

1 Truth-Conditional Semantics

This course (PLIN3004/PLING218 Advanced Semantic Theory) is an introduction to **formal semantics**. Formal semanticists study natural language semantics using formal/mathematical tools and techniques. It has its roots in philosophy and logic, which study formal/mathematical/logical languages such as Propositional Logic and Predicate Logic. The central tenet of formal semantics is to treat natural languages on a par with such formal languages.

In Lecture 1 we review the concept of **truth-condition** (which you must have learned in other semantics/pragmatics modules). The aspect of meaning pertinent to truth-conditions, or the **truth-conditional meaning**, is what we focus on in this course. It corresponds to what is often called the ‘literal meaning’.

Note that this is not to deny the existence of other types of meaning that natural language is capable of expressing, or to suggest that such meanings are not amenable to the approach we adopt here. We ignore non-truth-conditional meanings due to the limited time.

One of the reasons why we focus on truth-conditional meanings is because our intuitions about them are relatively clear. The clarity of data is very important for us, as we want to build a theory of natural language semantics and test it against empirical data. If the data are unclear, we can’t learn much from them.

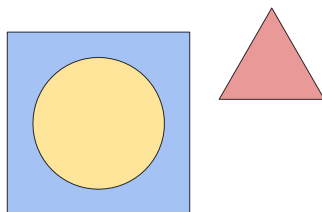
A particularly important fact about truth-conditions is that you, as a native speaker of English (or any other language, for that matter), know the truth-conditions of any declarative sentence in English (or whatever language you are a native speaker of). Let us state this fact as follows: *For any grammatical declarative sentence, a native speaker has truth-conditional intuitions about it, i.e. he or she knows in what situations it is true and in what situations it is false.*

Admittedly, sometimes your intuitions might not be very clear. For example, when is “There are many tall buildings in this city” true and when is it false? But, if you are given the sentence and a particular situation, you (often) can tell whether the sentence is true or false. Throughout this term we primarily look at sentences for which your truth-conditional intuitions are particularly clearcut.

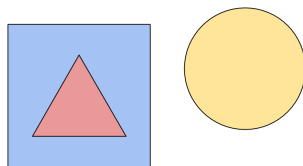
Here is an example illustrating the fact that you have truth-conditional intuitions.

- (1) There is a circle in a square.

If you are given a situation depicted below, you know that the sentence is true.



On the other hand, in the following situation, the sentence is false.



Of course these are not the only situations that you should consider, and in fact there are infinitely many situations where the sentence in (1) is true and infinitely many situations where it is false. However, it is not difficult to see that the sentence in (1) is true in any situation where there is a circle and there is a square and the circle is in the square, and is false in any situations where there isn't a circle in a square. We take this to be a good characterization of the truth-condition of the sentence. We will state the truth-conditions of various sentences in a similar manner.

Because writing such lengthy statements all the time would be too cumbersome, we usually abbreviate truth-conditions using 'iff' (short for 'if and only if'). For example, we write the truth-condition of (1) as follows:

- (2) "There is a circle in a square" is true iff there is a circle in a square.

More generally, the truth-condition of any declarative sentence ϕ can be written as:

- (3) " ϕ " is true iff ϕ .

This might look trivial but it is not. In particular, it is important to distinguish the **object language**—which is the language we study—and **metalanguage**—which is the language we use to state our analysis. The sentence in (3) in the above statement is part of the object language, while the rest belongs to the metalanguage. In our examples so far, we are using English both as our object language and as our metalanguage, so the truth-conditions deceptively look trivial, but consider the following Czech sentence instead:

- (4) Vlk zmrzl, zhltl hrst zrn.

Suppose your job as a semanticist is to figure out the truth-condition of this sentence. You ask your native speaker informants, and find out that the speakers judge it to be true whenever there is a wolf that froze and swallowed a handful of grains. Then, your analysis of the sentence can be stated as:

- (5) "Vlk zmrzl, zhltl hrst zrn" is true iff a wolf froze and swallowed a handful of grains.

As you can see, this is not at all trivial, because it can be a correct or incorrect description of the truth-condition of the sentence.

We take truth-conditional intuitions to be part of the native speaker's linguistic knowledge, on a par with other linguistic intuitions like grammaticality judgments. One of the primary questions in formal semantics is how the truth-conditional intuitions arise.

What is particularly important here is the fact that native speakers have truth-conditional intuitions about *all* grammatical declarative sentences. This is important because it entails that it's impossible to simply memorise all pairs of declarative sentences and their truth-conditions, because there would be infinitely many pairs to memorise, which is not possible!! Rather, there must be a finite mechanism that computes the truth-conditions of arbitrarily complex sentences.

One goal of formal semantics is to develop a finite semantic system that computes the truth-conditions of all grammatical declarative sentences in a given natural language. We'll start with a rather simple system in the next lecture, and gradually enrich it as we go along. In doing so, we learn interesting properties of natural language semantics.

2 Set Theory

Formal semantics uses various logical and mathematical concepts, and you need to be familiar with them. Here we review set theory.

- A **set** is a collection of objects.
 - A set containing $z, 9, \$$ and nothing else is written $\{z, 9, \$\}$.
 - Sets are defined by their **members** (alt.: elements). The members are not ordered. So, $\{z, 9, \$\} = \{9, z, \$\} = \{\$, 9, 9, z, 9, z, 9\}$
 - The set with no members $\{\}$ is called the **empty set** and written \emptyset .
 - A set can contain another set (including the empty set), e.g. $\{z, \emptyset, \{10, 3\}\}$.
 - The size of the set A is called the **cardinality** of A and written $|A|$. e.g. $|\{3, a, \emptyset\}| = 3, |\{7, \emptyset, \text{John}, \{a, 1\}, \$\}| = 5$
- Two ways of defining sets
 - Extensional definition (enumeration): Put $\{\}$ around the members, e.g. $\{d, 1, \emptyset, \text{Bill}\}$. Sets with infinite members cannot be defined this way.
 - Intensional definition (abstraction): Use a variable and state the conditions for the membership, e.g. $\{x \mid x \text{ is an even number}\}$ (\approx the set of x 's such that x is an even number). Sometimes $:$ is used in place of $|$. The variable name does not matter, e.g. $\{x \mid x \text{ is a natural number}\} = \{y \mid y \text{ is a natural number}\} = \mathbb{N}$.
- Relations among sets
 - If every member of A is also a member of B , we say A is a **subset** of B and write $A \subseteq B$. Notice for any set A , $A \subseteq A$.
 - \emptyset is a subset of every set, including \emptyset itself.
 - A and B are equivalent (written $A = B$) iff $A \subseteq B$ and $B \subseteq A$.
 - If $A \subseteq B$ and $A \neq B$, we say A is a **proper subset** of B and write $A \subset B$.
 - The set of all subsets of a set A is called its **power set**, $\wp(A)$ (or sometimes 'Pow(A)') e.g. $\wp(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- Useful operations on sets
 - **Intersection:** $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
 - **Union:** $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
 - **Set subtraction/Relative complementation:** $A - B = \{x \mid x \in A \text{ and } x \notin B\}$