $\quad$ 'every $\boldsymbol{m a n}_{\mathbf{w}^{\prime}} \in \operatorname{dom}\left(\llbracket[]^{g}\right)$ iff there is a man in $g\left(\mathbf{w}^{\prime}\right)$
b. If 'every $\left.\operatorname{man}_{\mathbf{w}^{\prime}} \in \operatorname{dom}(\llbracket \cdot]^{g}\right)$, then $\llbracket$ every $\operatorname{man}_{\mathbf{w}^{\prime}} \rrbracket^{g}$
$=\lambda P_{\langle e, t\rangle}$ : for each man $x$ in $g\left(\mathbf{w}^{\prime}\right), x \in \operatorname{dom}(P)$. for each man $x$ in $g\left(\mathbf{w}^{\prime}\right), P(x)=1$

$$
=\lambda f_{\langle e, e\rangle} . g(\mathbf{p})=\lambda w_{s}^{\prime}:\left[\begin{array}{l}
\text { there is a man in } w^{\prime} \text { and } \\
\text { for every man } x \text { in } w^{\prime}, \\
x \text { is male in } g(\mathbf{w}) \text { and } \\
g(\mathbf{f})(x) \text { is a relative of } x \text { 's in } g(\mathbf{w})
\end{array}\right] \cdot\left[\begin{array}{l}
\text { for every man } x \text { in } w^{\prime}, \\
x \text { hates } g(\mathbf{f})(x) \text { in } w^{\prime}
\end{array}\right]
$$

$$
\begin{equation*}
\llbracket \text { which }_{\langle e, e\rangle} \rrbracket^{g}=\lambda P_{\langle\langle e, e\rangle, t\rangle}:|\operatorname{dom}(P)|>0 \text {. for some } f \in D_{\langle e, e\rangle}, P(f)=1 \tag{51}
\end{equation*}
$$

More generally:

$$
\begin{align*}
& \llbracket \mathbf{w h i c h}_{\tau} \rrbracket^{g}=\lambda P_{\langle\tau, t\rangle}:|\operatorname{dom}(P)|>0 \text {. for some } v \in D_{\tau}, P(v)=1  \tag{52}\\
& \llbracket(45) \rrbracket^{g}=\lambda w_{s} \cdot \lambda p_{\langle s, t\rangle} .  \tag{53}\\
& \text { for some } f \in D_{\langle e, e\rangle}, p=\lambda w_{s}^{\prime}:\left[\begin{array}{l}
\text { there is a man in } w^{\prime} \text { and } \\
\text { for every man } x \text { in } w^{\prime}, \\
x \text { is male in } w \text { and } \\
f(x) \text { is a relative of } x \text { s in } w
\end{array}\right] \cdot\left[\begin{array}{l}
\text { for every man } x \text { in } w^{\prime}, \\
x \text { hates } f(x) \text { in } w^{\prime}
\end{array}\right]
\end{align*}
$$

This is almost the same as Engdahl 1986 except that the meaning of the restrictor figures in the presupposition of the question.

## 5 Functional vs. pair-list answers

Groenendijk \& Stokhof (1984: Ch. 3) identify three types of answers:
(54) Which woman does every man love?
(Groenendijk \& Stokhof 1984: p. 168)
a. Mary.
b. John loves Mary, Bill loves Suzy, ...
(individual answer)
(pair-list answer)
c. His mother.
(functional answer)
The functional answer is so-called because it seems to specify a function, rather than a particular individual. In the case of (54c), the function is meant to be [ $\lambda x_{e} . x^{\prime}$ 's mother], and the answer indicates that every man $x$ loves $\left[\lambda x_{e}\right.$. $x$ 's mother $](x)$.

It's easy to see that the individual answer is different from the other two. With a bound pronoun, an individual answer is not possible (unless all the relevant men all share relatives).
(55) Which relative of $\operatorname{his}_{i}$ does [every man] ${ }_{i}$ love?
a. \#Mary.
b. John loves his mother, Bill loves his father, ...
c. His mother.
(individual answer)
(pair-list answer)
(functional answer)

The distinction between pair-list and functional answers might not be obvious. Suppose the following son-mother relation:

| son | mother |
| :--- | :--- |
| Andy | Ann |
| Bob | Becky |
| Chris | Cate |
| Dan | Dorothy |

Suppose also that all these men love their mother. Then the following two answers seem to convey the exact same information extensionally.
(56) Which woman does every man love?
a. Andy loves Ann, Bob loves Becky, Chris loves Cate, and Dan loves Dorothy.
b. His mother.
(Groenendijk \& Stokhof 1984: p. 176f) remark as follows:

With many others, we believed for a long time that answers like his mother to questions like [(56)] are just a kind of abbreviation, a more economic way of expressing pair-list answers. [...] But can functional answers and pair-list ones really always be equated? There seem to be several reasons to doubt this.

Groenendjik \& Stokhof give three reasons that functional and pair-list answers are distinct readings that need to be captured with different denotations.

### 5.1 Argument 1

Someone who gives a functional answer might not be able to give a pair-list answer, and vice versa (see also Engdahl 1986, Heim 2012).
(57) John knows which woman every man loves.
a. John knows that every man loves his mother, but doesn't know who is the mother of who. $\quad \Rightarrow$ Only the functional reading is TRUE
b. John knows which man loves which woman, but doesn't know the fact that for each man, the woman he loves is his mother. $\quad \Rightarrow$ Only the pair-list reading is TRUE

I find this argument unconvincing. Maybe there is only one reading (i.e. one type of semantic object) that could be true in both of these situations? Notice that the same kind of 'ambiguity' obtains with individual answers.
(58) John knows which woman every Dutchman loves.
a. John knows that every Dutchman loves Beatrix. He's unaware that she is the former Queen of the Netherlands.
b. John knows that every Dutchman is crazy about the Dutch royal family and so loves the former Queen of the Netherlands, but he doesn't know his name.

Does this imply that there are two different individual-answer 'readings' of the embedded question? Not necessarily. Aloni (2001) points out that in some contexts not all extensionally equivalent individual answers are equally appropriate. ${ }^{1}$
(59) Context: It's 2001 and Your daughter Priscilla is doing her homework. She asks you:

Q: "Who is the president of Mali?"
$\mathrm{A}_{1}$ : "Konaré."
$\mathrm{A}_{2}$ : You fly to Mali, kidnap Konaré, bring him in your living room, and say "This guy." (adapted from Aloni 2001: p. 9)

Similarly: Suppose that Alex, a basketball player, scored the most in one game. You know nothing about basketball players, so I don't want to use names to talk about different players, and instead show you some pictures, one of them is a selfie I took with Alex after the match.
(60) Q1: Who scored the most?

Q2: Who did you take a selfie with after the match?
(61) A1: The player I took a selfie with.

A2: The player that scored the most.

[^0]Aloni also observes that this consideration applies to embedded questions as well, as illustrated by the following examples taken from Aloni (2001: p. 11)
(62) a. Someone killed Spiderman. You are at the Police department. you have just discovered that John Smith is the culprit. So you say:
John Smith did it. So I know who killed Spiderman.
b. You now want to arrest John Smith. He is attending a masked ball. You go there, but you don't know what he looks like. You say:
I don't know who killed Spiderman.
Aloni's idea: What counts as an appropriate answer in a given context depends on its intension (as well as its extension). Intensionally, the player that scored the most and the player I took a selfie with are distinct.

Coming back to functional vs. pair-list answers, I think the following story is not far-fetched:

- In some contexts, a pair-list answer is appropriate; in other contexts, a functional answer is appropriate.
- They may identify the same list, but they do so in intensionally distinct ways, just like a proper name and a demonstrative may identify the same individual but in intensionally different ways.


### 5.2 Argument 2

A complete pair-list answer and a complete functional answer can be different, e.g., Suppose John loves his mother Mary, Bill loves his mother Suzy and John's mother Mary, and Peter loves his mother Jane.
(63) Which woman does every man love?
a. Partial pair-list answer: John loves Mary, Bill loves Suzy, and Peter loves Jane.
b. Complete functional answer: His mother.

Groenendijk \& Stokhof claim that there's a sense in which (63a) is a partial answer but (63b) sounds like a complete answer, although they convey the same amount of information.

One confound: Is the singular marking on woman compatible with the complete pair-list answer? To control for this, let's make everything plural. Suppose that there are three Dutch men, Jeroen, Martin, and Adriaan. They are all married, and love their wife. They also love their mother. Jeroen and Martin love the former Queen Beatrix, but Adriaan doesn't.
(64) Which women does every Dutch man love?
a. Jeroen loves his wife Jenny and his mother Ineke, Martin loves his wife Maartje and his mother Maaike, and Adriaan loves his wife Anne and his mother Abby.
b. His wife and mother.
(64a) is a partial answer (doesn't mention Beatrix!), but (64b) sounds like a complete answer in this context.'

### 5.3 Argument 3

Certain quantifiers do not allow pair-list answers, while allowing functional answers.
(65) Which woman does $\left\{\begin{array}{l}\text { no man } \\ \text { few men } \\ \text { many men } \\ \text { a man } \\ \text { some men } \\ \text { most men } \\ \text { at least one man } \\ \text { exactly one man }\end{array}\right\}$ love?
a. Mary.
b. *John loves Mary, Bill loves Suzy, ...
c. *John doesn't love Mary, Bill doesn't love Suzy, ...
d. His/Their mother.

In fact, only universal quantifiers and wh-phrases allow pair-list readings.

### 5.4 Argument 4

One more argument from Preuss (2001): questions that allow for pair-list answers show quantificational variability effects, but those that only allow for functional answers don't.
(66) a. (John knows for the most part which boy likes which girl.)
b. John knows for the most part which girl every boy likes.
c. \#John knows for the most part which girl no boy likes.

### 5.5 Summary

Pair-list and functional answers should be given different analyses. We can regard Heim's 2012 analysis as an analysis of pair-list answers.

For functional answers:

- We want to account for the fact that the functional answer in (65d) cannot be paraphrased by (65b) or (65c), even if they are extensionally equivalent.
- We want to account for the fact that His wife and mother is a complete functional answer in (64).

A common idea is to quantify only over 'natural functions' (Groenendijk \& Stokhof 1984), but there's currently no insightful proposal about what counts as a natural function. It needs to be an intensional notion.

## References

Aloni, Maria. 2001. Quantification under conceptual covers. Universiteit van Amsterdam dissertation.
Engdahl, Elisabet. 1986. Constituent questions: The syntax and semantics of questions with special reference to Swedish. Dordrecht: D. Reidel.
von Fintel, Kai \& Irene Heim. 2021. Intensional semantics. Lecture notes, Massachusetts Institute of Technology.
Fox, Danny. 2000. Economy and semantic interpretation. Cambridge, MA: MIT Press.
Groenendijk, Jeroen \& Martin Stokhof. 1984. Studies on the semantics of questions and the pragmatics of answers. University of Amsterdam dissertation.

Heim, Irene. 1982. The semantics of definite and indefinite noun phrases. University of Massachusetts, Amherst dissertation.
Heim, Irene. 1983. On the projection problem for presuppositions. In Michael Barlow, Daniel P. Flickinger \& Michael Wescoat (eds.), Proceedings of WCCFL 2, 114-125. Stanford, CA: CSLI.
Heim, Irene. 2012. Functional readings without type-shifted noun phrases. Ms., MIT.
Heim, Irene \& Angelika Kratzer. 1998. Semantics in Generative Grammar. Oxford: Blackwell.
Percus, Orin. 2000. Constraints on some other variables in syntax. Natural Language Semantics 8(3). 173-229.
Preuss, Susanne. 2001. Issues in the semantics of questions with quantifiers. Rutgers University dissertation.
Rullmann, Hotze \& Sigrid Beck. 1998. Presupposition projection and the interpretation of whichquestions. In Devon Strolovitch \& Aaron Lawson (eds.), Proceedings of SALT 8, 215-232. Cornell University.
Schlenker, Philippe. 2008. Be Articulate! A pragmatic theory of presupposition projection. Theoretical Linguistics 34(3). 157-212. https://doi. org/0.1515/THLI. 2008.013.
Schlenker, Philippe. 2009. Local contexts. Semantics and Pragmatics 2(3). 1-78. https://doi . org/ 10.3765/sp.2.3.

Schwarz, Florian \& Ezra Keshet. 2019. De re/de dicto. In Jeanette Gundel \& Barbara Abbott (eds.), The Oxford handbook of reference, 168-202. Oxford University Press. https : / / doi . org/10.1093/ oxfordhb/9780199687305.013.10.


[^0]:    ${ }^{1}$ Alpha Oumar Konaré was President of the Republic of Mali between June 1992 and June 2002.

