1 λ-conversion

Simplify the following descriptions by λ-conversion.

i) $[\lambda x \in \mathbb{N}.\ x \times 5](12)$

ii) $[\lambda z \in \mathbb{N}.\ [\lambda y \in \mathbb{N}.\ [\lambda x \in \mathbb{N}.\ (x + y) \times z](8)]](5)$

iii) $[\lambda z \in \mathbb{N}.\ [\lambda y \in \mathbb{N}.\ [\lambda x \in \mathbb{N}.\ 1 \text{ iff } x > 3 \text{ and } x < 7](y)](z)]$

iv) $[\lambda x \in D.\ [\lambda y \in D.\ 1 \text{ iff } y \text{ likes } x]](\text{Ann})$

v) $[\lambda x \in D.\ [\lambda y \in D.\ 1 \text{ iff } y \text{ likes } x](\text{Ann})]$

2 Computation

Assume a model M such that $[\![\text{John}]\!]^M = j$ where j is some individual who didn't leave in M. Also, for any model M', $[\![\text{left}]\!]^{M'} = [\lambda x \in D.\ 1 \text{ iff } x \text{ left in } M']$. Then, we can compute the denotation of the sentence 'John left' in M as follows:

$$\left[\!\!\left[\begin{array}{cc} \multicolumn{2}{c}{\text{S}} \\ \text{DP} & \text{VP} \\ | & | \\ \text{John} & \text{left} \end{array} \right]\!\!\right]^M$$

$$= \left[\!\!\left[\begin{array}{c} \text{VP} \\ | \\ \text{left} \end{array} \right]\!\!\right]^M \left(\left[\!\!\left[\begin{array}{c} \text{DP} \\ | \\ \text{John} \end{array} \right]\!\!\right]^M \right) \qquad \text{(by Branching Node Rule)}$$

$$= [\![\text{left}]\!]^M \left(\left[\!\!\left[\begin{array}{c} \text{DP} \\ | \\ \text{John} \end{array} \right]\!\!\right]^M \right) \qquad \text{(by Non-Branching Node Rule)}$$

$$= [\![\text{left}]\!]^M([\![\text{John}]\!]^M) \qquad \text{(by Non-Branching Node Rule)}$$

$$= [\![\text{left}]\!]^M(j) \qquad \text{(because } [\![\text{John}]\!]^M = j\text{)}$$

$$= [\lambda x \in D.\ 1 \text{ iff } x \text{ left in } M](j) \qquad \text{(Lexicon)}$$

$$= 0 \qquad (\lambda\text{-conversion})$$

Compute the denotation of "Mary smokes" in M. Assume that $[\![\text{Mary}]\!]^M = m$, an individual who smokes in M, and for any model M', $[\![\text{smokes}]\!]^{M'} = [\lambda x \in D.\ 1 \text{ iff } x \text{ smokes in } M']$. You may use the bracket notation instead of tree diagrams.