1 Monotonicity

Another linguistically important property is *monotonicity*. This is a property that holds for many different types of functions but we will for now focus on the versions that apply to determiner denotations, i.e. type-$\langle et, \langle et, t\rangle\rangle$ functions. There are several variants of monotonicity. We will discuss them in turn.

1.1 Right Upward Monotonicity

Let us start with right upward monotonicity.

(1) A function $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is *right upward monotonic* iff for any functions $f, g, g' \in D_{\langle e,t\rangle}$ such that for each $x \in D_e$, if $g(x) = 1$ then $g'(x) = 1$, whenever $Q(f)(g) = 1$, $Q(f)(g') = 1$.

Let us re-state this in terms of sets. Take two functions $g, g' \in D_{\langle e,t\rangle}$ such that for each $x \in D_e$ such that $g(x) = 1$, we also have $g'(x) = 1$. This means set$(g) \subseteq$ set(g'). The above definition says, Q is right-upward monotonic, $Q(f)(g)$ entails $Q(f)(g')$ for any g' such that set$(g) \subseteq$ set(g').

This property is about the argument on the right, i.e. the VP denotation, so it is called *right* upward monotonicity. And it is upward, because you can replace the set set(g) with a superset of it, set(g'), while preserving the truth. The analogy here is that sets become bigger as you go upwards.

Here are some concrete examples. $[\![\text{every}]\!]^M$ is right upward monotonic.

(2) Every linguist is British.

Notice that set($[\![\text{British}]\!]^M$) $\subseteq$ set($[\![\text{European}]\!]^M$), or in other words, for each $x \in D_e$ such that $[\![\text{British}]\!]^M(x) = 1$, $[\![\text{European}]\!]^M(x) = 1$. Observe that (2) entails (3).

(3) Every linguist is European.

It is important to keep in mind that in checking monotonicity with concrete examples like these, you have to keep the NP part (e.g. 'linguist' in the above examples) constant across the two sentences. In the definition of right upward monotonicity in (1), the first argument f, which is the NP denotation, is held constant.

$[\![\text{some}]\!]^M$ is another right-upward monotonic determiner. This is illustrated by the entailment from (4a) to (4b).

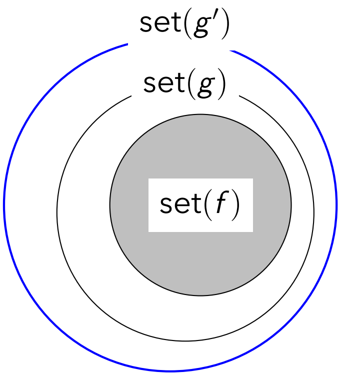
(4) a. Some linguist is British.
 b. Some linguist is European.

Keep in mind that in order to check monotonicity, you need to check all such sentences, and it is not sufficient to show the entailment with one pair to prove that a given determiner is right upward monotonic. Since there are in principle infinitely many such sentences, it is actually not possible to go through all examples. But instead, we can 'prove' right upward monotonicity of these determiners analytically as follows. Recall the set denotations of these determiners:

(5) a. $[\![\text{every}]\!]^M = [\lambda f \in D_{\langle e,t\rangle}. [\lambda f \in D_{\langle e,t\rangle}. 1 \text{ iff set}(f) \subseteq \text{set}(g)]]$
b. $[\![\text{some}]\!]^M = [\lambda f \in D_{\langle e,t\rangle}. [\lambda f \in D_{\langle e,t\rangle}. 1 \text{ iff set}(f) \cap \text{set}(g) \neq \varnothing]]$

Right upward monotonicity says, whenever $Q(f)(g) = 1$, we also have $Q(f)(g') = 1$ provided set$(g) \subseteq$ set(g'). Let us apply this for $Q = [\![\text{every}]\!]^M$. Suppose that $[\![\text{every}]\!]^M(f)(g) = 1$ for some arbitrary f and g. Then, we have set$(f) \subseteq$ set(g), because that's what the sentence states. Then for any g' such that set$(g) \subseteq$ set(g'), set$(f) \subseteq$ set(g') is also the case, because $A \subseteq B$ means that every member of A is also a member of B, and if every member of B is a member of C, then every member of A must be a member of C as well (or in other words, the subset relation is transitive). This is depicted in the following diagram.

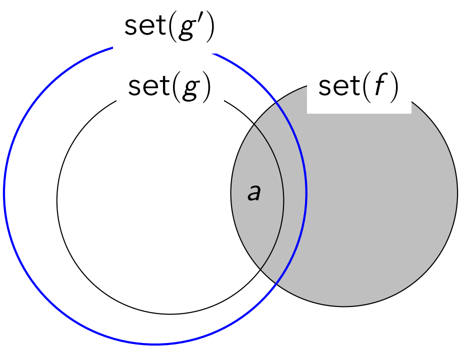
(6)



So we have set$(f) \subseteq$ set(g'). This means $[\![\text{every}]\!]^M(f)(g') = 1$. Since we are talking about arbitrary f and g, this reasoning applies to all NP and VP denotations. Therefore, $[\![\text{every}]\!]^M$ is right upward monotonic.

Similarly, if $[\![\text{some}]\!]^M(f)(g) = 1$, then we have set$(f) \cap$ set$(g) \neq \varnothing$. Then for any g' such that set$(g) \subseteq$ set(g'), we also have set$(f) \cap$ set$(g') \neq \varnothing$. This is because if set$(f) \cap$ set$(g) \neq \varnothing$, there must be at least one member of set$(f) \cap$ set(g). Call one such element a. Because set$(g) \subseteq$ set(g'), i.e. every member of the former is a member of the latter, it must be the case that $a \in$ set(g'). Then, a must be a member of set$(f) \cap$ set(g'). This is depicted in (7).

(7)



So set$(f) \cap$ set$(g') \neq \varnothing$. This means $[\![\text{some}]\!]^M(f)(g') = 1$.

But not all determiners are right upward monotonic. For example, $[\![\text{no}]\!]^M$ is not right upward monotonic. This is easy to demonstrate. Consider (8).

(8) a. No linguist is British.
b. No linguist is European.

Clearly, (8a) does not entail (8b). Concretely, in a situation where there are French linguists but no British linguists, (8a) is true but (8b) is false. Notice that in this case it is sufficient to raise one example to prove that $[\![\text{no}]\!]^M$ is not right upward monotonic. The definition requires entailment to hold for every f, g and g' such that set(g) $\subseteq$ set(g'), so one counter-example is enough to prove that the property does not hold.

Similarly, $[\![\text{exactly two}]\!]^M$ is not right upward monotonic. (9a) does not entail (9b) (what contexts make the former true and the latter false?).

(9) a. Exactly two linguists are British.
b. Exactly two linguists are European.

1.2 Right Downward Monotonicity

Right downward monotonicity is very similar to right upward monotonicity except that it uses subsets instead of supersets in the definition.

(10) A function $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is *right downward monotonic* iff for any functions $f, g, g' \in D_{\langle e,t\rangle}$ such that for each $x \in D_e$, if $g'(x) = 1$ then $g(x) = 1$, whenever $Q(f)(g) = 1$, $Q(f)(g') = 1$.

In terms of sets, we are now talking about those functions g' such that set(g') $\subseteq$ set(g). The idea is that if Q is right downward monotonic, we have entailment towards smaller sets, i.e. downwards. Let's go through some examples.

$[\![\text{No}]\!]^M$ is right downward monotonic. The following example illustrates the entailment pattern. Note that set($[\![\text{violinist}]\!]^M$) $\subseteq$ set($[\![\text{musician}]\!]^M$). Again, keep in mind that you only change the VP and leave everything else in the sentence intact.

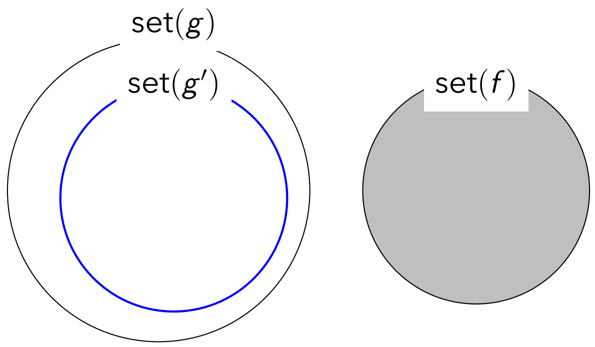
(11) a. No semanticist is a musician.
b. No semanticist is a violinist.

Clearly, if (11a) is true, (11b) needs to be true. But remember that in order to show that $[\![\text{no}]\!]^M$ is right downward monotonic, it is not sufficient to have an entailment between one pair of examples. Rather we need to show that the entailment goes through between all such pairs of sentences. As before, we can do this analytically, using the denotation of 'no' given in (12).

(12) $[\![\text{no}]\!]^M = [\lambda f \in D_{\langle e,t\rangle}.\ [\lambda f \in D_{\langle e,t\rangle}.\ 1 \text{ iff set}(f) \cap \text{set}(g) = \varnothing]]$

Suppose $[\![\text{no}]\!]^M(f)(g) = 1$ for some arbitrary f and g. We want to show that for any g' such that set(g') $\subseteq$ set(g), it follows that $[\![\text{no}]\!]^M(f)(g') = 1$. From the assumption that $[\![\text{no}]\!]^M(f)(g) = 1$, it follows that set(f) $\cap$ set(g) $= \varnothing$, which is to say that set(f) and set(g) are disjoint. Then, if you take any subset of set(g), it will be disjoint with set(f), because if set(g') and set(f) had a common member, that member would belong to set(g) as well, which would contradict the assumption that set(f) $\cap$ set(g) $= \varnothing$. The following diagram illustrates this.

(13)



Therefore, $[\![\text{no}]\!]^M(f)(g') = 1$ follows from $[\![\text{no}]\!]^M(f)(g) = 1$ for any g' such that $\text{set}(g') \subseteq \text{set}(g)$.

Not all determiners are right downward monotonic. Generally, right upward monotonic determiners are not right downward monotonic. The following pairs of sentences demonstrate that 'every' and 'some' are not right downward monotonic. There is no entailment from the (a)-example to the (b)-example (give concrete situations where (a) is true but (b) is false).

(14) a. Every semanticist is a musician.
b. Every semanticist is a violinist.

(15) a. Some semanticist is a musician.
b. Some semanticist is a violinist.

There are also determiners that are neither right upward monotonic nor right downward monotonic. For instance, (16a) does not entail and also is not entailed by (16b).

(16) a. Exactly two semanticists are British.
b. Exactly two semanticists are European.

If there are two British semanticists and one French semanticist, (16a) is true but (16b) is false. Suppose now that there is one British semanticist and one French semanticist, and no one else is a semanticist. Then (16b) is true but (16a) is false.

1.3 Left Upward Monotonicity

Not surprisingly, there are 'left' versions of monotonicity. Left upward monotonicity is defined as (17).

(17) A function $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is *left upward monotonic* iff for any functions $f, f', g \in D_{\langle e,t\rangle}$ such that for each $x \in D_e$, if $f(x) = 1$ then $f'(x) = 1$, whenever $Q(f)(g) = 1$, $Q(f')(g) = 1$.

In terms of sets, if Q is left upward monotonic, $Q(f)(g)$ entails $Q(f')(g)$ for any f' such that $\text{set}(f) \subseteq \text{set}(f')$. Let us go through some examples.

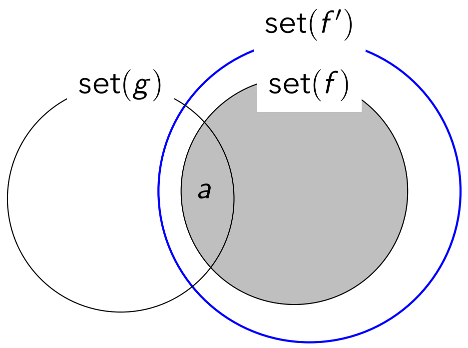
$[\![\text{some}]\!]^M$ is left upward monotonic, as illustrated by the following example. The two sets standing in the subset-superset relation here is $\text{set}([\![\text{phonologist}]\!]^M)$ and $\text{set}([\![\text{linguist}]\!]^M)$. This time, we only change the NP, and the VP is kept untouched.

(18) a. Some phonologist is happy.
b. Some linguist is happy.

As you can see, (18a) entails (18b).

As before, it is not sufficient show the entailment relation of one pair of sentences. We can prove the left upward monotonicity of $[\![\text{some}]\!]^M$ more generally as follows. Suppose $[\![\text{some}]\!]^M(f)(g) = 1$ for some arbitrary $f, g \in D_{\langle e,t\rangle}$. Then $\text{set}(f) \cap \text{set}(g) \neq \varnothing$. This means that there is at least one member in this intersection. Let's take one and call it a. Now take a superset $\text{set}(f')$ of $\text{set}(f)$. Since every member of $\text{set}(f)$ is a member of $\text{set}(f')$, it must be the case that $a \in \text{set}(f')$. We know that a belongs to $\text{set}(g)$ (as it's a shared member of $\text{set}(f)$ and $\text{set}(g)$), so $a \in \text{set}(f') \cap \text{set}(g)$. Then we have $\text{set}(f') \cap \text{set}(g) \neq \varnothing$, which means $[\![\text{some}]\!]^M(f')(g) = 1$. This reasoning is visualized in (19).

(19)



On the other hand, $[\![\text{every}]\!]^M$ is not left upward monotonic. This can be shown easily with an example: (20a) does not entail (20b), for example.

(20) a. Every phonologist is happy.
b. Every linguist is happy.

More concretely, if every phonologist is happy but there is an unhappy semanticist, (20a) is true but (20b) is false. So (20a) does not entail (20b), and hence $[\![\text{every}]\!]^M$ is not left upward monotonic.

1.4 Left Downward Monotonicity

Finally, left downward monotonicity is defined as (21).

(21) A function $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is *left downward monotonic* iff for any functions $f, f', g \in D_{\langle e,t\rangle}$ such that for each $x \in D_e$, if $f'(x) = 1$ then $f(x) = 1$, whenever $Q(f)(g) = 1$, $Q(f')(g) = 1$.

This time, $\text{set}(f')$ is a subset, rather than a superset, of $\text{set}(f)$.

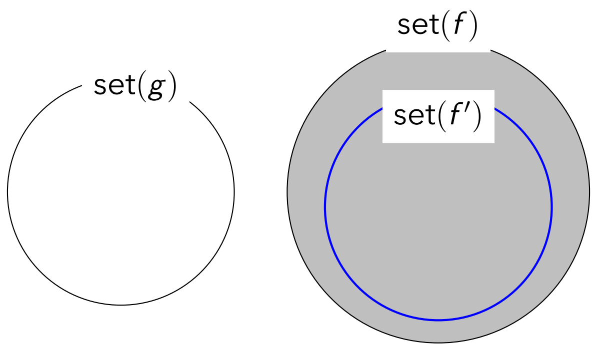
$[\![\text{No}]\!]^M$ is left downward monotonic, as illustrated by (22). Here we have the subset relation $\text{set}([\![\text{cat}]\!]^M) \subseteq \text{set}([\![\text{animal}]\!]^M)$.

(22) a. No animal is in the room.
b. No cat is in the room.

We have an entailment from (22a) to (22b). But again, to show that $[\![\text{no}]\!]^M$ is left downward monotonic, one example is not enough. Rather we have to reason about its meaning. Specifically, we will show that from the assumption that $[\![\text{no}]\!]^M(f)(g) = 1$ for some arbitrary $f, g \in D_{\langle e,t \rangle}$, it follows that $[\![\text{no}]\!]^M(f')(g) = 1$ for any $f' \in D_{\langle e,t \rangle}$ such that $\text{set}(f') \subseteq \text{set}(f)$.

Suppose $[\![\text{no}]\!]^M(f)(g) = 1$ for some $f, g \in D_{\langle e,t \rangle}$. This means that $\text{set}(f) \cap \text{set}(g) = \varnothing$. Then take a subset $\text{set}(f')$ of $\text{set}(f)$. $\text{set}(f')$ must be disjoint with $\text{set}(g)$, because if they overlapped, the common members of $\text{set}(f')$ and $\text{set}(g)$ would also belong to $\text{set}(f)$, and so it would contradict $\text{set}(f) \cap \text{set}(g) = \varnothing$. Thus, $\text{set}(f') \cap \text{set}(g) = \varnothing$, and therefore $[\![\text{no}]\!]^M(f')(g) = 1$. This is visualized in (23).

(23)



Some determiner denotations are not left downward monotonic, for example, 'some'. This is shown by the following examples.

(24) a. Some animal is in the room.
b. Some cat is in the room.

It is intuitively clear that (24a) does not entail (24b). More concretely, in a situation where there is a dog in the room but no cat is in the room, (24a) is true but (24b) is false. On the other hand, as we saw above, $[\![\text{some}]\!]^M$ is left upward monotonic.

There are also determiners that denote functions that are neither left upward monotonic nor right upward monotonic. Consider (25).

(25) a. Exactly two animals are brown.
b. Exactly two cats are brown.

Here, entailment doesn't hold in either direction. Concretely, if there are two brown dogs and all cats are gray, then (25a) is true but (25b) is false. Similarly, if there are two brown cats and three brown dogs, then (25b) is true but (25a) is false.

Here is a summary of the monotonicity properties of three quantificational determiners, 'every', 'some', 'no', and 'exactly two'.

(26)

	Left Upward	Left Downward	Right Upward	Right Downward
Every	No	Yes	Yes	No
Some	Yes	No	Yes	No
No	No	Yes	No	Yes
Exactly two	No	No	No	No

2 Negative Polarity Items

Monotonicity is an important concept for linguistics and used widely in analyzing a wide range of linguistic phenomena. For example, it is widely considered that the distribution of so-called *Negative Polarity Items* (NPIs) is sensitive to monotonicity.

NPIs are those items that require *negative licensors*. To illustrate, consider the following examples containing an NPI, 'ever'.

(27) a. *John has **ever** seen it.
b. *Everyone has **ever** seen it.
c. No one has **ever** seen it.

Roughly speaking, 'ever' needs a negative element in the same sentence. In (27a) and (27b), there is no negative item, so the sentences are ungrammatical. In contrast, in (27c), the quantifier is 'negative', and consequently the sentence is grammatical.

As we will see, our analysis of quantificational DPs allows us to refine the notion of 'negativity' relevant to NPI licensing as downward monotonicity.

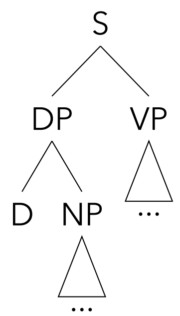
2.1 Fauconnier-Ladusaw Hypothesis

What are the licensing conditions for NPIs in English? Many theoretical linguists have been preoccupied with this question, and many different theoretical ideas have been put forward, including purely syntactic ones. Today, it is considered that at least part of the licensing conditions is semantic in nature, and many accept (a version of) the so-called *Fauconnier-Ladusaw Hypothesis*.

(28) *Fauconnier-Ladusaw Hypothesis*:
NPIs are licensed in downward monotonic contexts.

What are *downward monotonic contexts*? For sentences with quantificational subjects of the kind that we have been talking about, we can define downward monotonic contexts as follows (see the next section for a more general definition).

(29) In a sentence of the form



a. If $[\![D]\!]^M$ is left downward monotonic, NP is a downward monotonic context.
b. If $[\![D]\!]^M$ is right downward monotonic, VP is a downward monotonic context.

Let us go through some examples demonstrating this idea. Recall that $[\![\text{every}]\!]^M$ is left downward monotonic but right upward monotonic. Thus, in the following sentence, NP is a downward monotonic context, but VP is not (it is in fact an *upward* monotonic context).

(30) Every $\underbrace{[_{NP}\ \text{linguist who has lived in London}]}_{\text{Downward Monotonic Context}}\ \underbrace{[_{VP}\ \text{has been to Edinburgh}]}_{\text{Upward Monotonic Context}}$

According to the Fauconnier-Ladusaw Hypothesis, NPIs are licensed in the NP part of this sentence, but not in the VP part of the sentence. This prediction is borne out.

(31) a. Every linguist who has **ever** lived in London has been to Edinburgh.
b. *Every linguist who has lived in London has **ever** been to Edinburgh.

Keep in mind that we are talking here about licensing with 'every'. Unsurprisingly, if there's a separate licensor, e.g. negation, 'ever' can appear in VP, as in (32) (see Section 3 for an explanation how negation creates a downward entailing context).

(32) Every linguist who has lived in London has not **ever** been in Edinburgh.

Let us look at some more examples. Unlike $[\![\text{every}]\!]^M$, $[\![\text{no}]\!]^M$ is both left and right downward monotonic, so it licenses 'ever' in NP and VP, as predicted by the Fauconnier-Ladusaw Hypothesis.

(33) a. No linguist who has **ever** lived in London has been to Edinburgh.
b. No linguist who has lived in London has **ever** been to Edinburgh.

By contrast, $[\![\text{some}]\!]^M$ is both left and right upward monotonic, so the hypothesis predicts that it does not license 'ever' in any position. This is also correct.

(34) a. *Some linguist who has **ever** lived in London has been to Edinburgh.
b. *Some linguist who has lived in London has **ever** been to Edinburgh.

2.2 Other NPIs in English

English has a number of NPIs besides 'ever'. A particularly well-discussed one is 'any', as in (35).

(35) a. *Morris Halle read any of my papers.
b. *Every phonologist read any of my papers.
c. No phonologist read any of my papers.

The distribution of 'any' is considered to be similar to that of 'ever', but one caveat is that 'any' has a so-called *Free Choice reading*, under which it does not behave as an NPI. This is illustrated by the examples below.

(36) a. Chomsky will meet with any of my students, if I ask him.
b. Every syntactician will meet with any of my students, if I ask him.

The Free Choice reading of 'any' and similar items in other languages is also a well-studied topic in formal semantics. This might be a good topic for your essay.

In addition, there is a class of NPIs called *minimizers*, e.g. 'lift a finger', 'sleep a wink', 'budge an inch', '(have) a red cent', etc. (% indicates that only the literal meaning is available):

(37) a. %John lifted a finger for Mary.

b. %Every man lifted a finger for Mary.
c. No man lifted a finger for Mary.

It is known that there is a slight difference in distribution between 'ever' and minimizers. In particular, minimizers are not licensed in the NP argument of a quantificational determiner across the board:

(38) a. %Every boy who **lifted a finger** for Mary likes her.
b. %Every boy who likes Mary **lifted a finger** for her.

(39) a. %No boy who **lifted a finger** for Mary hates her.
b. No boy who hates Mary **lifted a finger** for her.

(40) a. %Some boy who **lifted a finger** for Mary likes her.
b. %Some boy who likes Mary **lifted a finger** for her.

For this reason, sometimes minimizers are called *strong NPIs* and NPIs like 'ever' are called *weak NPIs*. Generally, strong NPIs are licensed in a subset of environments where weak NPIs are licensed. There are two major hypotheses about the licensing conditions on strong NPIs:

- Zwarts (1998, 'Three types of polarity') claims that strong NPIs are licensed in a subset of downward monotonic contexts called anti-additive contexts.
- Gajewski (2011, 'Licensing strong NPIs') claims that strong NPIs are sensitive to the monotonicity of presuppositions.

A comparison of these two types of theories would make a good essay topic. It would also be a good essay project to investigate different types of negative polarity items in your native language.

3 Downward Monotonic Contexts and Generalized Entailment

In the above discussion, we did not discuss other licensors of NPIs than quantificational determiners, but it is obvious that quantificational determiners are not the only NPI licensors. For example, the following two sentences suggest that negation is an NPI licensor.

(41) a. *John has **ever** been to Paris.
b. John has not **ever** been to Paris.

In fact, the Fauconnier-Ladusaw Hypothesis is meant to capture the general distribution of NPIs, including but not limited to sentences with quantificational DPs. In order to capture (41), we need to define the notion of *downward monotonic contexts* more generally.

3.1 Rough Idea

Recall from above how right downward monotonicity is defined.

(10) A function $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is *right downward monotonic* iff for any functions $f, g, g' \in D_{\langle e,t\rangle}$ such that for each $x \in D_e$, if $g'(x) = 1$ then $g(x) = 1$, whenever $Q(f)(g) = 1$, $Q(f)(g') = 1$.

The idea here is if $Q(f)(g) = 1$, then the sentence obtained from it by replacing g with g' will also be true, provided that g' and g stand in a specific relation, namely, $\text{set}(g') \subseteq \text{set}(g)$.

Here, what is replaced is the VP denotation, a function of type $\langle e, t\rangle$, and the notion of downward monotonicity is defined for a function of type $\langle et, \langle et, t\rangle\rangle$. We will speak of the generalized version of the relation $\subseteq$–which is called *generalized entailment*–and then define the monotonicity properties of functions of any type.

3.2 Generalized Entailment

The standard notion of entailment is defined for sentences.

(42) *S entails S′* iff whenever *S* is true, *S′* is also true.

Regarding sentence denotations as truth-values, we can define the relation $\Rightarrow$ between truth-values as follows (cf. the logical connective $\rightarrow$ in Propositional Logic).

(43) If u, v are truth-values, $u \Rightarrow v$ iff $u = 0$ or $v = 1$ (i.e. $(u \rightarrow v)$ is true).

We will use (43) as the basic case, and define a similar notion for various types of functions.

Concretely, we will define a version of this relation that applies to functions of type $\langle e, t\rangle$ as follows.

(44) If f and g are functions of type $\langle e, t\rangle$, $f \Rightarrow g$ iff for each $x \in D_e$, $f(x) \Rightarrow g(x)$.

Notice that on the right-hand side of 'iff', $\Rightarrow$ is flanked by truth-values, while on the left-hand side it is flanked by functions of type $\langle e, t\rangle$.

Here is a concrete example. $[\![\text{British}]\!]^M \Rightarrow [\![\text{European}]\!]^M$, because for each $x \in D_e$, whenever $[\![\text{British}]\!]^M(x) = 1$, it is also the case that $[\![\text{European}]\!]^M(x) = 1$; or equivalently, either $[\![\text{British}]\!]^M(x) = 0$ or $[\![\text{European}]\!]^M(x) = 1$. Notice that $[\![\text{British}]\!]^M \Rightarrow [\![\text{European}]\!]^M$ iff $\text{set}([\![\text{British}]\!]^M) \subseteq \text{set}([\![\text{European}]\!]^M)$. But $\Rightarrow$ is a broader notion, as it applies to truth-values and functions of other types, as we will now defeine.

Using (44), we can define the version of $\Rightarrow$ for functions of type $\langle e, \langle e, t\rangle\rangle$ as follows.

(45) If f and g are functions of type $\langle e, \langle e, t\rangle\rangle$, $f \Rightarrow g$ iff for each $x \in D_e$, $f(x) \Rightarrow g(x)$.

It looks the same as before, but the semantic type of f and g is different. In particular, on the right-hand side of 'iff', $f(x)$ and $g(x)$ are both still functions. Specifically, they are functions of type $\langle e, t\rangle$. Thus in order to evaluate whether $f \Rightarrow g$ for functions of type $\langle e, \langle e, t\rangle\rangle$, one needs to refer to (44).

For example, $[\![\text{punch}]\!]^M \Rightarrow [\![\text{touch}]\!]^M$, because for each $x \in D_e$ and for each $y \in D_e$, if $[\![\text{punch}]\!]^M(x)(y) = 1$, then $[\![\text{touch}]\!]^M(x)(y) = 1$; or equivalently, $[\![\text{punch}]\!]^M(x)(y) = 0$ or $[\![\text{touch}]\!]^M(x)(y) = 1$.

Similarly, we can define $\Rightarrow$ for type-$\langle et, t\rangle$ functions as follows.

(46) If f and g are functions of type $\langle et, t\rangle$, $f \Rightarrow g$ iff for each $h \in D_{\langle e,t\rangle}$, $f(h) \Rightarrow g(h)$.

The idea is the same as above. And using this, one can define $\Rightarrow$ for type-$\langle et, \langle et, t\rangle\rangle$ functions.

(47) If f and g are functions of type $\langle et, \langle et, t\rangle\rangle$, $f \Rightarrow g$ iff for each $h \in D_{\langle e,t\rangle}$, $f(h) \Rightarrow g(h)$.

More generally, we can define $\Rightarrow$ for all semantic types that 'end in t', i.e. t and all types that look like $\langle \cdots \langle \sigma, t\rangle\rangle \cdots\rangle$, incluidng $\langle e, t\rangle$, $\langle e, \langle e, t\rangle\rangle$, $\langle et, t\rangle$, $\langle et, \langle et, t\rangle\rangle$, $\langle t, t\rangle$, etc. To be more precise, we define semantic types that end in t as follows.

(48) A semantic type τ *ends in t* if
   a. $\tau = t$ or
   b. $\tau = \langle \sigma_1, \sigma_2\rangle$ such that σ_1 is a semantic type and σ_2 a semantic type that ends in t.

Now, we can define $\Rightarrow$ for any semantic type that ends in t as in (49).

(49) *Generalized Entailment*
For any $x, y \in D_\tau$ where τ is a semantic type that ends in t,

$$x \Rightarrow y \text{ iff } \begin{cases} x = 0 \text{ or } x = y & \text{if } \tau = t \\ \text{for each } z \in D_{\sigma_1},\ x(z) \Rightarrow y(z) & \text{if } \langle \sigma_1, \sigma_2\rangle \end{cases}$$

As remarked above, $\subseteq$ can be seen as a special case of this when $\tau = \langle e, t\rangle$.

3.3 Generalized Monotonicity

Using the notion of generalized entailment, we can define monotonicity for any function of type that ends in t, as follows.

(50) a. A function f of type $\tau = \langle \sigma_1, \sigma_2\rangle$ that ends in t is *upward monotonic* iff for any $x, y \in D_{\sigma_1}$ such that $x \Rightarrow y$, $f(x) \Rightarrow y$.
   b. A function f of type $\tau = \langle \sigma_1, \sigma_2\rangle$ that ends in t is *downward monotonic* iff for any $x, y \in D_{\sigma_1}$ such that $x \Rightarrow y$, $f(y) \Rightarrow x$.

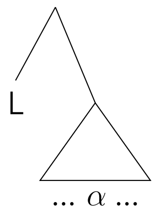
Let us zoom in on one particular case when $\tau = \langle et, t\rangle$. Recall, for any $x, y \in D_{\langle e,t\rangle}$, $x \Rightarrow y$ iff set(x) $\subseteq$ set(y). So f is upward monotonic iff for any x, y such that set(x) $\subseteq$ set(y), if $f(x) = 1$, $f(y) = 1$. That is, if $f(x) = 1$, the truth is preserved for any superset y of x. Downward monotonicity is the converse of this: $f(y) = 1$ guarantees that for any subset x, $f(x) = 1$.

The monotonicity properties of type-$\langle et, t\rangle$ functions are closely related to right upward/downward monotonicity of type-$\langle et, \langle et, t\rangle\rangle$ functions. That is, if $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is right upward monotonic, then $Q(f) \in D_{\langle et,t\rangle}$ is upward monotonic for any $f \in D_{\langle e,t\rangle}$. Likewise, if $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is right downward monotonic, then $Q(f) \in D_{\langle et,t\rangle}$ is downward monotonic for any $f \in D_{\langle e,t\rangle}$.

The left monotonic properties are simply monotonicity in the sense of (50) for type $\langle et, \langle et, t\rangle\rangle$ functions. $[\![\text{no}]\!]^M$ is downward monotonic. In order to see this, consider $[\![\text{no}]\!]^M(f)$ for an arbitrary $f \in D_{\langle e,t\rangle}$. Take any $f' \in D_{\langle e,t\rangle}$ such that $f \Rightarrow f'$, i.e. set(f) $\subseteq$ set(f'). Now take $[\![\text{no}]\!]^M(f)$ and $[\![\text{no}]\!]^M(f')$. These are functions of type $\langle et, t\rangle$. Take any $g \in D_{\langle e,t\rangle}$. If $[\![\text{no}]\!]^M(f')(g) = 1$, set($f'$) $\cap$ set(g) $= \varnothing$. Since set(f) $\subseteq$ set(f'), it is also the case that set(f) $\cap$ set(g) $= \varnothing$. Then we also have $[\![\text{no}]\!]^M(f)(g)$. So $[\![\text{no}]\!]^M(f')(g) \Rightarrow [\![\text{no}]\!]^M(f)(g)$, and since this is the case for any g, we have $[\![\text{no}]\!]^M(f') \Rightarrow [\![\text{no}]\!]^M(f)$. Furthermore, we started with arbitrary functions $f, f' \in D_{\langle e,t \rangle}$ such that set(f) $\subseteq$ set(f'), we can conclude $[\![\text{no}]\!]^M$ is downward monotonic.

Now we can state the Fauconnier-Ladusaw Hypothesis as follows.

(51) *Facuconnier-Ladusaw Hypothesis*
An NPI α is licensed if α occurs in the following configuration where $[\![\text{L}]\!]^M \in D_\tau$ where τ is a semantic type that ends in t and $[\![\text{L}]\!]^M$ is downward monotonic.



We call the instance of such L that is closest to the NPI α the *licensor* of α.

3.4 Negation

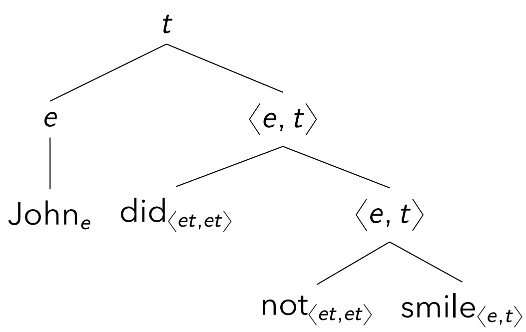
We can now show that negation can serve as an NPI licensor, or in other words, that it denotes a downward monotonic function. In sentences like (52), negation occurs between the subject and VP.

(52) a. John did not smile.
b. Mary does not like Bill.

It is a property of English syntax that whenever negation occurs, something overt must fill in the auxiliary position, e.g. 'did' and 'does' in (52). As it is beyond the scope of this course to discuss the semantics of auxiliaries (especially those that are called *modal auxiliaries*), we will not analyze the semantics of these items. For items like 'did' and 'does', let's assume that they are simply semantically vacuous, i.e. they denote identity functions.

As illustrated by the following tree diagram, we analyze $[\![\text{not}]\!]^M$ to be of type $\langle et, et \rangle$.

(53)



$[\![\text{not}]\!]^M$ takes a VP-denotation, a function of type $\langle e, t \rangle$ and says that it does not hold for the subject.

(54) For any model M,
$[\![\text{not}]\!]^M = [\lambda f \in D_{\langle e,t \rangle}.\ [\lambda x \in D_e.\ 1 \text{ iff } f(x) = 0]]$

This function is downward monotonic. First, its semantic type ends in t, i.e. after supplying all arguments, you will get a truth-value. Now take arbitrary functions $f, f' \in D_{\langle e,t \rangle}$ such that $f' \Rightarrow f$, i.e. set(f') is a subset of set(f). Now consider $[\![\text{not}]\!]^M(f)$. This is a function of type $\langle e, t \rangle$ such that for any $x \in D_e$ $[\![\text{not}]\!]^M(f)(x) = 1$ iff $f(x) = 0$, or equivalently, $x \notin$ set(f). Notice that whenever $x \notin$ set(f), $x \notin$ set(f'), because set(f') $\subseteq$ set(f) and so set(f') only contains entities that set(f) contains. So, for each $x \in D_e$, if $x \notin$ set(f), then $x \notin$ set(f'). This is equivalent to: for each $x \in D_e$, if $[\![\text{not}]\!]^M(f)(x) = 1$, $[\![\text{not}]\!]^M(f')(x) = 1$. So $[\![\text{not}]\!]^M(f) \Rightarrow [\![\text{not}]\!]^M(f')$. Since we are talking arbitrary $f, f' \in D_{\langle e,t \rangle}$ such that set(f') $\subseteq$ set(f), this proves that $[\![\text{not}]\!]^M$ is downward monotonic.