1 Quantificational DPs

For the rest of this course, we will discuss expressions called **quantificational determiner phrases (DPs)**. In addition to proper names like 'John' and 'Daniel', English have DPs like the ones in (1).

(1) a. every linguist
b. no angry cat
c. most students
d. few old people

These are syntactically DPs, just like proper names, as quantificational DPs and proper names show the same distributional properties. But as we will see, they have quite different semantic properties.

In this lecture we will explore the meanings of quantificational DPs and quantificational determiners (such as 'every', 'no', 'most', etc). Adopting *Generalized Quantifier Theory*, we will analyze the denotations of quantificational DPs to be **generalized quantifiers**. 'Generalized quantifiers' is just a different name for functions of type $\langle et, t\rangle$. But notice that this means that not all DPs are of type e.

2 Quantificational DPs do not refer to individuals

First we will convince ourselves that the denotations of quantificational DPs cannot be individuals/of type e. If they were individuals, there wouldn't be much difference between the following sentences.

(2) a. John smiled.
b. Every linguist smiled.

But we will see that we need to develop different accounts for these sentences. In particular, the proper name 'John' denotes an individual, but the quantificational DP 'every linguist' does not. Let us see why, by tentatively assuming the contrary, i.e. that quantificational DPs do denote individuals.

2.1 Which Individual?

If we assume that quantificational DPs denote individuals, just like proper names, then a question immediately arises: Which individuals do quantificational DPs denote?

Take, for example, 'every linguist'. Clearly, this phrase is not about a particular individual, but about all linguists. If it were to refer to an individual, then that individual has to somehow represent the totality of all linguists. It is unclear what that individual would look like.

The problem is worse with 'no linguist'. If this were to refer to an individual, that individual should somehow represent no linguist. Also, we would need to make sure that the referents of 'no linguist' and 'no student' could be different individuals, because the following sentences have different truth-conditions.

(3) a. No linguist smiled.
b. No student smiled.

Similarly, what would be an adequate type-*e* denotation for 'few linguists'?

Relatedly, the following passages from *Through the Looking-Glass, and What Alice Found There* by Lewis Carroll, which you might have read, illustrate the inadequacy of the hypothesis we are after. Pay attention to how the White King interprets 'nobody', which I emphasize here:

> 'Just look along the road, and tell me if you can see either of them.'
>
> 'I see **nobody** on the road,' said Alice.
>
> 'I only wish I had such eyes,' the King remarked in a fretful tone. 'To be able to see **Nobody**! And at the distance too! Why, it's as much as I can do to see real people, by this light!'
>
> ....
>
> 'Who did you pass on the road?' the King went on, holding out his hand to the Messenger for some more hay.
>
> '**Nobody**,' said the Messenger.
>
> 'Quite right,' said the King: 'this young lady saw him too. So of course **Nobody** walks slower than you.'
>
> 'I do my best,' the Messenger said in a sullen tone. 'I'm sure **nobody** walks much faster than I do!'
>
> 'He can't do that,' said the King, 'or else he'd have been here first.'

Here's another quote with a similar kind of double-meaning.

> This is a story about four people named Everybody, Somebody, Anybody and Nobody.
>
> There was an important job to be done and Everybody was sure that Somebody would do it.
>
> Anybody could have done it, but Nobody did.
>
> Somebody got angry about this, because it was Everybody's job. Everybody thought Anybody could do it, but Nobody realised that Everybody wouldn't do it.
>
> It ended up with that Everybody blamed Somebody when Nobody did what Anybody could have done.

2.2 Wrong Inferential Patterns

In addition to the question of which individuals they should refer to, quantificational DPs give rise to inferential patterns that are different from proper names. This would be unexpected, if they referred to individuals on a par with proper names.

Take the example in (4). Assuming that everyone speaks at least one language, the following statement is a tautology, i.e. it's guaranteed to be true.

(4) Alex is mono-lingual or Alex is multi-lingual.

Now replace the proper name *Alex* with a quantificational DP 'every linguist'.

(5) Every linguist is mono-lingual or every linguist is multi-lingual.

This is no longer a tautology. In fact, it is false in the actual state of affairs, given that there actually are both mono-lingual and multi-lingual linguists. If ‘every linguist’ denoted an individual, (5) should be tautological, because it should mean that the referent of the quantificational DP is either mono-lingual or multi-lingual, which is a tautology.

Similarly, consider the following pair of sentences.

(6) a. Alex lives in London or in Paris.
b. Alex lives in London or Alex lives in Paris.

These two sentences clearly have the same truth-conditions. But now replace the proper name with ‘every linguist’.

(7) a. Every linguist lives in London or in Paris.
b. Every linguist lives in London or every linguist lives in Paris.

These two sentences are not synonymous, although one of them (which?) entails the other.

Likewise, the sentences in (8) are synonymous, but the sentences in (9) are not.

(8) a. Alex went to Paris and Amsterdam.
b. Alex went to Paris and Alex went to Amsterdam.

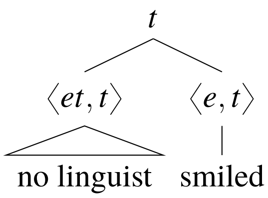
(9) a. No linguist went to Paris and Amsterdam.
b. No linguist went to Paris and no linguist went to Amsterdam.

These considerations point to the conclusion that quantificational DPs do not denote individuals. Then, what do they denote?

3 Generalized Quantifiers

Let us first figure out the semantic types of quantificational DPs. Take a simple sentence like ‘No linguist smiled’. If the quantificational DP is not of type e, there’s only one way to make the semantic composition work, namely, the quantificational DP must be of type $\langle et, t\rangle$.

(10)



This means that it is the quantificational DP that takes the VP as its argument, rather than the other way around.

Functions of type $\langle et, t\rangle$ are called **generalized quantifiers**. Let us now ask which generalized quantifier ‘no linguist’ should denote. In the above sentence, it will take $[\![\text{smiled}]\!]^{a,M}$, which is a function of type $\langle e, t\rangle$. We know the truth-condition of the sentence, i.e. ‘No linguist smiled’ is true if an only if no linguist smiled. Or to put it differently, the sentence is true in model M just in case $[\![\text{smiled}]\!]^{a,M}$ maps each linguist to 0. So, $[\![\text{no linguist}]\!]^{a,M}$ should say that for each linguist, $[\![\text{smiled}]\!]^{a,M}$ returns 0. This can be written as:

$$[\![\text{no linguist}]\!]^{a,M}([\![\text{smiled}]\!]^{a,M}) = 1 \text{ iff for each linguist } x \text{ in } M, [\![\text{smiled}]\!]^{a,M}(x) = 0$$

Abstracting over the VP meaning, we get the following as the denotation of ‘no linguist’.

(11) For any model M and assignment a,

$[\![\text{no linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff for each linguist } x \text{ in } M,\ f(x) = 0]$

There are actually many equivalent ways of stating the body of this function, for example:

(12) For any model M and assignment a,
$[\![\text{no linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff for no linguist } x \text{ in } M,\ f(x) = 1]$

We can give a similar analysis to 'every linguist'. This time, it should say that the VP denotation maps every linguist to 1, instead of 0, because 'Every linguist smiled' is true iff the denotation of 'smiled' maps every linguist to 1.

(13) For any model M and assignment a,
$[\![\text{every linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff for every linguist } x \text{ in } M,\ f(x) = 1]$

Likewise, for 'some linguist':

(14) For any model M and assignment a,
$[\![\text{some linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff for some linguist } x \text{ in } M,\ f(x) = 1]$

and for 'most linguists':

(15) For any model M and assignment a,
$[\![\text{most linguists}]\!]^{a,M} = [\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff for most linguists } x \text{ in } M,\ f(x) = 1]$

So generally, we can state the meaning of a quantificational DP as follows.

(16) For any quantificational DP, QP: For any model M and assignment a,
$[\![\text{QP}]\!]^{a,M} = [\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff for QP } x \text{ in } M,\ f(x) = 1]$

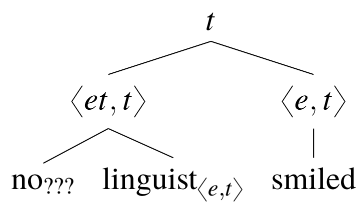
This looks trivial, but the triviality is only apparent, arising due to the fact that we are using English to analyze English (recall the distinction between metalanguage and object language mentioned in Lecture 1). If you analyze quantificational DPs in a different language (using English as your metalanguage), it will look less trivial as we can't state a generalization like (16).

So, in words, quantificational DPs take the VP denotation, which is a function of type $\langle e, t \rangle$ and say what kind of individuals it should map to 1 to make the entire sentence true. In the case of $[\![\text{every linguist}]\!]^{a,M}$, it says that the entire sentence is true, if the VP denotation maps every linguist to 1; $[\![\text{no linguist}]\!]^{a,M}$ says that the entire sentence is true if the VP denotation maps no linguist to 1; $[\![\text{some linguist}]\!]^{a,M}$ says that the entire sentence is true if the VP denotation maps some linguist to 1; and so on.

4 Determiners

From the above analysis of quantificational DPs as generalized quantifiers, we can deduce the denotations of determiners. We know that nouns like *linguist* are of type $\langle e, t \rangle$, so we have the following semantic types.

(17)



We can apply Functional Application, if $[\![\text{no}]\!]^{a,M}$ is of type e, but the output won't be of type $\langle et, t\rangle$, so this is not a viable option. Then, the only possibility is that $[\![\text{no}]\!]^{a,M}$ is a function that takes $[\![\text{linguist}]\!]^{a,M}$ and returns the generalized quantifier $[\![\text{no linguist}]\!]^{a,M}$.

$$[\![\text{no linguist}]\!]^{a,M} = [\![\text{no}]\!]^{a,M}([\![\text{linguist}]\!]^{a,M})$$

This means that $[\![\text{no}]\!]^{a,M}$ is of type $\langle et, \langle et, t\rangle\rangle$.

Now, let's figure out which function of type $\langle et, \langle et, t\rangle\rangle$ it is. Replacing $[\![\text{no linguist}]\!]^{a,M}$ and $[\![\text{linguist}]\!]^{a,M}$ in the above equation, we get:

$$\begin{aligned}[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for no linguist } x \text{ in } M,\ f(x) = 1] &= [\![\text{no}]\!]^{a,M}([\![\text{linguist}]\!]^{a,M}) \\ &= [\![\text{no}]\!]^{a,M}([\lambda x \in D_e.\ 1 \text{ iff } x \text{ is a linguist in } M])\end{aligned}$$

In words, $[\![\text{no}]\!]^{a,M}$ takes the type-$\langle e, t\rangle$ function $[\lambda x \in D_e.\ 1 \text{ iff } x \text{ is a linguist in } M]$ and returns a generalized quantifier on the left of $=$. Abstracting over this particular function, we get the following as the denotation of 'no'.

(18) For any model M, and any assignment a,
$[\![\text{no}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for no individual } x \text{ such that } g(x) = 1,\ f(x) = 1]]$

Here g is the NP-denotation and f is the VP-denotation. g determines which individuals the generalized quantifier will be about, namely the individuals that g map to 1. Call these individuals g-individuals. The determiner says how many of the g-individuals f needs to map to 1 to make the sentence true. In this case, f maps zero g-individuals to 1, the sentence will be true.

It is easy to generalize this analysis to other quantificational determiners.

(19) For any model M, and any assignment a,

a. $[\![\text{every}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for every individual } x \text{ such that} \\ g(x) = 1, f(x) = 1\end{array}\right]\right]$

b. $[\![\text{some}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for some individual } x \text{ such that} \\ g(x) = 1, f(x) = 1\end{array}\right]\right]$

c. $[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for most individuals } x \text{ such that} \\ g(x) = 1, f(x) = 1\end{array}\right]\right]$

All of these determiners say how many g-individuals f needs to map to 1 to make the sentence true. Specifically, (19a) says the entire sentence will be true if f maps all g-individuals to 1; (19b) says the entire sentence will be true if f maps some g-individuals to 1; (19c) says the entire sentence will be true if f maps most g-individuals to 1.

Note that for 'every', the first argument g will be a singular noun, while for 'most', it is a plural noun, and for 'some', it can be either.

(20) a. Every linguist is tall.
b. Some linguist is tall.
c. *Most linguist is tall.

(21) a. *Every linguists are tall.
b. Some linguists are tall.
c. Most linguists are tall.

We are disregarding the difference between singular and plural nouns. In fact, we haven't really discussed the semantics of plural nouns. For this course, let us assume that they have the same denotations. For example:

(22) $[\![\text{linguist}]\!]^{a,M} = [\![\text{linguists}]\!]^{a,M} = [\lambda x \in D_e.\ x \text{ is a linguist in } M]$

As you can guess, this is inadequate, since, obviously, the number information should have semantic consequences. The singular noun 'linguist' is about one individual, while the plural noun 'linguists' is about more than one individual. However, the semantics of plural nouns gives rise to a set of very complex (and extremely interesting) issues, and we cannot deal with it in this course. So we tentatively assume that singular and plural nouns have the same denotation. Also, we cannot talk about mass nouns, which also give rise to issues related to the semantics of plural nouns.

5 Sets and Their Characteristic Functions

When thinking about generalized quantifiers, the notion of *characteristic functions* becomes handy. Take a function of type $\langle e, t\rangle$, say $[\![\text{smokes}]\!]^{a,M}$.

(23) $[\![\text{smokes}]\!]^{a,M} = [\lambda x \in D_e.\ 1 \text{ iff } x \text{ smokes in } M]$

This function maps anybody in M to 1 if he or she smokes in M, and to 0 if not. So it divides the inhabitants of M into two groups, smokers and non-smokers. We can represent them as sets:

(24) a. $\{\, x \mid x \in D_e \text{ and } x \text{ smokes in } M \,\}$
b. $\{\, x \mid x \in D_e \text{ and } x \text{ doesn't smoke in } M \,\}$

$[\![\text{smokes}]\!]^{a,M}$ maps the individuals in (24a) to 1 and the individuals in (24b) to 0.

In this situation, we say $[\![\text{smokes}]\!]^{a,M}$ *characterizes* the set in (24a). Or equivalently, $[\![\text{smokes}]\!]^{a,M}$ is the *characteristic function* of the set in (24a).

Generally, any function of type $\langle e, t\rangle$ characterizes some set. More precisely, any function f of type $\langle e, t\rangle$ characterizes the set $\{\, x \mid x \in D_e \text{ and } f(x) = 1 \,\}$. Generalizing this further, we can say that each function f of type $\langle \sigma, t\rangle$ characterzes the set $\{\, x \mid x \in D_\sigma \text{ and } f(x) = 1 \,\}$ for any semantic type σ. Keep in mind that only functions of type $\langle \sigma, t\rangle$ characterize sets. So, for instance, functions of type $\langle e, \langle e, t\rangle\rangle$, like $[\![\text{loves}]\!]^{a,M}$, do not characterize sets. The output type needs to be t.

Correspondingly, when a set whose members are all of the same type is given, e.g. a set of individuals, we can tell which function characterizes the set. For instance, take a set of individuals $\{\, a, b, c \,\}$. This is a set whose members are all of type e. This set is characterized by the function: $[\lambda x \in D_e.\ 1 \text{ iff } x = a \text{ or } x = b \text{ or } x = c]$. More generally, any set of individuals S is characterized by a function of type $\langle e, t\rangle$, namely $[\lambda x \in D_e.\ 1 \text{ iff } x \in S]$. Generalizing over semantic types, any set S whose members are all of type σ is characterized by the function of type $\langle \sigma, t\rangle$, $[\lambda x \in D_\sigma.\ 1 \text{ iff } x \in S]$.[1]

It is important to notice that when any function of type $\langle \sigma, t\rangle$ is given (for any semantic type σ), we can uniquely determine the set it characterizes. If f is a function of type $\langle \sigma, t\rangle$, the set it characterizes is $\{\, x \mid x \in D_\sigma \text{ and } f(x) = 1 \,\}$. It doesn't characterize any other set. Furthermore, for any set whose members are of the same type, there is a unique function that characterizes

[1]We will not talk about sets whose members are not of a uniform type. Technically it is possible to define characteristic functions of such sets, but such sets and their characteristic functions do not play a role in our semantic theory.

it. Specifically, for a set S whose members are all of type σ, i.e. $S \subseteq D_\sigma$, its characteristic function is $[\lambda x \in D_\sigma.\ 1 \text{ iff } x \in S]$, and no other function of type $\langle \sigma, t \rangle$ characterizes S. Notice also that for any function f of type $\langle \sigma, t \rangle$, the characteristic function of the set f characterizes is f itself (maybe not surprisingly).

This means that there is a *one-to-one correspondence* between functions of type $\langle \sigma, t \rangle$ and the sets they characterize. If a function f of type $\langle \sigma, t \rangle$ is given, one can uniquely determine which set it characterizes and from that set, one can reconstruct the function f. This is important, because it means that these two kinds of objects, functions and sets, actually carry the same amount of information. That is to say, although formally distinct, they are indistinguishable at some level, and we can treat them as the 'same thing' for certain purposes.

Now, going back to the above examples, we can regard the denotation of 'smokes', $[\![\text{smokes}]\!]^{a,M}$, as either a function of type $\langle e, t \rangle$ or alternatively as the set in (24a). Likewise, we defined $[\![\text{boy}]\!]^{a,M}$ and $[\![\text{blond}]\!]^{a,M}$ as functions of type $\langle e, t \rangle$, but alternatively we can treat them as the set of boys in M, and the set of blond people in M, respectively. Such 'set talk' is harmless, because we are not losing any information by converting these type-$\langle e, t \rangle$ functions into sets. We can always go back to the original functions, as ensured by the one-to-one correspondence.

But why do we do this? Because it allows us to look at the same thing from a different angle and it can be quite informative, as we will see below.

Before moving on, let us introduce some notations. For any function f of type $\langle \sigma, t \rangle$, we denote the set it characterizes by $\mathsf{set}(f)$. Similarly, for any set S such that $S \subseteq D_\sigma$, we denote its characteristic function by $\mathsf{func}(S)$. To stress the main point, we can regard f and $\mathsf{set}(f)$ as the 'same thing', because $f = \mathsf{func}(\mathsf{set}(f))$, and similarly, S and $\mathsf{func}(S)$ as the 'same thing' because $S = \mathsf{set}(\mathsf{func}(f))$.

6 Determiners as Relations between Sets

Recall the denotations of quantificational determiners.

(25) For any model M, and any assignment a,

a. $$[\![\text{every}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t \rangle}.\ \left[\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for every individual } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$$

b. $$[\![\text{no}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t \rangle}.\ \left[\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for no individual } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$$

c. $$[\![\text{some}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t \rangle}.\ \left[\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for some individual } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$$

d. $$[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t \rangle}.\ \left[\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for most individuals } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$$

Let's re-state these in terms of sets. The determiners themselves are of type $\langle et, \langle et, t \rangle \rangle$ so they don't characterize sets. But their arguments are of type $\langle e, t \rangle$, so they characterize sets of individuals.

Take the denotation of 'every' in (25a). We can rewrite it using sets as follows.

(26) For any model M, and any assignment a,

$$[\![\text{every}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t \rangle}.\ \left[\lambda f \in D_{\langle e,t \rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for every individual } x \text{ such that}\\ x \in \mathsf{set}(g), x \in \mathsf{set}(f)\end{array}\right]\right]$$

This is not so different from the representation in (26a). But now notice that this is essentially saysing that $\mathsf{set}(g)$ is a subset of $\mathsf{set}(f)$, because every member of the former is a member of the latter. So we can write the denotation more economically as follows.

(27) For any model M, and any assignment a,
$[\![\text{every}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \subseteq \mathsf{set}(f)]]$

So 'every' expresses the subset relation. This is intuitively correct. If you take a concrete example like 'Every linguist smiled', its truth-condition can be paraphrased as 'The set of linguists is a subset of the set of people who smiled'.

What about 'some'? Using the sets, we have:

(28) For any model M, and any assignment a,
$[\![\text{some}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for some individual } x \text{ such that}\\ x \in \mathsf{set}(g), x \in \mathsf{set}(f)\end{array}\right]\right]$

This means that $\mathsf{set}(g)$ and $\mathsf{set}(f)$ have some member in common. So using symbols from Set Theory, we can re-state it as:

(29) For any model M, and any assignment a,
$[\![\text{some}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \cap \mathsf{set}(f) \neq \varnothing]]$

Take a concrete example, say, 'Some boy is blond'. This is indeed paraphrased by 'The set of boys and the set of blond people have a non-empty intersection.'

'No' is essentially the converse of 'some'. It says that the intersection is empty.

(30) For any model M, and any assignment a,
$[\![\text{no}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \cap \mathsf{set}(f) = \varnothing]]$

Again, consider a concrete example. 'No semanticist is left-handed'. This is the same as 'The intersection of the set of semanticists and the set of left-handed people is empty'.

The meaning of 'most' is more complex, but it is possible to express it in terms of sets as well. Let us first simply re-write the denotation we came up with earlier using sets:

(31) For any model M, and any assignment a,
$[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for most individuals } x \text{ such that}\\ x \in \mathsf{set}(g), x \in \mathsf{set}(f)\end{array}\right]\right]$

Let us now consider an example, say 'Most dogs are brown'. When is this true? In terms of sets, it means: 'The majority of members of the set of dogs are also members of the set of brown things.' That is, more members of the set of dogs are in the set of brown things, than not. We can express this using symbols from Set Theory as follows. Recall that $|A|$ is the cardinality of the set A, which is the number of distinct members of A, and $A - B$ is the complement of A relative to B, defined as $\{\, x \mid x \in A \text{ and } x \notin B \,\}$.

$$|\,\{\, x \mid x \text{ is a dog} \,\} \cap \{\, x \mid x \text{ is brown} \,\}\,| > |\,\{\, x \mid x \text{ is a dog} \,\} - \{\, x \mid x \text{ is brown} \,\}\,|$$

The left-hand side of $>$ is the cardinality of the set of brown dogs, and the right-hand is the cardinality of the set of non-brown dogs. Now using this, we can re-write (31) as follows.

(32) For any model M, and any assignment a,
$[\![\text{most}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| > |\mathsf{set}(f) - \mathsf{set}(g)|]]$

In words, $|\mathsf{set}(g) \cap \mathsf{set}(f)|$ is the number of common members of g and f. In our example, this is the number of brown dogs. $|\mathsf{set}(f) - \mathsf{set}(g)|$ is the number of members of f that are not in

g. In our example, this is the number of dogs that are not brown. And the inequality says that the former number is greater than the latter.

We can write the same relationship in different ways. For instance, the following means the same thing as above.

(33) For any model M, and any assignment a,
$$[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } |\mathsf{set}(g) \cap \mathsf{set}(f)| > \frac{|\mathsf{set}(f)|}{2}\right]\right]$$

If the number of brown dogs is more than half the number of all dogs, the majority of the dogs must be brown. So this is actually the same statement as before. You can further transform it to:

(34) For any model M, and any assignment a,
$$[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \frac{|\mathsf{set}(g) \cap \mathsf{set}(f)|}{|\mathsf{set}(f)|} > \frac{1}{2}\right]\right]$$

These are all equivalent.

A note on the meaning of 'most': According to the present analysis, it means the same thing as 'more than half', but you might have a quibble about that. In fact, if 53 out of 100 dogs are brown, it's a bit strange to say 'Most of the dogs are brown', if not outright false, while 'More than half of the dogs are brown' is true. So 'most' might require the fraction $\frac{|\mathsf{set}(g)\cap\mathsf{set}(f)|}{|\mathsf{set}(f)|}$ to be much larger than $\frac{1}{2}$. But at the same time, we do not have clear intuitions about which fraction it needs to be larger than. We just can't really name such a threshold fraction for 'Most dogs are brown'. Rather, it seems to be inherently vague. Our semantic system so far has no resources to deal with vague expressions like this, because in our semantics, every statement is clearly either true or false. To deal with this and related issues of 'vagueness', we need to enrich our model. Several such ideas have been proposed, but this is outside of the scope of this course.

7 Summary

We analyzed quantificational DPs as Generalized Quantifiers, which are functions of type $\langle et, \langle et, t\rangle\rangle$.

(35) For any model M and assignment a,

a. $[\![\text{every linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for every linguist } x \text{ in } M,\ f(x) = 1]$

b. $[\![\text{no linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for no linguist } x \text{ in } M,\ f(x) = 1]$

c. $[\![\text{some linguist}]\!]^{a,M} = [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for some linguist } x \text{ in } M,\ f(x) = 1]$

d. $[\![\text{most linguists}]\!]^{a,M} = [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for most linguists } x \text{ in } M,\ f(x) = 1]$

This analysis is general enough to cover all sorts of quantificational DPs.

(36) For any quantificational DP, QP: For any model M and assignment a,
$[\![\text{QP}]\!]^{a,M} = [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff for QP } x \text{ in } M,\ f(x) = 1]$

Based on this, we arrived at the denotations of quantificational determiners.

(37) For any model M, and any assignment a,

a. $$[\![\text{every}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for every individual } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$$

b. $[\![\text{no}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for no individual } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$

c. $[\![\text{some}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for some individual } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$

d. $[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \begin{array}{l}\text{for most individuals } x \text{ such that}\\ g(x) = 1, f(x) = 1\end{array}\right]\right]$

These can be re-rewritten as follows, using sets.

(38) For any model M, and any assignment a,

a. $[\![\text{every}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \subseteq \mathsf{set}(f)]]$

b. $[\![\text{no}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \cap \mathsf{set}(f) = \varnothing]]$

c. $[\![\text{some}]\!]^{a,M} = [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \mathsf{set}(g) \cap \mathsf{set}(f) \neq \varnothing]]$

d. $[\![\text{most}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff } \dfrac{|\mathsf{set}(g) \cap \mathsf{set}(f)|}{|\mathsf{set}(f)|} > \dfrac{1}{2}\right]\right]$

Next week, we'll discuss formal properties of these determiners, and how they might be linguistically relevant.

7.1 One Word Quantificational DPs

English has several words that function as quantificational DPs. They include:

(39) a. somebody, everybody, nobody, anybody
b. something, everything, nothing, anything
c. somewhere, everywhere, nowhere, anywhere
d. somehow, anyhow

We can analyze the ones in (39a) and (39b) as follows. Take 'somebody' as an example. It is made up of the determiner 'some' and '-body'. Essentially, 'somebody' means the same thing as 'some person'. Thus, we can keep the analysis of 'some' above, and simply analyze '-body' as meaning the same thing as person.

(40) Fro any assignment function a and for any model M,
$[\![\text{somebody}]\!]^{a,M} = [\![\text{some person}]\!]^{a,M} = \left[\lambda f \in D_{\langle e,t\rangle}.\ \begin{array}{l}1 \text{ iff for some person } x \text{ in } M\\ f(x) = 1\end{array}\right]$

Similarly for other words in (39a) and (39c).

For the words in (39c) and (39d), as they (can) function as adverbs, we need a theory of how to analyze adverbs, which actually requires a non-trivial extension of the semantics we've been developing here. It is an absolutely fascinating topic, but we don't have enough time to introduce it in this module.

It is also interesting to notice that (39c) and (39d) involve *wh*-words like 'where' and 'how'. Cross-linguistically, it is very common to use *wh*-words to form quantificational words like these. And there must be some deep semantic reason for this. This is another intriguing topic that we need to leave open here.

7.2 Indefinite Article 'a(n)'

And here's one more open issue. Recall that in Lecture 5, we analyzed the indefinite article 'a' as a semantically vacuous item. Recall also that semantically vacuous items denote identity functions.

(41) Fro any assignment function a and for any model M,
$[\![\text{a}]\!]^{a,M} = [\lambda f \in D_{\langle e,t\rangle}.\ f]$

However, (as some of you have pointed out to me), in sentences like (42), we need a different lexical entry for 'a'.

(42) A linguist smiled.

In fact, in (42), 'a' means something very similar to 'some'.

(43) Some linguist smiled.

Although you might feel that (43) has an extra connotation that the identity of the linguist is unknown (which is itself a very interesting phenomenon), the two sentences have very similar truth-conditions, namely, they are true if there is at least one linguist who smiled, and false otherwise. Then, we should analyze 'a' in (42) as a quantificational determiner, as in (44).

(44) Fro any assignment function a and for any model M,
$$[\![\text{a}_{\text{det}}]\!]^{a,M} = \left[\lambda g \in D_{\langle e,t\rangle}.\ \left[\lambda f \in D_{\langle e,t\rangle}.\ \begin{array}{c}\text{1 iff for an individual } x \in D_e \\ \text{such that } g(x) = 1,\ f(x) = 1\end{array}\right]\right]$$
$$= [\lambda g \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ \mathsf{set}(g) \cap \mathsf{set}(f) \neq \varnothing]]$$

Having two lexical entries is certainly theoretically unsatisfactory, so one should try to explain away one of them using the other. We will not attempt to do it here, but if you are interested, this might be a good topic for your Long Essay.