It's not always redundant to assert what can be presupposed*

Comments on Mayr & Romoli (2016)

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Abstract

Mayr & Romoli (2016) point out that the felicity of sentences of the form $(\neg\phi \vee (\phi \wedge \psi))$ is problematic for theories of redundancy, as the second occurrence of ϕ is predicted to be *redundant*. I call this the *Disjunction Puzzle*. They propose two solutions to it that make crucial use of (i) incrementality, and (ii) a grammatical mechanism of exhaustification, and conclude that the Disjunction Puzzle supports (i) and (ii). I take issue with their conclusion and argue that the Disjunction Puzzle does not necessarily motivate (i) and (ii), because the puzzle only arises under a specific assumption about presupposition satisfaction and redundancy of assertion, namely, that they refer to the same notion of entailment. It is natural to make this assumption in possible worlds semantics, but is not necessarily motivated in a more fine-grained theory of meaning. Adopting Situation Semantics, I will demonstrate that the Disjunction Puzzle disappears once it is assumed that redundancy is computed with reference to a stronger notion of entailment than presupposition satisfaction.

1 Introduction

Mayr & Romoli (2016) point out that the felicity of disjunctive sentences like (1) poses a puzzle for theories of redundancy.

(1) Either Mary isn't pregnant, or she is and she is expecting a daughter.
(Mayr & Romoli 2016:2)

Specifically, the conjunct *she is (pregnant)* is redundant here. I will define the relevant notion of redundancy below, but the idea behind this can easily be illustrated when (1) is compared to the version of the sentence without this conjunct, (2), which means the same thing.

(2) Either Mary isn't pregnant, or she is expecting a daughter.

Theories of redundancy assume that such redundant expressions give rise to infelicity, and

*[acknowldgments to be added]

therefore incorrectly predict (1) to be infelicitous.[1] I will call this puzzle the *Disjunction Puzzle*.

To solve the Disjunction Puzzle, Mayr & Romoli (2016) propose two ways to make *she is (pregnant)* in (1) non-redundant. Both of their solutions make essential use of (i) incrementality and (i) a grammatical mechanism of exhaustification, which they call Exh (Chierchia, Fox & Spector 2012, among others). Mayr & Romoli conclude that the Disjunction Puzzle motivates (i) and (ii).

I would like to take issue with this conclusion. I claim that Mayr & Romoli's puzzle only arises under a specific assumption about presupposition satisfaction and redundancy of assertion, namely that they refer to the same notion of entailment. It is natural to make this assumption in possible worlds semantics, which is assumed by Mayr & Romoli and more generally by many theories of redundancy, but it becomes non-trivial, once we move to a more fine-grained theory of meaning. I point out that if we assume that redundancy of assertion is computed with respect to a stronger notion of entailment than presupposition satisfaction, the Disjunction Puzzle disappears. I will offer a formal implementation of this idea in a version of Update Semantics that uses Situation Semantics (Kratzer 1998, 2002, 2016). Since my explanation does not make use of the mechanisms mentioned above, (i) and (ii), I conclude that the Disjunction Puzzle does not necessarily support them.

This paper is organized as follows. In Section 2 and Section 3, I will review the Stalnakerian theory of redundancy of assertion and why (1) is problematic for it. In Section 4, I will discuss the aforementioned tacit but crucial assumption that redundancy of assertion and presupposition satisfaction refer to the same notion of entailment, and claim that once it is dropped, the Disjunction Puzzle disappears. I will then make this proposal concrete in Section 5, by formalizing it by augmenting Stanakerian Update Semantics with Situation Semantics (Kratzer 1989, 2002, 2016). In Section 6, I will critically review Mayr & Romoli's two solutions, and compare them to my alternative solution to the Disjunction Puzzle.

2 The Stalnakerian Theory of Redundancy

2.1 Basics

Regardless of one's theoretical taste in pragmatics, it should come across as a natural assumption that redundant assertions are to be avoided, as they would convey redundant information. Although redundant information does not make the conversation inconsistent, it is costly for both the speaker and hearer, and therefore should be eschewed. Let us state this as a principle:

(3) *Principle of Non-Redundant Assertion*: An assertion is infelicitous if it is redundant.

What does it mean for an assertion to be redundant? Stalnaker (1978) puts forward a very influential view about what counts as redundant, which I call the Stalnakerian theory of redundancy.

The Stalnakerian theory of redundancy is built upon possible worlds semantics where a (declarative) sentence S denotes a proposition $[\![S]\!]$, and a proposition is a set of possible worlds.

[1] Mayr & Romoli (2016) discuss two types of theories of redundancy, and correctly point out that the Disjunction Puzzle is problematic for both. Globalist theories (Singh & Katzir 2014, Meyer 2013) compute redundancy at the global level by comparing sentences and their *simplifications*. A sentence is said to be redundant if it could be simplified without changing the overall meaning. Localist theories, on the other hand, (Stalnaker 1978, Singh 2007, Fox 2008, Schlenker 2009) compute redundancy with respect to *local contexts* in the standard dynamic sense (see below). Mayr & Romoli propose solutions under both types of theories. Here, I will first focus on the localist approach, as the new solution I will propose is a variant of it. I will discuss Mayr & Romoli's globalist and localist solutions in Section 6.

Asserting S against a conversational context c can be seen as adding the proposition p to the common ground of c. What does it mean to add a proposition to the common ground? Stalnaker (1978) introduces the notion of the *context set* of c, which is the set of possible worlds that are compatible with what the discourse participants commonly believe to be true in the context of utterance.[2] Since the only aspect of context that matters for us is the context set, I will henceforth equate the two, and just say context to mean its context set.

According to Stalnaker, assertion of a (declarative) sentence S with respect to context c is a proposal to update c so as to create a new context c' that encompasses the information conveyed by S. This update of context c to another context c' is modeled as intersecting c with $[\![S]\!]$: $c' = \{ w \in c \mid w \in [\![S]\!] \}$, or equivalently, $c' = c \cap [\![S]\!]$.

Stalnaker introduces a pragmatic principle stating that assertion of sentence S with respect to context c is felicitous only if it results in a non-trivial update of c. There are two kinds of trivial update. One is when assertion of S eliminates all possible worlds, i.e. $c \cap [\![S]\!] = \varnothing$. In such a case we say S is *contradictory* with respect to c, and it is infelicitous. Another kind of trivial update is when assertion of S does not eliminate any possible worlds, i.e. $c \cap [\![S]\!] = c$. In this case we say S is *redundant* with respect to c, and it is also assumed to results in infelicity. Let us state this as a principle:

(4) **Stalnaker's Principle of Assertion**: Assertion of sentence S with respect to context c is infelicitous if S is contradictory or redundant with respect to c.
  a. S is *contradictory* with respect to c if $c \cap [\![S]\!] = \varnothing$.
  b. S is *redundant* with respect to c if $c \cap [\![S]\!] = c$.

In this paper I will not be concerned with contradictory sentences, and mostly only talk about cases redundant sentences.

To see how (4) works concretely, consider the following example, modeled after Mayr & Romoli's (2016).

(5) Mary is expecting a girl. #She is pregnant.

The infelicity of the second sentence is explained as follows. Suppose that the first sentence is uttered with respect to context c. This assertion adds to c information that Mary is pregnant and also is having a girl. Let us assume that this resulted in a non-trivial update of c, so the proposition that Mary is expecting a girl is only compatible with some of the possible worlds in c. We call the set of these possible worlds c', which is the context that results from updating c with the meaning of the first sentence. Now, the second sentence is processed against c'. Then, in c', it is already common ground that Mary is pregnant, and its intersection with the proposition that Mary is pregnant returns c'. Consequently, *She is pregnant* in (5) is redundant in c' in the sense defined above.

2.2 Sentence-Internal Redundancy

One complication that arises here is that the above notion of redundancy does not explain the observation that redundancy of assertion may arise within a single sentence, as in (6).

(6) a. #Mary is expecting a girl, and she is pregnant and is happy.

[2] This actually is a simplification. For Stalnaker the context set is the set of possible worlds compatible with the *speaker presupposition*, which is essentially what the speaker believes (or pretends to believe) to be the common ground. This complication does not concern us here, so we simplify and regard the context set as a representation of the common ground itself.

b. #If Mary is expecting a girl, then she is pregnant and is happy.

Quite intuitively, these sentences are infelicitous, because they involve redundant expressions within themselves, and it seems desirable to account for their infelicity by the same principle as the infelicity of (5) above, namely the principle in (3). But the definition of redundancy above does not explain it. Specifically, if it is not commonly known whether Mary is pregnant or not, then asserting (6a) should have a non-trivial effect on the context, because the first conjunct ensures that the update will eliminate at least some possible worlds form the context.

Stalnaker suggests an ingenious way of accounting for (5) and (6) uniformly, namely, by understanding the *context* in the definition of redundancy as the *local context* for S. The notion of local context is a standard one (see Appendix for a rigorous syntactic definition in the system laid out in the next section). Roughly put, *she is pregnant* in (6a) is used to update a context that results from updating the initial context with the first conjunct *Mary is expecting a girl*, and its redundancy is computed with respect to that derived context—its local context—rather than the initial context—the global context. Then the infelicity of (6a) and the infelicity of (5) will be given exactly the same explanation: in both cases, *she is pregnant* is redundant with respect to the context they are updating. Similarly, (6b) involves a hypothetical update of the context with the antecedent clause *Mary is expecting a girl*, which results in a new context that serves as the local context for the sentence *she is pregnant*. Again, this will be infelicitous for the same reason as (5). I will make these points more precise in the next section.

3 Stalnakerian Update Semantics

For the sake of clarity, let us consider a propositional language $\mathcal{L}$ with $\neg$, $\wedge$, $\vee$, and $\rightarrow$. Throughout this paper, p^i denotes the ith atomic proposition in $\mathcal{L}$; ϕ, ψ and χ are meta-variables ranging over formulae of $\mathcal{L}$; c, c' and c'' range over contexts (or more precisely context sets); and $\equiv$ denotes syntactic identity between two formulae. Proofs of the formal properties mentioned in this section can be found in Appendix.

3.1 Update Rules

Assertion of formula ϕ in a context c is modelled by *updated rules*. We denote the context that results after accepting assertion of ϕ with respect to c by $c[\phi]$.

Definition 1. (Update rules)

$$\begin{aligned}
c[p^i] &\coloneqq \{\, w \in c \mid w \vDash p^i \,\} \\
c[\neg\phi] &\coloneqq c - c[\phi] \\
c[(\phi \wedge \psi)] &\coloneqq c[\phi][\psi] \\
c[(\phi \rightarrow \psi)] &\coloneqq c - (c[\phi] - c[\phi][\psi]) \\
c[(\phi \vee \psi)] &\coloneqq c[\phi] \cup c[\neg\phi][\psi]
\end{aligned}$$

The update rules for $\neg$, $\wedge$ and $\rightarrow$ are standard (Heim 1982, 1983, Beaver 2001, Schlenker 2009). Mayr & Romoli (2016) follow Beaver (2001) and Schlenker (2009), among others, and assume the above update rule for $\vee$, which we will discuss in detail later.

Here are some useful formal properties of this system (cf. Groenendijk & Stokhof 1991, Rothschild & Yalcin 2016). Firstly, it is *eliminative* in the sense that for any context c and $\phi \in \mathcal{L}$, $c[\phi] \subseteq c$. This means that updates never result in larger context. Assuming that Stalnaker's Principle of Assertion in (4b) is obeyed, this means that any one update will result in a non-empty proper subset of the input context. This update semantics is also *distributive* in the sense that for any context c and $\phi \in \mathcal{L}$, $c[\phi] = \bigcup_{w \in c}(\{ w \} [\phi])$. An immediate consequence of distributivity is *monotonicity*: for any context c and $\phi \in \mathcal{L}$, if $c' \subseteq c$, then $c'[\phi] \subseteq c[\phi]$.

3.2 Entailment

Now, let us define the notion of redundancy in this update semantics. Firstly, the central idea behind it is that assertion of S is redundant with respect to c if $[\![S]\!]$ is already known to be true. In such a situation, we say c *entails* $[\![S]\!]$ and write $c \vDash S$.

Definition 2. (Entailment) c *entails* ϕ ($c \vDash \phi$) if $c[\phi] = c$.

Entailment is persistent in the sense that if $c \vDash \phi$, any contexts obtained by updating c will always entail ϕ. Given eliminativity, persistence can be stated as follows: If $c \vDash \phi$, then for any $c' \subseteq c$, $c' \vDash \phi$. It should also be noted that a context updated with ϕ, $c[\phi]$, always entails ϕ.

3.3 Stalnakerian Redundancy in Update Semantics

As mentioned in the previous section, Stalnakerian redundancy is a stronger notion than entailment. In terms of the formal language here, this can be illustrated as follows. Whenever $c \vDash \phi$, we want $(\phi \wedge \psi)$ to be redundant in c, but if $c \nvDash \psi$, it also follows that $c \nvDash (\phi \wedge \psi)$. Rather, we want to compute redundancy at each update. This leads to the following recursive definition of redundancy:

Definition 3. (Stalnakerian redundancy) ϕ *redundant* with respect to c ($c \blacktriangleright \phi$) if any of the following is the case:

$$\begin{array}{ll}
\phi \equiv p^i \text{ and} & c \vDash p^i \\
\phi \equiv \neg\psi \text{ and} & c \blacktriangleright \psi \text{ or } c \vDash \phi \\
\phi \equiv (\psi \wedge \chi) \text{ and} & c \blacktriangleright \psi \text{ or } c[\psi] \blacktriangleright \chi \text{ or } c \vDash \phi \\
\phi \equiv (\psi \rightarrow \chi) \text{ and} & c \blacktriangleright \psi \text{ or } c[\psi] \blacktriangleright \chi \text{ or } c \vDash \phi \\
\phi \equiv (\psi \vee \chi) \text{ and} & c \blacktriangleright \psi \text{ or } c \blacktriangleright \neg\psi \text{ or } c[\neg\psi] \blacktriangleright \chi \text{ or } c \vDash \phi
\end{array}$$

It should be stressed that this notion of Stalnakerian redundancy is derived from the update rules based on the idea that the sentence used in each update must not be redundant with respect to the context it is used to update, its local context.

It follows from persistence of entailment that Stalnakerian redundancy is also persistent. That is, once ϕ has been commonly believed to be true, asserting it again in a later context will always be redundant. Given eliminativity, persistence of Stalnakerian redundancy can be stated in terms of subsets as follows: For any context c and $\phi \in \mathcal{L}$, if $c \blacktriangleright \phi$, then for any $c' \subseteq c$, $c' \blacktriangleright \phi$. It also holds that for any context c and $\phi \in \mathcal{L}$, $c[\phi] \blacktriangleright \phi$. We can furthermore prove that for any context c and $\phi \in \mathcal{L}$, if $c \blacktriangleright \phi$, then $c \blacktriangleright \Gamma[\phi]$ where $\Gamma[\phi]$ is any formula in $\mathcal{L}$ containing at least one occurrence of ϕ.

3.4 Conjunction and Implication

Let us now go through some concrete examples. The update semantics accounts for the infelicity of the following conjunctive example due to Mayr & Romoli (2016).

(7) #Mary is pregnant, and she is and it doesn't show.

Translating this sentence into $(p \wedge (p \wedge (\neg s)))$, we can prove that it is redundant in every context c. More generally, we can prove the following.

Proposition 1. *For every context c, $c \blacktriangleright (\phi \wedge (\phi \wedge \psi))$.*

Proof. By definition, $c \blacktriangleright (\phi \wedge (\phi \wedge \psi))$ if at least one of the following is the case: (i) $c \blacktriangleright \phi$; (ii) $c[\phi] \blacktriangleright (\phi \wedge \psi)$; or (iii) $c \vDash (\phi \wedge (\phi \wedge \psi))$. Similarly, (ii) is the case if at least one of the following is the case: (iv) $c[\phi] \blacktriangleright \phi$; (v) $c[\phi][\phi] \blacktriangleright \psi$; or (vi) $c[\phi] \vDash (\phi \wedge \psi)$. Since for any context c' and $\chi \in \mathcal{L}$, $c'[\chi] \blacktriangleright \chi$, (iv) is always the case. □

Let us consider another example from Mayr & Romoli (2016), reproduced in (8). Assuming that (8) is translated by $(p \rightarrow (p \wedge \neg s))$, we can also account for its infelicity.

(8) #If Mary is pregnant, then she is and it doesn't show.

More generally, we can prove the following. The proof is essentially identical to the above case of conjunction.

Proposition 2. *For every context c, $c \blacktriangleright (\phi \rightarrow (\phi \wedge \psi))$.*

Proof. By definition, $(\phi \rightarrow (\phi \wedge \psi))$ is redundant in c, if any of the following is the case: (i) $c \blacktriangleright \phi$; (ii) $c[\phi] \blacktriangleright (\phi \wedge \psi)$; (iii) $c \vDash (\phi \rightarrow (\phi \wedge \psi))$. Similarly, (ii) is the case if at least one of the following is the case: (iv) $c[\phi] \blacktriangleright \phi$; (v) $c[\phi][\phi] \blacktriangleright \psi$; or (vi) $c[\phi] \vDash (\phi \wedge \psi)$. Since for any context c' and $\chi \in \mathcal{L}$, $c'[\chi] \blacktriangleright \chi$, (iv) is always the case. □

3.5 Disjunction Puzzle

Now, recall Mayr & Romoli's Disjunction Puzzle. The example in (1) is repeated here.

(1) Either Mary isn't pregnant, or she is and she is expecting a daughter.

Translating this sentence as $(\neg p \vee (p \wedge d))$, we can show that the present theory incorrectly predicts it to be redundant in every context. More generally, the following result obtains.

Proposition 3. *$(\neg\phi \vee (\phi \wedge \psi))$ is redundant in every context.*

Proof. By definition, $c \blacktriangleright (\neg\phi \rightarrow (\phi \wedge \psi))$ if at least one of the following is the case: (i) $c \blacktriangleright [\neg\phi]$; (ii) $c \blacktriangleright [(\neg\neg\phi)]$; (iii) $c[(\neg\neg\phi)] \blacktriangleright (\phi \wedge \psi)$; (iv) $c \vDash (\neg\phi \rightarrow (\phi \wedge \psi))$. Note that $c[(\neg\neg\phi)] = c - c[\neg\phi] = c - (c - c[\phi]) = c[\phi]$. Then (iii) is always the case for the same reasons as above. □

Thus, the present theory predicts (1) to be infelicitous, contrary to fact. This is the Disjunction Puzzle.

Now it is important to note that the Disjunction Puzzle arises because of the update rule for disjunction, which states:

$$c[(\phi \vee \psi)] := c[\phi] \cup c[\neg\phi][\psi]$$

One might think that it is more natural to assume the following rule, which does not involve the update with the negation of ϕ in the second disjunct. To distinguish the two update rules, I will temporarily introduce another connective $\dot{\vee}$.

$$c[(\phi \dot{\vee} \psi)] := c[\phi] \cup c[\psi]$$

In fact, with this connective, $(\neg\phi \dot{\vee} (\phi \wedge \psi))$ would not be redundant in every context. Concretely, suppose that neither ϕ nor ψ is redundant with respect to c. It is easy to see that $(\neg\phi \dot{\vee} (\phi \wedge \psi))$ is not redundant with respect to c.

Why do Mayr & Romoli assume the above update rule for $\vee$, rather than the update rule for $\dot{\vee}$? This has to do with the behavior of presupposition in disjunctive sentences.

3.6 Presuppositional Update Semantics

Stalnaker (1974, 1978) proposes that presuppositions are those inferences that need to be satisfied by virtue of being commonly believed to be true in the context of utterance. Assuming that presuppositions are propositions, this amounts to a requirement that they be true in all the possible worlds in the current context. Stalnaker (1974) furthermore proposes that presuppositions of atomic sentences need to be satisfied with respect to their local contexts, an influential idea further pursued by Heim (1983), among others.

In order to incorporate this, we will extend the update semantics as follows. We take presuppositions to be properties of atomic sentences, and define a new language $\mathcal{L}^p$ where presuppositions are represented as subscripts $\pi \in \mathcal{L}$ on atomic propositions, e.g. p^i_π.[3] The update rules are as in $\mathcal{L}$, including the update rule for atomic sentences, i.e. $c[p^i_\pi] = \{ w \in c \mid w \vDash p^i \}$.

The only aspect of presuppositions that we are concerned with is whether they are satisfied with respect their local contexts. Presupposition satisfaction is thus defined recursively as follows.

Definition 4. (Presupposition Satisfaction) The presupposition of ϕ is *satisfied* with respect to c ($c \triangleright \phi$) if any of the following is true:

$$\begin{array}{ll} \phi \equiv p^i_\pi \text{ and} & c \vDash \pi \\ \phi \equiv \neg\psi \text{ and} & c \triangleright \psi \\ \phi \equiv (\psi \wedge \chi) \text{ and} & c \triangleright \psi \text{ and } c[\psi] \triangleright \chi \\ \phi \equiv (\psi \rightarrow \chi) \text{ and} & c \triangleright \psi \text{ and } c[\psi] \triangleright \chi \\ \phi \equiv (\psi \vee \chi) \text{ and} & c \triangleright \psi \text{ and } c \triangleright \neg\psi \text{ and } c[\neg\psi] \triangleright \chi \end{array}$$

It should be remarked that presupposition satisfaction is conceived of as the 'dual' of Stalnakerian redundancy in the sense that assertions must not be redundant in any update step, while presuppositions must be redundant in each update step. Let us state this as a principle:

(9) *Principle of Presupposition Satisfaction*: An assertion of sentence S is infelicitous in c unless the presupposition of S is satisfied with respect to c.

It should also be remarked that just like Stalnakerian redundancy, presupposition satisfaction for any formula can be uniquely determined by the relevant update rules based on the idea that presuppositions must be redundant at each update.

What is important for our discussion is that the above rule for $(\psi \vee \chi)$ says $c[\neg\psi] \triangleright \chi$, which means that the presupposition of the second disjunct χ is evaluated against the context resulting from update with the negation of the first disjunct ψ. This is due to the assumption that the update rule for disjunction involves the corresponding update with the negation of the first disjunct ψ, before evaluating the second disjunct χ, which, you may recall, is the culprit of the Disjunction Puzzle. Mayr & Romoli (2016) assume this update rule for disjunction, precisely because they wanted to obtain the above rule of presupposition satisfaction, which, they claim, is empirically motivated. More concretely, consider the following example from Mayr & Romoli (2016).

[3] $\mathcal{L}^p$ is the smallest language such that: If p^i is an atomic proposition in $\mathcal{L}$, and $\pi \in \mathcal{L}$, then $p^i_\pi \in \mathcal{L}^p$; If $\phi, \psi \in \mathcal{L}^p$, then $\neg\phi, (\phi\Box\psi) \in \mathcal{L}^p$ for $\Box \in \{ \wedge, \rightarrow, \vee \}$. Incidentally, a two-dimensional representation of presupposition like this is known to suffer from the so-called *Binding Problem*, which has to do with existential quantification (cf. Karttunen & Peters 1979, Beaver 2001, Sudo 2012). As we are only dealing with a propositional language here, the Binding Problem does arise. More importantly, nothing crucial here hinges on the two-dimensional representation of presuppositions. If one wishes, it is not at all difficult to redefine presupposition satisfaction in terms of trivalent logic.

(10) Either John doesn't smoke, or Mary does too.

The second disjunct presupposes that someone other than Mary smokes, but the entire sentence does not presuppose this. As Mayr & Romoli discuss in great detail, this is correctly predicted by the above definition of presupposition satisfaction, because the presupposition of the second disjunct is evaluated with respect to the context that has been updated with the negation of the first conjunct, which therefore entails that John smokes.

Conversely, this reading of (10) would not be explained with $\dot{\vee}$, as the rule for presupposition satisfaction would look like:

$$c \triangleright (\phi \dot{\vee} \psi) \text{ iff } c \triangleright \phi \text{ and } c \triangleright \psi$$

Here, the second disjunct is directly evaluated against the global context c, which is to say that c must entail that someone other than Mary smokes.

To summarize, given the behavior of presupposition, we assume that the update rule for disjunction as follows:

$$c[(\phi \vee \psi)] = c[\phi] \cup c[\neg\phi][\psi]$$

But then it incorrectly predicts sentences of the form $(\neg\phi \vee (\phi \wedge \psi))$ to be Stalnakerian redundant and hence infelicitous, which is the Disjunction Puzzle.

4 An Implicit Assumption about Entailment

Here I would like to first point out that Mayr & Romoli's Disjunction Puzzle crucially hinges on the assumption that presupposition satisfaction and redundancy of assertion refer to the same notion of entailment. To make it more explicit, they assume:

(11) a. c satisfies the presupposition of p^i_π ($c \triangleright p^i_\pi$) iff c entails π ($c \vDash \pi$).
  b. c makes assertion of p^i_π redundant ($c \blacktriangleright p^i_\pi$) iff c entails p^i ($c \vDash p^i$).

Mayr & Romoli are not the only authors who make this assumption. In fact, it comes from Stalnaker (1978). And as far as I know, it has never been questioned before in the context of Stalnakerian update semantics. However, that does not mean that it is correct.

Now suppose that presupposition satisfaction and redundancy of assertion are computed with respect to two different notions of entailment. In particular, let us assume that redundancy of assertion is computed in terms of a stronger notion of entailment, which I call *strict entailment*.

(12) a. c satisfies the presupposition of p^i_π ($c \triangleright p^i_\pi$) iff c entails π ($c \vDash \pi$).
  b. c makes assertion of p^i_π redundant ($c \blacktriangleright p^i_\pi$) iff c *strictly entails* p^i ($c \Vdash p^i$).

The rationale behind this modification is that presupposition conveys backgrounded information that is typically not the main point of the utterance, while assertion is foregrounded information, a main point of the utterance. Then, it seems to be not so farfetched to assume that presupposition can be satisfied by virtue of being entailed by backgrounded, tacit common belief, while redundancy of assertion only tracks foregrounded information, which is what has been explicitly mentioned or paid explicit attention to by the discourse participants. It is not my purpose here to make explicit proposals about what backgrounded and foregrounded information is, so I will not do full justice to these notions here. Rather, my main objective here is to show that the above change leads to dissolution of the Disjunction Puzzle.

I will make my proposal concrete in the following section, but the explanation proceeds roughly as follows. The update rule for $(\phi \vee \psi)$ is the same as before, so when $(\neg\phi \vee (\phi \wedge \psi))$

is uttered in c, the second disjunct $(\phi \wedge \psi)$ is used to update $c[\neg\neg\phi]$. Now assume that the $\neg\neg\phi$ conveys enough information to satisfy presupposition that ϕ (for the same reasons as before), but does not necessarily make ϕ redundant, which is to say, $c[\neg\neg\phi]$ entails ϕ but does not strictly entail ϕ. Using the informal characterization of backgrounded and foregrounded information above, $c[\neg\neg\phi]$ does not tell discourse participants to pay attention to ϕ, but nonetheless contains enough backgrounded information to entail ϕ . So uttering ϕ in $c[\neg\neg\phi]$ will not be completely redundant, as it has some effects, namely to bring ϕ to the foreground.

5 Update Semantics with Situations

Let us now make the above idea concrete. The simple version of possible world semantics that we have been assuming so far is a relatively coarse model of information and does now allow us to have the necessary two notions of entailment. There are currently many alternative frameworks that are more fine-grained and offer multiple notions of entailment, which include, among others, Truth-Maker Semantics (Fine 2012, 2014) and Inquisitive Semantics (Ciardelli 2009, Mascarenhas 2009, Ciardelli, Groenendijk & Roelofsen 2009, 2013, 2014, 2015, Ciardelli & Roelofsen 2011, Roelofsen 2013). Here I will use Situation Semantics as proposed by Kratzer (1998, 2002, 2016). The main reason why I choose Situation Semantics is because it allows for a rather straightforward extension of Stalnakerian Update Semantics presented in Section 3 by simply adding parts of possible world, or *situations*, to the context. As my aim here is to offer proof of concept for the idea put forward in the previous section, I leave open whether it can be reformulated in other frameworks.

5.1 Situation Semantics

Situations are parts of possible worlds. I denote situations by s, s', etc. Situations are partially ordered by the part-whole relation $\sqsubseteq$: $s \sqsubseteq s'$ means that s is part of s'. It is assumed that some situations are maximal and not part of other situations. They are called possible worlds. Each situation s is assumed to be part of one and only one possible world, so for each s, there is exactly one w such that $s \sqsubseteq w$ and w is a maximal situation. For any situation s, I denote the possible world it is part of by $\mathsf{w}(s)$.

We assume that propositions are sets of situations, rather than sets of possible worlds. Furthermore, propositions are assumed to be upward-closed sets (or equivalently, filters) of situations, i.e. whenever $s \in p$ and $s \sqsubseteq s'$, $s' \in p$. The idea here is that if s makes p true, any situation containing s should also make p true.

Kratzer (2002, 2016) introduces a notion of *exemplifying situation* of proposition p, which roughly means an exact witness of the truth of p. It is defined as follows:

Definition 5. Situation s *exemplifies* proposition p ($s \Vdash p$) if either $s' \in p$ for all $s' \sqsubseteq s$, or s is a minimal situation in p.

See Kratzer (2002, 2016) for examples. There can well be multiple exemplifying situations in a single possible world, depending on the proposition.

5.2 Rebuilding Update Semantics with Situations

We will now define update semantics with situations. Recall that in Stalnakerian Update Semantics, a context was a set of possible worlds. We replace them with situations, but do so by keeping the structure intact. That is, we will have these possible worlds but we will represent each of them as a set of situations, rather than a single possible world. That is, a context c is a set i of sets of situations such that all the situations in i belong to the same possible world. Let us call sets of situations from the same possible world *homogeneous sets*. Thus, a context is a set of homogeneous sets of situations. I denote the common possible world of a homogeneous set i by $\mathsf{w}(i)$.[4] Notice that each $i \in c$ acts as a representation of a possible world via $\mathsf{w}(i)$. For any context c, we denote the set of possible worlds $\{ \mathsf{w}(i) \mid i \in c \}$ by $\mathsf{W}(c)$.

Now we define the update rules. One big change from Stalnakerian Update Semantics is that the new system will be non-eliminative (though still distributive and monotonic). This is due to the update rule for atomic formulae which add situations. More specifically, it adds the exemplifying situations of the proposition and as well as the sub-parts of these exemplifying situations:

$$c[p^i_\pi] \coloneqq \{ i \cup {\downarrow}\mathsf{E}_{\mathsf{w}(i)} \mid i \in c \wedge \mathsf{E}_{\mathsf{w}(i)} \neq \varnothing \}$$

Here $\mathsf{E}_w(p)$ denotes the set of exemplifying situations that are parts of possible world w of the proposition that an atomic formula p represents, i.e. $\mathsf{E}_w(p) \coloneqq \{ s \mid s \Vdash p \wedge s \sqsubseteq w \}$, and ${\downarrow}S$ denotes the downset $\{ s' \mid \exists s \in S\colon\ s' \sqsubseteq s \}$.

In words, the above update rule does the following: in each set i of situations in c, form a set with the exemplifying situations of the proposition belonging to $\mathsf{w}(i)$. If this set is empty, this means p is false in $\mathsf{w}(i)$, so throw away i. If it is not empty, then p is true in $\mathsf{w}(i)$. Then, add the exemplifying situations of p belonging to $\mathsf{w}(i)$ to i, as well as its subsituations (the reason why we add subsituations will be explained below).[5] Notice that it is guaranteed that the resulting set of situations will be homogeneous, because we are only talking about situations in $\mathsf{w}(i)$ for each $i \in c$.

Thus, the above update rule does both elimination and addition. The underlying idea behind this addition component is that update with p^i tells the discourse participants to pay attention to exemplifying situations of p^i as well as its subparts, which one could regard as representations of foregrounded information.

Now, we define update rules for connectives. The rules for $\wedge$ and $\vee$ are as before, but we have to re-define negation because set subtraction no longer works, due to non-eliminativity. We also replace the update rule for $\rightarrow$ with an equivalent rule with $\neg$ and $\wedge$ (see Appendix).

Definition 6. (Update rules)

$$\begin{aligned}
c[p^i_\pi] &\coloneqq \{ i \cup {\downarrow}\mathsf{E}_{\mathsf{w}(i)}(p) \mid i \in c \wedge \mathsf{E}_{\mathsf{w}(i)}(p) \neq \varnothing \} \\
c[\neg\phi] &\coloneqq \{ i \in c \mid \{ i \} [\phi] = \varnothing \} \\
c[(\phi \wedge \psi)] &\coloneqq c[\phi][\psi] \\
c[(\phi \rightarrow \psi)] &\coloneqq c[\neg(\phi \wedge \neg(\phi \wedge \psi))] \\
c[(\phi \vee \psi)] &\coloneqq c[\phi] \cup c[\neg\phi][\psi]
\end{aligned}$$

5.3 Two Notions of Entailment

We are ready to define the two notions of entailment.

Definition 7.

[4] Note that a homogeneous set i might not contain $\mathsf{w}(i) \notin i$, but nothing I will say below would change even if every $i \in c$ always contained $\mathsf{w}(i)$. Assuming so in fact might be convenient in modeling the initial state of a context where nothing has been said yet.

[5] There is a theoretical choice here. Instead of adding all exemplifying situations (and their subsituations), we could add at least one exemplifying situation (and its subsituations). This makes a difference if $\mathsf{E}_w(p)$ contains more than one member. For the purposes of this paper, this more complex update rule is not necessary, and I will stick to the simpler formulation.

- c *entails* ϕ ($c \vDash \phi$) if $\mathsf{W}(c[\phi]) = \mathsf{W}(c)$.
- c *strictly entails* ϕ ($c \Vdash \phi$) if $c[\phi] = c$.

Notice that whenever $c \Vdash \phi$, $c \vDash \phi$, but not vice versa. In terms of the intuitive idea behind this proposal mentioned in the previous section, the situations in the sets i of situations in context c are meant to represent foregrounded information, whereas the possible worlds in $\mathsf{W}(c)$ are meant to represent backgrounded information. Oridnary entailment is entailment by backgrounded information (i.e. at the level of possible worlds), while strict entailment is entailment by foregrounded information (i.e. at the level of sets of situations).

The definition of presupposition satisfaction need no change here, except that the relevant notion of entailment is the new one above. We will redefine redundancy of assertion in terms of strict entailment:

Definition 8. (Redundancy) ϕ *redundant* in c ($c \blacktriangleright \phi$) if any of the following is the case:

$\phi \equiv p^i_\pi$ and	$c \Vdash p^i$
$\phi \equiv \neg\psi$ and	$c \blacktriangleright \psi$ or $c \Vdash \phi$
$\phi \equiv (\psi \wedge \chi)$ and	$c \blacktriangleright \psi$ or $c[\psi] \blacktriangleright \chi$ or $c \Vdash \phi$
$\phi \equiv (\psi \rightarrow \chi)$ and	$c \blacktriangleright \psi$ or $c[\psi] \blacktriangleright \chi$ or $c \Vdash \phi$
$\phi \equiv (\psi \vee \chi)$ and	$c \blacktriangleright \psi$ or $c \blacktriangleright \neg\psi$ or $c[\neg\psi] \blacktriangleright \chi$ or $c \Vdash \phi$

This update semantics has some of the properties of Stalnakerian Update Semantics, but not all. Above all, it is no longer eliminative, as mentioned above. However, a restricted form of eliminativity still holds, namely eliminativity with respect to possible worlds: for any context c and for any $\phi \in \mathcal{L}^p$, $\mathsf{W}(c[\phi]) \subseteq \mathsf{W}(c)$. Also, as in Stalnakerian Update Semantics, entailment is persistent, and so is strict entailment. Since we don't have eliminativity, we cannot state persistence in terms of subset, but it can be stated as follows: for any context c and $\phi \in \mathcal{L}^p$, if $c \vDash \phi$, then $c[\psi_1] \cdots [\psi_n] \vDash \phi$ for any $\psi_1, \ldots, \psi_n \in \mathcal{L}^p$. Similarly for strict entailment. It also holds that for any context c and $\phi \in \mathcal{L}^p$, $c[\phi] \vDash \phi$ and $c[\phi] \Vdash \phi$.

These results for strict entailment imply that the new notion of redundancy is persistent, and also that $c[\phi] \blacktriangleright \phi$ for any context c and $\phi \in \mathcal{L}^p$, as before. This in turn means that we can prove the following. The proofs are parallel to the proofs of Propositions 1 and 2 and make crucial use of the fact that for any context c and $\phi \in \mathcal{L}^p$, $c[\phi]$ makes ϕ redundant (in Appendix, I prove stronger propositions that subsume cases like (13)).

Proposition 4.

- *For every context c, $c \blacktriangleright (\phi \wedge (\phi \wedge \psi))$.*
- *For every context c, $c \blacktriangleright (\phi \rightarrow (\phi \wedge \psi))$.*

In terms of natural language, this means that the present system, just like Stalnakerian Update Semantics, accounts for the infelicity of the following examples.

(7) #Mary is pregnant, and she is and it doesn't show.

(8) #If Mary is pregnant, then she is and it doesn't show.

We also predict the infelicity of the following sentences, which are variants of these.

(13) a. #Mary is expecting a girl, and she is pregnant and it doesn't show.
b. #If Mary is expecting a girl, then she is pregnant and it doesn't show.

The crucial, but arguably natural, assumption here is that a situation exemplifies the proposition that Mary is expecting a girl necessarily includes a situation where Mary is pregnant. Recall that the update rule for atomic formulae not only adds exemplifying situations but also their parts. This allows us to account for these examples.

5.4 Solving the Disjunction Puzzle

Now, we are ready to solve the Disjunction Puzzle. The relevant sentence is of the form $(\neg\phi \vee (\phi \wedge \psi))$. Suppose a context c that does not make ϕ redundant or contradictory. Then $c[\neg\neg\phi]$ will not necessarily make it redundant either, as ϕ might add some new situations. More concretely, consider $(\neg p \vee (p \wedge d))$ as a translation of (1), repeated here.

(1) Either Mary isn't pregnant, or she is and she is expecting a daughter.

Assume that at least some (possibly all) sets $i \in c$ of situations contain no exemplifying situations of the proposition p that Mary is pregnant. Now, $c[\neg\neg p]$ is a subset of c, as $\neg\phi$ by assumption does not add new situations. So there will still be sets i in c that do not contain exemplifying situations of p there either. Then, $c[\neg\neg p][p]$ will be non-redundant, because p will add new situations, namely exemplifying situations of p.

On the other hand, take r_p that presupposes p occurring in $(\neg p \wedge (r_p \wedge d))$. According to the update rule for $\neg$, $c[\neg\neg p]$ is those $i \in c$ whose possible worlds are members of p. That is to say, for each $i \in c[\neg p]$, $\mathsf{w}(i)$ is a world where Mary is pregnant. Therefore, $c[\neg\neg p] \vDash p$ and hence $c[\neg\neg p] \triangleright r_p$.

To reiterate the main point, the crucial feature of the present update semantics is non-eliminativity. Because assertion of p is non-eliminative, while its double-negation counterpart $\neg\neg p$ is eliminative, they are not equivalent. Consequently, $c[\neg\neg p][p]$ is not always redundant. This solves the Disjunction Puzzle, as we have just seen. Furthermore, the notion of eliminativity restricted to possible worlds, which this update semantics does have, accounts for how presuppositions projection in disjunctive sentences in exactly the same manner as under Stalnakerian Update Semantics.

5.5 Negation

There is, however, one remaining problem. The present system predicts the following sentence to be infelicitous.

(14) Either Mary is pregnant, or she isn't and just quit smoking.

The reason why this is predicted to be infelicitous is because a negative sentence like *she isn't (pregnant)* is eliminative and does not add any new situations.

In order to account for this, we have to make negative sentences non-eliminative as well, so that update with *she isn't (pregnant)* would be non-trivial in (14). To achieve this, I propose that a natural language negation does not translate into $\neg$ but into $\sim$.

What does $\sim$ do? I will make the semantics two-dimensional in that we not only keep track of exemplifying situations of atomic propositions, but also anti-exemplifying situations. Recall that exemplifying situations of propositions are exact verifiers of the propositions. Anti-exemplifying situations are exact *falsifiers* of the propositions (cf. similar notions in Truth-Maker Semantics). We assume that each atomic proposition is assigned two sets of situations, a positive extension p^+ and a negative extension p^-, both of which are upward closed sets of situations. Exemplifying and anti-exemplifying situations are now defined as follows:

Definition 9.

- Situation s *exemplifies* proposition p ($s \Vdash p$) if either $s' \in p^+$ for all $s' \sqsubseteq s$, or s is a minimal situation in p^+.
- Situation s *anti-exemplifies* proposition p ($s \dashv\mid p$) if either $s' \in p^-$ for all $s' \sqsubseteq s$, or s is a minimal situation in p^-.

Now, a negative formula $\sim p^i_\pi$ will add anti-exemplifying situations, rather than exemplifying situations. I denote the set of anti-exemplifying situations of p belonging to possible world w by $\mathsf{A}_w(p)$:

$$\mathsf{A}_w(p) \coloneqq \{\, s \mid s \dashv\mid p \wedge s \sqsubseteq w \,\}$$

Using this, I define two modes of update as follows.

Definition 10. (Update rules)

$$\begin{aligned}
c[p^i_\pi]^+ &\coloneqq \{\, i \cup {\downarrow}\mathsf{E}_{\mathsf{w}(i)}(p) \mid i \in c \wedge \mathsf{E}_{\mathsf{w}(i)}(p) \neq \varnothing \,\} \\
c[p^i_\pi]^- &\coloneqq \{\, i \cup {\downarrow}\mathsf{A}_{\mathsf{w}(i)}(p) \mid i \in c \wedge \mathsf{A}_{\mathsf{w}(i)}(p) \neq \varnothing \,\} \\
\\
c[\sim\phi]^+ &\coloneqq c[\phi]^- \\
c[\sim\phi]^- &\coloneqq c[\phi]^+ \\
\\
c[\neg\phi]^+ &\coloneqq \{\, i \in c \mid \{\, i \,\}\, [\phi]^+ = \varnothing \,\} \\
c[\neg\phi]^- &\coloneqq \{\, i \in c \mid \{\, i \,\}\, [\phi]^- = \varnothing \,\} \\
\\
c[(\phi \wedge \psi)]^+ &\coloneqq c[\phi]^+[\psi]^+ \\
c[(\phi \wedge \psi)]^- &\coloneqq c[\phi]^- \cup c[\phi]^+[\psi]^- \\
\\
c[(\phi \rightarrow \psi)]^+ &\coloneqq c[\neg(\phi \wedge \neg(\phi \wedge \psi))]^+ \\
c[(\phi \rightarrow \psi)]^- &\coloneqq c[\neg(\phi \wedge \neg(\phi \wedge \psi))]^- \\
\\
c[(\phi \vee \psi)]^+ &\coloneqq c[\phi]^+ \cup c[\neg\phi]^+[\psi]^+ \\
c[(\phi \vee \psi)]^- &\coloneqq c[\phi]^-[\psi]^-
\end{aligned}$$

Now we can show that $(p \vee (\sim p \wedge s))$ is not always redundant. By assumption, when an assertion of ϕ in context c is interpreted as $c[\phi]^+$ (as assertion of a sentence is conceived of as asserting its truth). Let us assume that both p and $\sim p$ are non-trivial in c. Then, the local context for the second occurrence of p will be $c[\neg p]^+$. Update with $\neg\phi$ is always eliminative, so $c[\neg p]^+$ is a subset of c where for each $i \in c[\neg p]^+$, $\mathsf{w}(i) \notin p$. However, $\sim p$ is still non-redundant in $c[\neg p]^+$, because it can add anti-exemplifying situations of p.

5.6 Section Summary

What I have shown above is a solution to the Disjunction Puzzle that does not make use of the twos mechanism that Mayr & Romoli (2016) crucially refer to, namely incrementality and the grammatical mechanism of exhaustification. Therefore, I conclude that the Disjunction Puzzle does not necessarily motivate them.

The most important components of my proposal are non-eliminativity and the update rule for disjunction $(\phi \vee \psi)$ that involves update with $\neg\phi$, which results in entailment to the negation of ϕ but not strict entailment to it. Among the connectives we discussed—namely, conjunction, conditional and disjunction—only disjunction gives rise to such a discrepancy between entailment and strict entailment.[6] Therefore, the Disjunction Puzzle never arises with conjunction or conditional, which seems to me to be a desirable prediction. As we will see in the next section, Mayr & Romoli's solutions differ from my in this respect.

Before leaving this section, it should be stressed that my main goal here is a modest one and the formal system presented in this section is meant to be proof of concept for the solution to the Disjunction Puzzle based on the idea of strict vs. ordinary entailment. As such, it is not meant to be a general framework for analyzing various pragmatic phenomena, although if it could be used for such purposes it would be ideal. In particular, a version of the update semantics proposed here might be of some use for phenomena like propositional anaphora, but investigation of such phenomena is beyond the scope of this paper and left for another occasion.

6 Mayr & Romoli's Two Solutions

In this final section, I will critically review the two solutions to the Disjunction Puzzle Mayr & Romoli (2016) put forward. In both cases they make crucial use of incrementality and the grammatical mechanism of exhaustification Exh. The two solutions differ in the notion of incrementality. Their common component, Exh, is defined as follows (Chierchia et al. 2012, Fox 2007).

(15) a. $[\![\textsc{Exh}]\!](p)(w) = p(w) \wedge \forall q \in \mathsf{Excl}(p, \mathsf{Alt}(p))[\neg q(w)]$
b. $\mathsf{Excl}(p, P) = \{\, q \in P \mid p \nsubseteq q \wedge \neg r \in P[(p \wedge \neg q) \subseteq r]\,\}$

(Mayr & Romoli 2016:10)

As they themselves explain how this works with examples, I will not repeat them here. Let us now review their two solutions in turn.

6.1 Solution 1: Global Redundancy

Let us assume for the moment that the only alternative in $\mathsf{Excl}(\phi \vee \psi, Alt(\phi \vee \psi))$ is $(\phi \wedge \psi)$. Mayr & Romoli observe that the following two sentences will not be equivalent:

(16) a. Exh(Mary isn't pregnant or she is and she is happy)
b. Exh(Mary isn't pregnant or she is happy)

As they explain on pp.10–11, while Exh in (16b) leads to an exclusive inference that not both disjuncts are true, Exh in (16a) is vacuous, because one of the alternatives in this case, namely *Mary isn't pregnant and she is and she is happy*, is contradictory and its exclusion is trivial. Based on this result, Mayr & Romoli claim that *she is (pregnant)* in (16a) is actually not vacuous, because its presence affects what inference Exh generates.

However, they point out that a simple notion of vacuity will not be able to account for the pair in (17).

(17) a. Exh(Mary isn't pregnant or she is and she is expecting a daughter)
b. Exh(Mary isn't pregnant or she is expecting a daughter)

[6] There might be connectives in natural language that have not been considered here that do refer to $\neg\phi$, e.g. *unless*. I leave this issue open here.

Notice that in this case, the two sentences mean the same thing, because the exclusive inference of (17b) that not both conjuncts are true is vacuous anyway. To solve this, Mayr & Romoli make use of an incremental notion of redundancy. Firstly, they define *global redundancy* as follows:

(18) *Global Redundancy*

a. ψ is globally redundant in ϕ given a context c if ϕ is contextually equivalent to ϕ', where ϕ' is a simplification of ϕ without ψ.

b. ψ is a simplification of ϕ if ψ can be derived from ϕ by replacing nodes in ϕ with their subconstinuents, without deleting any instance of EXH present in ϕ.

(Mayr & Romoli 2016:12)

They use this to define the following notion of incremental redundancy:

(19) *Incremental Redundancy*

a. ψ is incrementally redundant in ϕ given a context c if it is globally redundant in all ϕ', where ϕ' is a possible continuation of ϕ at point ψ

b. ϕ' is a possible continuation of ϕ at point ψ iff it is like ϕ in its structure and number of constituents, but the constinuents pronounced after ψ are possibly different. (Mayr & Romoli 2016:6)

What is crucial here is reference to all possible continuations of ϕ. As Mayr & Romoli explain, this makes *she is (pregnant)* in (17a) non-redundant, because although it is redundant with the particular continuation in this particular sentence (namely, *she is expecting a daughter*), there is a possible continuation which makes it non-redundant, e.g. *she is happy* as in (16a).

In Section 5 of their paper, Mayr & Romoli (2016) actually question the validity of this reasoning, because intuitively, EXH in (16a) does not sound vacuous. That is, it seems to have an exclusive inference that either Mary isn't pregnant or she is happy and not both as much as (16b) does. Then, the crucial interpretive difference between (16a) and (16b) disappears. Since their second solution does not run into this problem, Mayr & Romoli says it is more preferable.

Before moving onto the second solution, I would like to note that my alternative solution to the Disjunction Puzzle does not suffer from this problem either. Although I did not say anything about the exclusive inference of disjunction under my solution, my solution simply does not rely on the non-equivalence of (16a) and (16b). In principle, one can take one's favorite theory of scalar implicature to derive the desired exclusive inference of (16a) without losing our earlier results.

I would also like to point out another problem for Mayr & Romoli's solution under discussion, which has to do with the effect of EXH on the interpretation of *and* occurring in negative contexts. Mayr & Romoli (2016) themselves discuss one such case in Section 5.6, which is reproduced here:

(20) #It's not the case that Mary is expecting a daughter and she is pregnant.

As they themselves point out, this is potentially problematic, because EXH should generate a scalar inference due to the stronger disjunctive alternative in (21).

(21) It's not the case that Mary is expecting a daughter or she is pregnant.

Since the simplification of (20), *It's not the case that Mary is expecting a daughter* has no scalar inference, *she is pregnant* in (20) is rendered non-redundant.

Mayr & Romoli suggest a solution here. The scalar inference is the negation of (21), which says that one of the conjuncts of (20) is true. The assertion of (20) says that not both of the conjuncts are true. But the first conjunct cannot be true while the second conjunct is false, so the overall meaning will simply be Mary is pregnant. And they suggest that this is why the sentence is infelicitous. In their words:

> it is natural to think that a negated conjunct $\neg(p \wedge q)$—like its corresponding disjunction $\neg p \vee \neg q$—should not be assertable unless both $\neg p$ and $\neg q$ have a chance of being true. (p.38)

This constraint, however, would wrongly rule out (22), if Exh is present.

(22) It's not the case that Mary is pregnant and she's expecting a daughter.

To counter this, Mayr & Romoli could capitalize on the fact that the parse without Exh is predicted to be felicitous. But this problem of conjunction in negative environments is not confined to conjunction under negation. Consider the following sentences:

(23) a. Every woman who is expecting a girl is happy.
b. #Every woman who is expecting a girl and is pregnant is happy.

Intuitively, the infelicity of (23b) should be due to the redundant conjunct *is pregnant*, but this is not accounted for by Mayr & Romoli's first solution for the same reason as the conjunction-under-negation example above. Specifically, the simpler sentence in (23a) has no scalar item that could introduce a scalar inference, so Exh would be vacuous. By contrast, (23b) contains *and*, to which *or* is an alternative that would derive a stronger meaning, so Exh will generate a non-trivial inference that not every women who is expecting a girl or is pregnant is happy, which is to say, not every pregnant woman is happy. Then, *is pregnant* is not redundant and (23b) is wrongly predicted to be felicitous. Now compare this to the following felicitous sentence:

(24) Every woman who is pregnant and is expecting a girl is happy.

This seems to (be able to) have a scalar inference that not every pregnant woman is happy, so a parse with Exh exists here, meaning that one conjunct entailing the other is not a problem.[7] Then, (23b) cannot be ruled out by such a constraint.

On the other hand, my analysis has no problem accounting for the infelicity of (24b). Although I only presented a propositional version of the analysis, if it is to be extended to predicate conjunction, *is expecting a girl and is pregnant* will be predicted to be redundant for the same reason as *Mary is expecting a girl and she is pregnant* is redundant. Or in other words, my analysis is completely localist and the presence of Exh higher in the clause would not affect what the local context is. Therefore, it predicts that a conjunction like *Mary is expecting a girl and she is pregnant* is always redundant, no matter where it appears. This seems to me to be a desirable prediction, as the Disjunction Puzzle appears to me to be something particular about disjunction, a point that Mayr & Romoli 2016's first solution misses. For them, redundancy is non-compositional in the sense that operators higher in the clause could in principle affect redundancy. As we will see, the same problem arises with their second solution, to which we now turn.

[7]Note that (24) might have another scalar inference that not every woman is happy, which could be derived in competition with the alternative *Every woman is happy*, but this is a weaker inference, entailed by the inference that not every pregnant woman is happy, so it is unclear whether it is an independent inference. In any case, it does not matter for the reasoning here.

6.2 Solution 2: Schlenkerian local contexts + Exh

Mayr & Romoli's second solution to the Disjunction Puzzle makes use of an incremental definition of local context due to Schlenker (2009):

(25) Local context of d in adb ($\mathsf{lc}(c, d, a_b)$)

$$= \text{the strongest element of} \left\{ x \;\middle|\; \begin{array}{l} x \text{ is an object of the type specified by } d \text{ such that} \\ \text{for any } d' \text{ of the same type as } d \text{ and} \\ \text{every grammatical } b' \text{ that can linearly follow } d, \\ a[c' \wedge d']b' \Leftrightarrow_c ad'b', \\ \text{where } x \text{ is the semantic value of } c' \end{array} \right\}.$$

(Mayr & Romoli 2016:22)

Instead of the global notion of redundancy in the first solution above, computation of redundancy is now done with respect to local contexts, just as in the case of Stalnakerian Update Semantics and Update Semantics with Situations. The only difference to these two systems is how local contexts are defined.

Now, how does this solve the Disjunction Problem? As Schlenker (2009) observes, the local contexts for the disjuncts in $(p \vee q)$ will come out as the same as in Stalnakerian Update Semantics.

(26) a. $\mathsf{lc}(c, p, (\_ \vee q)) = c$
b. $\mathsf{lc}(c, q, (p \vee \_)) = c - [\![p]\!]$

Then, the Disjunction Problem rears its head here. Specifically, for $(\neg p \vee (p \wedge q))$, we have:

$$\mathsf{lc}(c, p, (\neg p \vee (\_ \wedge q))) = c - [\![\neg p]\!] = c \cap [\![p]\!]$$

Consequently, the second occurrence of p is bound to be redundant with respect to its local context $c \cap [\![p]\!]$.

Mayr & Romoli's solution crucially relies on Exh again. That is, they observe that with Exh, the local context for p in Exh$(\neg p \vee (p \wedge q))$ will not be $c - [\![p]\!]$ but simply c:

$$\mathsf{lc}(c, p, \textsc{Exh}(\neg p \vee (\_ \wedge q))) = c$$

Then, the Disjunction Problem simply does not arise. This is Mayr & Romoli's second solution.

Recall, however, that there is empirical reason to think that the local context for the second disjunct is not the same as the global context, but the result of updating the global context with the negation of the first disjunct, namely, the behavior of presupposition in sentences like (10), repeated here.

(10) Either John doesn't smoke, or Mary does too.

For such cases, Mayr & Romoli (2016) suggest that the parse without Exh is used, because without it, the local context for the second disjunct will include the negation of the first disjunct.

This, however, makes a prediction that when we combine the two phenomena, the sentence should be infelicitous. This, however, does not seem to be the case. Consider the following sentence:

(27) Either no student is both smart and aware that they are smart, or some are aware that they are smart but are feeling uneasy about it.

Here, the factive presupposition of the second disjunct that some students are smart is entailed by the negation of the first conjunct, so does not become part of the entire disjunction, but its assertive meaning, then, would be redundant.

Mayr & Romoli could say that the factive presupposition is relatively easy to locally accommodate and the relevant reading of (27) does involve local accommodation, an analytical possibility that they themselves discuss in some detail. I could not construct a more convincing example that is also natural.[8] However, there is another problem for their second solution.

Recall the problem of conjunction in negative contexts. It turns out to be problematic for this account as well. As they themselves discuss, (20), repeated here, is predicted to be felicitous with Exh.

(20) #It's not the case that Mary is expecting a daughter and she is pregnant.

As mentioned before, they suggest a constraint, but the felicity of the version of this sentence where the order of the conjuncts is reversed, (22), must be due to the reading without Exh. This might be so, but the example with *every* (23b) poses a more serious problem. I refrain from going into the details of the computation of Schlenkarian local contexts here, but essentially for the same reason as (20), (23b) will be predicted to be felicitous:

(23b) #Every woman who is expecting a girl and is pregnant is happy.

This is a more serious problem, because as we have already seen, when the order of the two conjuncts is reversed, the sentence becomes felicitous and also seems to be able to have a scalar inference.

Again, to stress, my localist analysis predicts that scalar inferences should not affect redundancy in the relevant examples. In particular, *Mary is expecting a daughter and she is pregnant* is always predicted to be redundant, no matter where it appears. Furthermore, it is a prediction of my analysis that disjunction, but not conjunction or conditional, can give rise to the Disjunction Puzzle, because what is essential is the presence of $\neg$, which creates entailment but not strict entailment.

6.3 Hurford Disjunction

Before closing, I will remark on what Mayr & Romoli (2016) call the problem of Hurford disjunction. In Section 5.4 of their paper, they discuss the following example:

(28) #Either Mary is expecting a daughter, or she is pregnant.

As they observe, this sentence is infelicitous, and its infelicity seems to be due to the redundancy of the disjunct *she is pregnant*. However, neither of their solutions predicts *she is pregnant* to be redundant here. As they explain this in detail, I will not repeat their explanations here.

My analysis also fails to account for the infelicity of (28). Specifically, the local context for the second disjunct is obtained from the global context by subtracting those sets i of situations such that in $\mathsf{w}(i)$, Mary is not expecting a daughter. Then there can well be some sets i such that i contains no exemplifying situations of the proposition that Mary is pregnant (whose worlds

[8] In particular, additive presuppositions of focus particles like *too*, which Mayr & Romoli (2016) assume cannot be easily locally accommodated, do not result in very natural examples. The relevant example would look like *Either not both of Alice and Becky like linguistics, or Becky likes it too but doesn't want to say so.*. It seems to me that this example seems to already presuppose that Alice likes linguistics (which one could see as a case of the proviso problem), and I suspect this has to do with information structural considerations that arise in conjunction with not pronouncing (and hence not focusing) *Alice* in the second disjunct.

can be ones where Mary is not even pregnant). Then, *she is pregnant* should be able to update this local context non-redundantly.

However, I do not think this is necessarily probematic for my analysis. Rather, my analysis can be augmented with a simple global structural notion of redundancy, along the lines of what Mayr & Romoli (2016) call Global Redundancy. Specifically, update with *Mary is expecting a daughter* would add exemplifying situations of the proposition that Mary is expecting a daughter and all their subsituations. Some of these subsituations will be exemplifying situations of the proposition that Mary is pregnant. Then, the overall update effect of (28) will be identical to that of a simpler sentence *Mary is expecting a daughter*, which we could blame for the infelicity of (28).

If this idea is on the right track, it implies that not all cases of redundancy are given the same explanation. This is perhaps a non-null hypothesis, but I do not think it is conceptually unappealing. I followed Stalnaker in defining a semantic notion of which is only concerned with the redundancy of information with respect to local contexts. It seems to me that the kind of redundancy illustrated by (28) is of a different nature, namely, it has to do with the complexity of the uttered linguistic expression (although, needless to say, the meaning it expresses is not irrelevant). It does not seem to me to be farfetched to assume that such metalinguistic considerations are regulated by a separate principle.

A Proofs

In what follows, $\Box$ is a meta-variable ranging over binary connectives (i.e. $\wedge, \vee, \rightarrow$), and IH stands for induction hypothesis. I will only show proofs for $\mathcal{L}$, but the results will carry over to $\mathcal{L}^p$, because presupposition has no effect on assertion.

We define the ranks of formulae in the standard way.

Definition 11. (Rank)

$$\begin{aligned} \mathsf{rank}(p^i) &= 0 \\ \mathsf{rank}(\neg\phi) &= \mathsf{rank}(\phi) + 1 \\ \mathsf{rank}((\phi \Box \psi)) &= \max(\mathsf{rank}(\phi), \mathsf{rank}(\psi)) + 1 \qquad \text{for } \Box \in \{\wedge, \vee, \rightarrow\} \end{aligned}$$

Definition 12. (Local context) For an occurrence o of a formula ψ in ϕ, we define the *local context* $\mathsf{lc}^{\phi}_{c}(o)$ *of* o *with respect to* $c[\phi]$ recursively as follows:

- if $\phi \equiv p^i$, then $\mathsf{lc}^{\phi}_{c}(o) = c$.
- if $\phi \equiv (\neg\chi)$, then $\mathsf{lc}^{\chi}_{c}(o)$.
- if $\phi \equiv (\chi_1 \Box \chi_2)$, then $\mathsf{lc}^{\chi_i}_{c}(o)$, where o is in χ_i for $i \in \{1, 2\}$.

A.1 Stalnakerian Update Semantics

I will first show that $(\phi \rightarrow \psi)$ is definable in terms of $\neg$ and $\wedge$. This means that we only need to prove theorems for the version of $\mathcal{L}$ without $\rightarrow$. From now on, I will mean that language with $\rightarrow$ by $\mathcal{L}$.

Theorem 1. *For any context c and $\phi, \psi \in \mathcal{L}$,* $c[(\phi \rightarrow \psi)] = c[\neg(\phi \wedge \neg\psi)]$.

Proof. $c[(\phi \rightarrow \psi)] = c - (c[\phi] - c[\phi][\psi]) = c - (c[\phi][\neg\psi]) = c - c[(\phi \wedge \neg\psi)] = c[\neg(\phi \wedge \neg\psi)]$. ∎

Theorem 2. *(Eliminativity) For any context c and $\phi \in \mathcal{L}$, $c[\phi] \subseteq c$.*

Proof. Proof by induction on ϕ.

- If $\phi \equiv p^i$, trivial.
- IH: For any ψ such that $\mathsf{rank}(\psi) < n$, $c[\psi] \subseteq c$. Suppose $\mathsf{rank}(()\phi) = n$.
  - If $\phi \equiv (\neg\psi)$, trivial.
  - If $\phi \equiv (\psi \wedge \chi)$, then $c[\phi] = c[\psi][\chi]$. By IH, $c[\psi] \subseteq c$ and $c[\psi][\chi] \subseteq c[\psi]$. By transitivity, $c[\psi][\chi] \subseteq c$.
  - If $\phi \equiv (\psi \vee \chi)$, then $c[\phi] = c[\psi] \cup c[\neg\psi][\chi]$. By IH, $c[\psi] \subseteq c$, and $c[\neg\psi][\chi] \subseteq c[\neg\psi]$, and since the case of negation is trivial, it follows that $c[\neg\psi] \subseteq c$. Then, by transitivity, $c[\neg\psi][\chi] \subseteq c$. Then $c[\psi] \cup c[\neg\psi][\chi] \subseteq c$.

□

Theorem 3. *(Distributivity) For any context c and $\phi \in \mathcal{L}$, $c[\phi] = \bigcup_{w \in c}(\{ w \} [\phi])$.*

Proof. Proof by induction on ϕ.

- When $\phi \equiv p^i$, obvious.
- IH: For all ψ such that $\mathsf{rank}(\psi) < n$, $c[\psi] = \bigcup_{w \in c}(\{ w \} [\psi])$. Suppose $\mathsf{rank}(\phi) = n$.
  - When $\phi \equiv (\neg\psi)$, $c[\phi] = c - c[\psi]$. By IH, $c - c[\phi] = c - \bigcup_{w \in c}(\{ w \} [\psi])$. Since $\{ w \} [\psi]$ is either $\{ w \}$ or $\varnothing$, $c - \bigcup_{w \in c}(\{ w \} [\psi]) = \bigcup_{w \in c}(\{ w \} - \{ w \} [\psi]) = \bigcup_{w \in c}(\{ w \} [\neg\psi]) = \bigcup_{w \in c}(\{ w \} [\phi])$.
  - When $\phi \equiv (\psi \wedge \chi)$, $c[\phi] = c[\psi][\chi]$. By IH, this is $(\bigcup_{w \in c}(\{ w \} [\psi]))[\chi] = \bigcup_{w' \in \bigcup_{w \in c}(\{ w \} [\psi])}(\{ w' \} [\chi]) = \bigcup_{w \in c}(\{ w \} [\psi][\chi]) = \bigcup_{w \in c}(\{ w \} [(\psi \wedge \chi)]) = \bigcup_{w \in c}(\{ w \} [\phi])$.
  - When $\phi \equiv (\psi \vee \chi)$, $c[\psi] \cup c[\neg\psi][\chi] = \bigcup_{w \in c}(\{ w \} [\psi]) \cup \bigcup_{w \in c}(\{ w \} [\neg\psi][\chi]) = \bigcup_{w \in c}(\{ w \} [\psi] \cup \{ w \} [\neg\psi][\chi]) = \bigcup_{w \in c}(\{ w \} [(\psi \vee \chi)]) = \bigcup_{w \in c}(\{ w \} [\phi])$.

□

Theorem 4. *(Monotonicity) For any context c and $\phi \in \mathcal{L}$, if $c' \subseteq c$, then $c'[\phi] \subseteq c[\phi]$.*

Proof. $c'[\phi] = \bigcup_{w \in c'}(\{ w \} \phi) \subseteq \bigcup_{w \in c}(\{ w \} \phi) = c[\phi]$. □

Theorem 5. *(Persistence of entailment) For any context c and $\phi \in \mathcal{L}$, if $c \vDash \phi$, then for any $c' \subseteq c$, $c' \vDash \phi$.*

Proof. Let $c \vDash \phi$ and $c' \subseteq c$. By distributivity, $\bigcup_{w \in c}(\{ w \} [\phi]) = c$. Then $\bigcup_{w \in c'}(\{ w \} [\phi]) = c'$, so $c' \vDash \phi$. □

Theorem 6. *For any context c and $\phi \in \mathcal{L}$, $c[\phi] \vDash \phi$. Or equivalently, $c[\phi] = c[\phi][\phi]$.*

Proof. Proof by induction on ϕ.

- If $\phi \equiv p^i$, then $c[\phi] = \{ w \in c \mid w \vDash p^i \} = \{ w \in \{ w \in c \mid w \vDash p^i \} \mid w \vDash p^i \} = c[\phi][\phi]$.
- IH: if $\mathsf{rank}(\phi) < n$, then $c[\phi] \vDash \phi$. Suppose $\mathsf{rank}(\phi) = n$:

– If $\phi \equiv \neg\psi$, then $c[\phi][\phi] = (c - c[\psi]) - (c - c[\psi])[\psi]$. By eliminativity and monotonicity, $(c-c[\psi])[\psi] \subseteq c[\psi]$, so $(c-c[\psi])-(c-c[\psi])[\psi] = c-c[\psi] = c[\phi]$.
– If $\phi \equiv (\psi \wedge \chi)$, then $c[\phi] = c[\psi][\chi]$. By IH, $c[\psi] \vDash \psi$. By Theorem 5, $c[\psi][\chi] \vDash \psi$. Similarly, by IH,$c[\psi][\chi] \vDash \chi$, and by Theorem 5, $c[\psi][\chi][\psi] \vDash \chi$. Then $c[\phi] = c[\psi][\chi] = c[\psi][\chi][\psi] = c[\psi][\chi][\psi][\chi] = c[\phi][\phi]$.
– if $\phi \equiv (\psi \vee \chi)$, then $c[\phi][\phi] = c[\psi][\psi] \cup c[\phi][\neg\psi][\chi] = (c[\psi] \cup c[\neg\psi][\chi])[\psi] \cup (c[\psi] \cup c[\neg\psi][\chi])[\neg\psi][\chi]$. By distributivity, this is equivalent to, $c[\psi][\psi] \cup c[\neg\psi][\chi][\psi] \cup c[\psi][\neg\psi][\chi] \cup c[\neg\psi][\chi][\neg\psi][\chi]$. By IH, we have $c[\psi][\psi] = c[\psi]$ and also by IH and monotonicity, $c[\neg\psi][\chi][\neg\psi][\chi] = c[\neg\psi][\chi]$. Furthermore, since $c[\psi][\neg\psi][\chi] \subseteq c[\psi]$ and $c[\neg\psi][\chi][\psi] \subseteq c[\psi]$, this is equivalent to $c[\psi] \cup c[\neg\psi][\chi] = c[\phi]$.

□

Theorem 7. *(Persistence of redundancy) For any context c and $\phi \in \mathcal{L}$, if $c \blacktriangleright \phi$, then for any $c' \subseteq c$, $c' \blacktriangleright \phi$.*

Proof. Follows from Theorem 5. □

Theorem 8. *For any context c and $\phi \in \mathcal{L}$, $c[\phi] \blacktriangleright \phi$.*

Proof. Follows from Theorem 6. □

Theorem 9. *For any context c and $\phi \in \mathcal{L}$, if $c \blacktriangleright \phi$, then $c \blacktriangleright \Gamma[\phi]$ where $\Gamma[\phi]$ is a formula containing an occurrence of ϕ.*

Proof. Proof by induction on Γ.

- When $\mathsf{rank}(\Gamma) = 0$, $\Gamma = \phi = p^i$. Then it is trivially true that if $c \blacktriangleright \phi$, $c \blacktriangleright \Gamma[\phi]$.
- IH: If $\mathsf{rank}(\Gamma) < n$, then if $c \blacktriangleright \phi$, then $c \blacktriangleright \Gamma[\phi]$. Suppose $\mathsf{rank}(\Gamma) = n$, and $c \blacktriangleright \phi$.
  – If $\Gamma \equiv \neg\psi$, then by IH, $c \blacktriangleright \psi$. Then by definition, $c \blacktriangleright \Gamma$.
  – If $\Gamma \equiv (\psi \wedge \chi)$, there are two cases. If ϕ occurs in ψ, by IH, $c \blacktriangleright \psi$. Then by definition, $c \blacktriangleright \Gamma$. If ϕ occurs in χ, by IH, $c \blacktriangleright \chi$. By the persistence of redundancy and eliminativity, we also have $c[\psi] \blacktriangleright \chi$. Therefore, $c \blacktriangleright \Gamma$. by IH, $c \blacktriangleright \chi$. By the persistence of redundancy and eliminativity, we also have $c[\psi] \blacktriangleright \chi$. Therefore, $c \blacktriangleright \Gamma$.
  – If $\Gamma \equiv (\psi \vee \chi)$, there are two cases. If ϕ occurs in ψ, then by IH, $c \blacktriangleright \psi$. Then by definition, $c \blacktriangleright \Gamma$. If ϕ occurs in χ, by IH, $c \blacktriangleright \chi$. By the persistence of redundancy and eliminativity, we also have $c[\neg\psi] \blacktriangleright \chi$. Therefore, $c \blacktriangleright \Gamma$.

□

A.2 Update Semantics with Situations

Since $(\phi \rightarrow \psi)$ is defined in terms of $\wedge$ and $\neg$, $\rightarrow$ will be ignored.

Theorem 10. *(Distributivity) For any context c and $\phi \in \mathcal{L}$, $c[\phi] = \bigcup_{i \in c} \{ i \} [\phi]$.*

Proof. Proof by induction on ϕ.

- If $\phi \equiv p^i$, obvious.

- IH: for any ψ such that $\mathsf{rank}(\psi) < n$, $c[\psi] = \bigcup_{i \in c} \{ i \} [\psi]$. Suppose $\mathsf{rank}(\phi) = n$.
  - If $\phi \equiv \neg\psi$, obvious.
  - If $\phi \equiv (\psi \wedge \chi)$, then $c[\phi] = c[\psi][\chi]$. By IH, this is $\bigcup_{j \in c[\psi]} \{ j \} [\chi] = \bigcup_{j \in \bigcup_{i \in c} \{ i \} [\psi]} \{ j \} [\chi] = \bigcup_{i \in c} \{ i \} [\psi][\chi] = \bigcup_{i \in c} \{ i \} [\phi]$.
  - If $\phi \equiv (\psi \vee \chi)$, then $c[\phi] = c[\psi] \cup c[\neg\psi][\chi]$. By IH, this is $\bigcup_{i \in c} \{ i \} [\psi] \cup \bigcup_{i \in c} \{ i \} [\neg\psi][\chi] = \bigcup_{i \in c} (\{ i \} [\psi] \cup \{ i \} [\neg\psi][\chi]) = \bigcup_{i \in c} \{ i \} [\phi]$.

□

It follows from this that we have monotonicity.

Theorem 11. *(Monotonicity) For any context c and $\phi \in \mathcal{L}$, if $c' \subseteq c$, then $c'[\phi] \subseteq c[\phi]$.*

Proof. Same as the proof of Theorem 4. □

Theorem 12. *(Eliminativity with respect to possible worlds) For any context c for any $\phi \in \mathcal{L}$,* $\mathsf{W}(c[\phi]) \subseteq \mathsf{W}(c)$.

Proof. Induction on ϕ.

- If $\phi \equiv p^i$, for any $i \in c$, $\{ i \} [p^i]$ is either $\varnothing$ or $\{ j \}$ for some $j \supseteq i$. Since it is guaranteed that when $\{ i \} [p^i] = \{ j \}$, $\mathsf{w}(i) = \mathsf{w}(j)$, it follows that $\mathsf{W}(c[p^i]) \subseteq \mathsf{W}(c)$.
- IH: for any $\psi \in \mathcal{L}$ such that $\mathsf{rank}(\psi) < n$, $\mathsf{W}(c[\psi]) \subseteq \mathsf{W}(c)$. Suppose $\mathsf{rank}(\phi) = n$.
  - If $\phi \equiv \neg\psi$, $c[\phi] = \{ i \in c \mid \{ i \} [\psi] = \varnothing \} \subseteq c$. Therefore, $\mathsf{W}(c[\phi]) \subseteq \mathsf{W}(c)$.
  - The rest of the proof is parallel to the poof of Theorem 2.

□

Theorem 13. *(Persistence of entailment) For any context c and $\phi \in \mathcal{L}$, if $c \vDash \phi$, then $c[\psi_1] \ldots [\psi_n] \vDash \phi$ for any $\psi_1, \ldots, \psi_n \in \mathcal{L}$.*

Proof. Because $c[\psi_1] \ldots [\psi_n] = c[(\ldots(\psi_1 \wedge \psi_2) \cdots \wedge \psi_n)]$, it is sufficient to prove that if $c \vDash \phi$, then $c[\psi] \vDash \phi$ for any $\psi \in \mathcal{L}$. By definition, $c \vDash \phi$ means $\mathsf{W}(c[\phi]) = \mathsf{W}(c)$. Then, by monotonicity and eliminativity with respect to possible worlds, we have $\mathsf{W}(c[\psi][\phi]) = \mathsf{W}(c[\phi])$, which means $c[\psi] \vDash \phi$. □

Before proving the persistence of strict entailment, I will first prove the following lemma.

Lemma 14. For any set i of situations and $\phi \in \mathcal{L}$, if $\{ i \} [\phi] = \{ i \}$, then for any $j \supseteq i$, $\{ j \} [\phi] = \{ j \}$.

Proof. Proof by induction on ϕ.

- If $\phi \equiv p^i$ and $\{ i \} [p^i] = \{ i \}$, i already contains all the exemplifying situations of p^i in $\mathsf{w}(i)$ and their subsituations. Then, for any $j \supseteq i$, $\{ j \} [p^i] = \{ j \}$.
- IH: For any $\psi \in \mathcal{L}$ such that $\mathsf{rank}(\psi) < n$, if $\{ i \} [\psi] = \{ i \}$, then for any $j \supseteq i$, $\{ j \} [\psi] = \{ j \}$. Suppose $\mathsf{rank}(\phi) = n$.
  - If $\phi \equiv \neg\psi$ and $\{ i \} [\neg\psi] = \{ i \}$, then $\{ i \} [\psi] = \varnothing$. Adding more situations to i will not make ψ true, so for any $j \supseteq i$, $\{ j \} [\psi] = \varnothing$, which means $\{ j \} [\neg\psi] = \{ j \}$.

– Suppose $\phi \equiv (\psi \wedge \chi)$ and $\{i\}[(\psi \wedge \chi)] = \{i\}$. $\{i\}[(\psi \wedge \chi)] = \{i\}[\psi][\chi] = \{i\}$, then it must be the case that $\{i\}[\psi] = \{i\}$ and also $\{i\}[\chi] = \{i\}$. Then by IH, for any $j \supseteq i$, $\{j\}[\psi] = \{j\}$ and $\{j\}[\chi] = \{j\}$, which means $\{j\}[(\psi \wedge \chi)] = \{j\}$.

– Suppose $\phi \equiv (\psi \vee \chi)$ and $\{i\}[(\psi \vee \chi)] = \{i\}$. $\{i\}[(\psi \vee \chi)] = \{i\}[\psi] \cup \{i\}[\neg\psi][\chi] = \{i\}$. If $\{i\}[\psi] = \{i\}$, it follows from IH that for any $j \supseteq i$, $\{j\}[\psi] = \{j\}$ and hence $\{j\}[(\psi \vee \chi)] = \{j\}$. Similarly, If $\{i\}[\neg\psi][\chi] = \{i\}$, then $\{i\}[\neg\psi] = \{i\}$ and $\{i\}[\chi] = \{i\}$. $\{i\}[\neg\psi] = \{i\}$ iff $\{i\}[\psi] = \varnothing$. Then, for any $j \subseteq i$, $\{j\}[\psi] = \varnothing$, so $\{j\}[\neg\psi] = \{j\}$. Also, by IH, for any $j \subseteq i$, $\{j\}[\chi] = \{j\}$. Then in this case too, for any $j \subseteq i$, $\{j\}[(\psi \vee \chi)] = \{j\}$.

$\square$

Theorem 15. *(Persistence of strict entailment) For any context c and $\phi \in \mathcal{L}$, if $c \Vdash \phi$, then $c[\psi_1]\ldots[\psi_n] \Vdash \phi$ for any $\psi_1, \ldots, \psi_n \in \mathcal{L}$.*

Proof. For the same reason as the previous theorem, it is sufficient to prove that if $c \Vdash \phi$, then $c[\psi] \Vdash \phi$ for any $\psi \in \mathcal{L}$. Proof by induction on ψ.

- Suppose $\psi \equiv p^i$. Let $c \Vdash \phi$, or equivalently, $c[\phi] = c$. By distributivity, for any $i \in c$, $\{i\}[\phi] = \{i\}$. Now for each $j \in c[p^i]$, there is exactly one $i \in c$ such that $i \subseteq j$. Then, by Lemma 14, for each $j \in c[p^i]$, $\{j\}[\phi] = \{j\}$, which by distributivity means $c[\phi] = c$.

- IH: For $\chi \in \mathcal{L}$ such that $\mathsf{rank}(\chi) < n$, if $c \Vdash \phi$, then $c[\chi] \Vdash \phi$. Suppose $\mathsf{rank}(\psi) = n$.

  – If $\psi \equiv \neg\chi$, $c[\psi] \subseteq c$. If $c \Vdash \phi$, $c[\phi] = c$ and by distributivity, for each $i \in c$, $\{i\}[\phi] = \{i\}$. Then for each $i \in c[\psi]$, $\{i\}[\phi] = \{i\}$ and therefore $c[\psi][\phi] = c[\psi]$, which means $c[\psi] \Vdash \phi$.

  – If $\psi \equiv (\chi_1 \wedge \chi_2)$, $c[\psi] = c[\chi_1][\chi_2]$. Suppose $c \Vdash \phi$. By IH, $c[\chi_1] \Vdash \phi$. Then by IH again, $c[\chi_1][\chi_2] \Vdash \phi$, so $c[\psi] \Vdash \phi$.

  – If $\psi \equiv (\chi_1 \vee \chi_2)$, $c[\psi] = c[\chi_1] \cup c[\neg\chi_1][\chi_2]$. Suppose $c \Vdash \phi$. By IH, $c[\chi_1] \Vdash \phi$. $c[\neg\chi_1] = \{i \in c \mid \{i\}[\chi_1] = \varnothing\}$. Since $c[\chi_1] \Vdash \phi$, by distributivity, for each $i \in c$, $\{i\}[\chi_1][\phi] = \{i\}[\chi_1]$. Then given that $c[\neg\chi_1] \subseteq c$, for each $i \in c[\neg\chi_1]$, $\{i\}[\chi_1][\phi] = \{i\}[\chi_1] = \varnothing$. Then, trivially, $c[\neg\chi_1] \Vdash \phi$. Then by IH, $c[\neg\chi_1][\chi_2] \Vdash \phi$. Since $c[\chi_1] \Vdash \phi$ and $c[\neg\chi_1][\chi_2] \Vdash \phi$, by distributivity, we have $c[\psi] \Vdash \phi$.

$\square$

Theorem 16. *For any context c and $\phi \in \mathcal{L}$, $c[\phi] \vDash \phi$.*

Proof. Proof by induction on ϕ.

- If $\phi \equiv p^i$, obvious.

- IH: For any context c and $\psi \in \mathcal{L}$ such that $\mathsf{rank}(\psi) < n$, $c[\psi] \vDash \psi$. Suppose $\mathsf{rank}(\phi) = n$.

  – If $\phi \equiv \neg\psi$, obvious.

  – If $\phi \equiv (\psi \wedge \chi)$, $c[\phi] = c[\psi][\chi]$. By IH, $c[\psi] \vDash \psi$. By Theorem 13, $c[\psi][\chi] \vDash \psi$. This means $\mathsf{W}(c[\psi][\chi]) = \mathsf{W}(c[\psi][\chi][\psi])$. Similarly, by IH, $c[\psi][\chi] \vDash \chi$ and by Theorem 13, $c[\psi][\chi][\psi] \vDash \chi$. This means $\mathsf{W}(c[\psi][\chi][\psi]) = \mathsf{W}(c[\psi][\chi][\psi][\chi])$. Then we have $\mathsf{W}(c[(\psi \wedge \chi)]) = \mathsf{W}(c[\psi][\chi]) = \mathsf{W}(c[\psi][\chi][\psi][\chi]) = \mathsf{W}(c[(\psi \wedge \chi)][(\psi \wedge \chi)])$. Therefore, $c[\phi] \vDash \phi$.

– If $\phi \equiv (\psi \vee \chi)$, $c[\phi] = c[\psi] \cup c[\neg\psi][\chi]$. By IH, $c[\psi] \vDash \psi$, which means $\mathsf{W}(c[\psi]) = \mathsf{W}(c[\psi][\psi])$. Note also that $c[\psi][\neg\psi] = \varnothing$ and hence $c[\psi][\neg\psi][\chi] = \varnothing$. Now, it is obvious that $c[\neg\psi] \vDash \neg\psi$, and by Theorem 13, $c[\neg\psi][\chi] \vDash \neg\psi$. Thus, $\mathsf{W}(c[\neg\psi][\chi]) = \mathsf{W}(c[\neg\psi][\chi][\neg\psi])$. Also by IH, $c[\neg\psi][\chi] \vDash \chi$ and by Theorem 13 $c[\neg\psi][\chi][\neg\psi] \vDash \chi$, which means $\mathsf{W}(c[\neg\psi][\chi][\neg\psi]) = \mathsf{W}(c[\neg\psi][\chi][\neg\psi][\chi])$. It is obvious that $c[\neg\psi][\chi][\psi] = \varnothing$. Now, by distributivity, $\mathsf{W}(c[\phi]) = \mathsf{W}(c[\psi] \cup c[\neg\psi][\chi]) = \mathsf{W}(c[\psi]) \cup \mathsf{W}(c[\neg\psi][\chi]) = \mathsf{W}(c[\psi][\psi]) \cup \mathsf{W}(c[\neg\psi][\chi][\neg\psi][\chi]) = \mathsf{W}(c[\psi][\psi] \cup c[\neg\psi][\chi][\neg\psi][\chi]) = \mathsf{W}((c[\psi][\psi] \cup c[\neg\psi][\chi][\psi]) \cup (c[\psi][\neg\psi][\chi] \cup c[\neg\psi][\chi][\neg\psi][\chi])) = \mathsf{W}(c[\phi][\psi] \cup c[\phi][\neg\psi][\chi]) = \mathsf{W}(c[\phi][\phi])$. Therefore, $c[\phi] \vDash \phi$.

□

Theorem 17. *For any context c and $\phi \in \mathcal{L}$, $c[\phi] \Vdash \phi$.*

Proof. Proof by induction on ϕ.

- If $\phi \equiv p^i$, obvious.
- IH: For any context c and $\psi \in \mathcal{L}$ such that $\mathsf{rank}(\psi) < n$, $c[\psi] \Vdash \psi$. Suppose $\mathsf{rank}(\phi) = n$.
  – If $\phi \equiv \neg\psi$, obvious.
  – If $\phi \equiv (\psi \wedge \chi)$, $c[\phi] = c[\psi][\chi]$. By IH, $c[\psi] \Vdash \psi$. By Theorem 15, $c[\psi][\chi] \Vdash \psi$. By IH, $c[\psi][\chi] \Vdash \chi$, and by Theorem 15, $c[\psi][\chi][\psi] \Vdash \chi$. Then $c[\phi] = c[\psi][\chi] = c[\psi][\chi][\psi] = c[\psi][\chi][\psi][\chi] = c[\phi][\phi]$. Therefore $c[\phi] \Vdash \phi$.
  – If $\phi \equiv (\psi \vee \chi)$, $c[\phi] = c[\psi] \cup c[\neg\psi][\chi]$. By IH, $c[\psi] \Vdash \psi$, so $c[\psi] = c[\psi][\psi]$. Since $c[\psi][\neg\psi] = \varnothing$, $c[\psi][\neg\psi][\chi] = \varnothing$. It is easy to see that $c[\neg\psi] \Vdash \neg\psi$. Then by Theorem 15, $c[\neg\psi][\chi] \Vdash \neg\psi$. By IH, $c[\neg\psi][\chi] \Vdash \chi$ and by Theorem 15, $c[\neg\psi][\chi][\neg\psi] \Vdash \chi$, so $c[\neg\psi][\chi][\neg\psi] = c[\neg\psi][\chi][\neg\psi][\chi]$. Notice also that $c[\neg\psi][\chi][\psi] = \varnothing$. Then, $c[\phi] = c[\psi] \cup c[\neg\psi][\chi] = (c[\psi][\psi] \cup c[\neg\psi][\chi][\psi]) \cup (c[\psi][\neg\psi][\chi] \cup c[\neg\psi][\chi][\neg\psi][\chi]) = c[\phi][\psi] \cup c[\phi][\neg\psi][\chi] = c[\phi][\phi]$.

□

Theorem 18. *(Persistence of redundancy) For any context c and $\phi \in \mathcal{L}$, if $c \blacktriangleright \phi$, then for any $\psi_1, \ldots, \psi_n \in \mathcal{L}$, $c[\psi_1]\cdots[\psi_n] \blacktriangleright \phi$.*

Proof. Follows from Theorem 15. □

Theorem 19. *For any context c and $\phi \in \mathcal{L}$, $c[\phi] \blacktriangleright \phi$.*

Proof. Follows from Theorem 17. □

Proposition 5. *Suppose that $\phi, \phi^+ \in \mathcal{L}$, and for any context c', $c'[\phi^+] \Vdash \phi$. Then, the following are true.*

1. *For every context c, $c \blacktriangleright (\phi^+ \wedge (\phi \wedge \psi))$.*
2. *For every context c, $c \blacktriangleright (\phi^+ \to (\phi \wedge \psi))$.*

Proof. 1. $c \blacktriangleright (\phi^+ \wedge (\phi \wedge \psi))$ if (i) $c \blacktriangleright \phi^+$ or (ii) $c[\phi^+] \blacktriangleright (\phi \wedge \psi)$ or (ii) $c \Vdash (\phi^+ \wedge (\phi \wedge \psi))$. $c[\phi^+] \blacktriangleright (\phi \wedge \psi)$ if (iv) $c[\phi^+] \blacktriangleright \phi$ or (v) $c[\phi^+][\phi] \blacktriangleright \psi$ or (vi) $c[\phi^+] \Vdash (\phi \wedge \psi)$. (iv) follows from the assumption that $c[\phi^+] \Vdash \phi$. The proof for 2. is parallel. □

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