# Domain variables, homogeneity projection, and two notions of entailment

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## 1 Homogeneity and monotonicity

### 1.1 Simple cases

**Homogeneity effect**: Generally, plural definites are interpreted (quasi-)universally in UE environments, (1a), and (quasi-)existentially in DE environments, (1b).

(1) a. Every student likes **these three professors**.
$\approx$ Every student likes each of these three professors

b. No student likes **these three professors**.
$\approx$ No student likes any of these three professors

### 1.2 A caveat: non-maximal readings

We are not interested in non-maximal readings in this talk, but it should be kept in mind that the non-maximal and existential reading in UE contexts are often hard to distinguish (similarly for the non-maximal and universal reading in DE contexts).

(2) a. The windows are open. Burglars might get in!!
b. The final-year students are unhappy about my course.

Luckily for us, non-maximal readings become harder to access with numerals, as in (1), although this might not be a foolproof way of blocking non-maximal readings (see Križ 2016: §4.6). We also need to be careful with non-maximal readings under negation, as in (3).

(3) a. This participant didn't answer the demographic questions. b. John didn't remember the four keywords.

### 1.3 Non-monotonic environments

Generally, when a plural definite is placed in a non-monotonic environment, both universal and existential readings are simultaneously observed: The positive part of the meaning is based on its universal reading and the negative part of the meaning is based on its existential reading (Križ 2015, Križ & Chemla 2015, Križ & Spector 2021).[1]

(4) a. Exactly one student liked **these three courses**.
≈ One student liked all three courses, the other students didn't like any of them.
b. Only John has been to **these three countries**.
≈ John has been to all three countries, the other relevant people hasn't been to any of them.

NB: We don't want to test plural quantifiers, because they may license cumulative readings, which are hard to distinguish from existential readings for positive meanings and from universal readings for negative meanings.

(5) <u>Exactly 10 students</u> took **the two advanced courses** in their first year.

Polar questions are another non-monotonic context:

(6) Does John like **those three professors**?
a. Yes = John likes all of them b. No = John doesn't like any of them

The same generalisation applies to non-monotonicity that arises due to presuppositions:

(7) The professor is <u>unaware</u> that Mary took **the three intro courses** last semester.

[1]More specifically, we are interested in 'bi-polar environments', which have both positive and negative entailments. Not all non-monotonic environments are bi-polar, e.g., arguments of collective predicates.

a. Presupposition (+): Mary took all three intro courses last semester.
b. Assertion (-): The professor doesn't know of any of the three intro courses that Mary took it.

(8) John forgot to call **his three sisters** today.

a. Presupposition (+): John was supposed to call all of his three sisters.
b. Assertion (-): John did not call any of his three sisters.

(9) John started liking **these three professors**.

a. Presupposition (-): Before the ref time, John didn't like any of the three professors.
b. Assertion (+): Since the ref time, he likes all three professors.

### 1.4 Towards an analysis

Take the implicative verb *forgot*. We adopt the Sauerland notation for presuppositions $\frac{\text{presupposition}}{\text{assertion}}$, and simplify intensionality. We omit the presuppositions triggered in $P$ as well.

(10) $[\![\textbf{forgot}]\!] = \lambda P_{\langle e,t\rangle}.\lambda x_e.\ \frac{\Box P(x)}{\neg P(x)}$

It is possible to develop a theory of homogeneity such that the definite plural in $P$ will give rise to a universal reading whenever $P$ is evaluated in a UE context, and an existential reading whenever $P$ is evaluated in a DE context (cf., Križ 2015). More on this later.

## 2 Restrictors of *every* and *no*

The restrictors of *every* and *no* are non-monotonic contexts with an UE existence presupposition, and a DE assertion, but plural definites there don't behave as expected from the above generalisation.

### 2.1 *Every*

(11) Every student who took **the two intro courses** in their first year took my seminar in their second year.

The ‘mixed interpretation’ expected from the generalisation doesn’t seem to be available.

(12) a. Presupposition (+): There is a student who took both intro courses in their first year.
b. Assertion (-): Every student who took one or both of the intro courses in their first year took my seminar in their second year.

Rather, the most natural interpretation is ‘doubly universal’:

(13) a. Presupposition: There is a student who took both intro courses in their first year.
b. Assertion: Every student who took both intro courses in their first year took my seminar in their second year.

We think a ‘doubly existential reading’ is also available:[2]

(14) a. Presupposition: There is a student who took one or both of the intro courses in their first year.
b. Assertion: Every student who took one or both of the intro courses in their first year took my seminar in their second year.

That such doubly existential readings are available is clearer with examples like (15).[3] (Doubly universal readings are available too.)

(15) a. In this oblast, every resident who speaks these two Tungusic languages is over 70.
b. Every immigrant who lives in the five Nordic countries is worried.

These doubly existential readings are different from non-maximal readings. Compare (15) with:

(16) a. At least one resident who speaks these two Tungusic languages is over 70.
b. Ivan speaks these two Tungusic languages.
c. The rich man who speaks these two Tungusic languages is over 70.

(17) a. At least one immigrant who lives in the five Nordic countries is worried.
b. ??John lives in the five Nordic countries.
c. ??The rich man who lives in the five Nordic countries is worried.

[2]Note that this reading seems to presuppose that both intro courses were taken by some students, which is more than just an existential reading of the plural definite. This is clearer for (15a). We’ll come back to this point in the end.

[3]We avoid *everyone*, as it might license a cumulative reading, given its compatibility of collective predicates like *gather*.

**Observation**: A plural definite in the restrictor of *every* gives rise to ambiguity between a doubly universal reading and a doubly existential reading; a mixed reading is not available.

Note: The absence of the mixed reading is actually not so easy to demonstrate convincingly, given the (optional) availability of the doubly existential reading and the entailment relation among the three readings, but at least it is not the first interpretation one perceives, unlike other non-monotonic examples we saw above.

### 2.2 Notes on inverse linking (to be skipped)

In some cases what looks like a doubly existential reading can be derived via the universal reading + inverse linking + distributivity (as well).

(18) a. Every speaker of these two Tungusic languages is over 70.
b. Every immigrant in the five Nordic country is worried.

(19) [these two Tungusic languages]
$\Delta$ [$\lambda x$ [every speaker of $x$ is over 70]]

Comparison with upward monotonic cases suggests that this derivation should be possible:

(20) a. At least one speaker of these two Tungusic languages is below 70.
b. At least one immigrant in the five Nordic countries is happy.

Such derivations are unavailable for cases involving a finite relative clause, as in (15) vs. (16)/(17).

So for the doubly existential reading of (20), we don't want to scope out the definite plural.

(But note that it's not simply existential, as remarked in fn. 2, because it presupposes that both languages are still spoken.)

### 2.3 *No*

Definite plurals in the restrictor of *no* behave similarly:

(21) No student who took **the two intro courses** in their first year took my seminar in their second year.

Doubly universal reading:

(22) a. Presupposition: There is a student who took both intro courses in their first year.
b. Assertion: No student who took both intro courses in their first year took my seminar in their second year.

Doubly existential reading:

(23) a. Presupposition: There is a student who took one of both intro courses in their first year.
b. Assertion: No student who took one or both intro courses in their first year took my seminar in their second year.

Similarly:

(24) a. No resident who speaks these two Tungusic languages is under 70.
b. No immigrant who lives in the five Nordic countries is happy.

The mixed reading that seems to be absent is:

(25) a. Presupposition (+): There is a student who took both intro courses in their first year.
b. Assertion (-): No student who took one or both intro courses in their first year took my seminar in their second year.

### 2.4 More examples (to be skipped)

Doubly universal reading prominent:

(26) a. Every student who correctly answered these ten questions passed.
b. Every linguist that has met my two supervisors likes one of them but hates the other.

(27) a. No student who correctly answered these ten questions failed.
b. No linguist who has met my two PhD students likes both of them.

Doubly existential reading prominent:

(28) a. Every employee who was in these three buildings at the time of the explosion was killed.
b. Every linguist who went to these two graduate schools eventually landed a descent job.

(29) a. No employee who was in these three buildings at the time of the explosion survived.
b. No linguist who went to these two graduate schools has a shitty job.

Ambiguous:

(30) a. Every single student who is taking **my two semantics courses** is smart.
b. No student who is taking **my two semantics courses** is dumb.

(31) Every time I have a meeting with **these three guys**, I get depressed afterwards.

### 2.5 Interim summary

In the restrictors of *every* and *no*, definite plurals give rise to the same reading in the presupposition and assertion: either universal in both or existential in both. Mixed readings do not seem to be available.

We hereafter assume that only doubly universal and doubly existential readings are possible.

The readings of definite plurals in the restrictors of *every* and *no* are unlike the readings of definite plurals in other non-monotonic contexts.

Recall the implicative verb *forgot*:

(32) Jimmy forgot to call **his three sisters**.
a. Presupposition: Jimmy (has sisters) and was supposed to call all of his three sisters.
b. Assertion: Jimmy did not call any of his three sisters.

(33) $[\![\textbf{forgot}]\!] = \lambda P_{\langle e,t\rangle}.\lambda x_e.\ \dfrac{\Box P(x)}{\neg P(x)}$

Now, compare the 'classical' lexical entry for *every* (ignoring the presuppositions triggered in $Q$):

(34) $[\![\textbf{every}]\!] = \lambda P_{\langle e,t\rangle}. \dfrac{\exists x[P(x) = 1]}{\lambda Q_{\langle e,t\rangle}.\text{ for each } x \in \{\, x \mid P(x) = 1 \,\}, Q(x) = 1}$

Suppose that the restrictor $P$ contains an (unembedded) definite plural. According to (34), $P$ is evaluated once in the presupposition and once in the assertion. Then the definite plural should receive a universal reading in the presupposition, and an existential reading in the assertion, in a way parallel to (33), which will be a mixed reading.

**Puzzle**: Why do plural definites in the restrictors of *every* and *no* cannot receive mixed readings? How do we derive doubly universal and doubly existential readings?

## 3 Positive presuppositions are necessary for universal readings

**Claim**: A doubly universal readings is available when there is an existence presupposition that makes the position of the definite plural UE.

### 3.1 Weak *no*

*No* tends to have an existence presupposition in subject position, (35a); it sometimes does in object position too, (35b).

(35) a. No student who managed to solve this difficult problem has ever failed my exam.
b. I've failed no student who managed to solve this difficult problem.

In these cases, doubly universal readings are available (so are doubly existential readings):

(36) a. No student who managed to solve these three problems has ever failed my exam.
b. I've failed no student who managed solve these three problems.

Under the reading of *no* without an existence presupposition ('weak *no*'), a doubly universal reading doesn't seem to be available.

(37) a. There was no student who could solve these three problems in my class.
b. I've so far had no student who could solve these three problems.

But note that universal readings of plural definites are not entirely absent in DE contexts, e.g., (3) (see also Križ & Chemla 2015, Augurzky et al. 2022). The claim here is that (37) is at the baseline level, while universal readings are more prominent in (36).

### 3.2 Positive assertion and negative presupposition

By inserting negation, we can turn the position of the plural definite DE in the presupposition and UE in the assertion.

(38) Every student who didn't solve **the three easy problems** failed.

The prominent reading is doubly existential:

(39) a. Presupposition (-): There was a student who didn't solve any of the three easy problems.
b. Assertion (+): Every student who didn't solve any of the three easy problems failed.

Here, again, a mixed reading is unavailable:

(40) a. Presupposition (-): There was a student who didn't solve any of the three easy problems.
b. Assertion (+): Every student who didn't solve all of the three easy problems failed.

The doubly universal reading seems to be difficult to get too.

(41) a. Presupposition (-): There was a student who didn't solve all of the three easy problems.
b. Assertion (+): Every student who didn't solve all of the three easy problems failed.

This reading might not be impossible, but again, a universal reading is not completely unavailable in sentences like this, regardless of the monotonicity profile (Križ & Chemla 2015, Augurzky et al. 2022).

(42) a. At least one student who didn't solve **the three easy problems** failed.
b. The boy who didn't solve **the three easy problems** failed.

The observations above suggest that the doubly universal reading is facilitated by an existence presupposition of *no/every* that makes the position of the definite plural UE, but not by an assertion that does so.

## 4 Domain variables

**Idea**: The presupposition and assertion of a strong quantifier (including *every* and 'strong *no*) refer to one and the same domain of quantification (cf. Geurts & van der Sandt 1999). Homogeneity is computed only once in fixing the domain.

### 4.1 Strong quantifiers and their domains

Let us (tentatively) postulate a domain variable $X$ in the LF representation. For the sake of concreteness we assume that it refers to a set of individuals (we could alternatively assume its referent to be a plurality of individuals). We ignore the presuppositions triggered in $Q$ to simplify.

(43) $$[\![\textbf{every}_X]\!]^g = \lambda P_{\langle e,t\rangle}.\ \frac{X \in \text{dom}(g) \text{ and } g(X) \neq \varnothing \text{ and for each } x \in g(X), P(x) = 1}{\lambda Q_{\langle e,t\rangle}.\ \text{for each } x \in g(X), Q(x) = 1}$$

Remarks:

- We focus on the distributive reading, so we've built in distributivity (You could move it to $P$ and $Q$, depending on how you account for the distributivity of *every*).
- Crucially, the restrictor denotation $P$ is referenced only once in the presupposition, unlike in the classical entry in (34).

'Strong *no*' can be analyzed similarly.

(44) $$[\![\textbf{no}_X]\!]^g = \lambda P_{\langle e,t\rangle}.\ \frac{X \in \text{dom}(g) \text{ and } g(X) \neq \varnothing \text{ and for each } x \in g(X), P(x) = 1}{\lambda Q_{\langle e,t\rangle}.\ \text{for each } x \in g(X), Q(x) = 0}$$

'Weak *no*' will be:

(45) $$[\![\textbf{no}]\!]^g = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\ \text{for each } x \in D_e \text{ such that } P(x) = 1, Q(x) = 0$$

As discussed below, this account runs into an issue with NPI licensing. We'll propose a dynamic semantic solution to it.

But first, we will use the above entries to derive doubly universal readings using particular one theory of homogeneity. We will deal with the doubly existential reading after talking about NPI licensing.

To save space, we'll focus on *every* from now on.

### 4.2 Homogeneity in restrictors

There are several competing theories of homogeneity (Križ 2015, 2016, Križ & Spector 2021, Bar-Lev 2018; see Križ 2019 for an overview).

We will adopt Križ's idea that the homogeneity of definite plurals is due to the (non-trivially) trivalent meaning they give rise to.

**Twist**: In order to account for homogeneity effects in presuppositions and assertions separately, we have to assume that each dimension of meaning has trivalent meaning.

If a definite plural occurs in the nuclear scope of *every*, it triggers a homogeneity effect in the assertion = makes the assertion trivalent:

(46) Every$_X$ student solved the three problems.

a. Presupposition is $\begin{cases} 1 & \text{if } g(X) \neq \varnothing \text{ and each member of } g(X) \text{ is a student} \\ 0 & \text{if } g(X) = \varnothing \text{ or some member of } g(X) \text{ is not a student} \end{cases}$

b. Assertion is $\begin{cases} 1 & \text{if each member of } g(X) \text{ solved all three problems} \\ 0 & \text{if at least one member of } g(X) \text{ solved none of the three problems} \\ \# & \text{otherwise} \end{cases}$

Križ accounts for non-maximal readings by assuming that some cases of $\#$ can be regarded practically as $1$. We are not interested in non-maximal readings here, so we'll ignore this aspect of his theory.

Crucially, if the restrictor contains a definite plural, the homogeneity effect manifests itself only in the presupposition.

(47) Every$_X$ student who solved the three problems passed.

a. Presupposition is $\begin{cases} 1 & \text{if } g(X) \neq \varnothing \text{ and each member of } g(X) \text{ is a student that solved all three probems} \\ 0 & \text{if } g(X) = \varnothing \text{ or at least one member of } g(X) \text{ is not a student or didn't solve any of the three problems} \\ \# & \text{otherwise} \end{cases}$

b. Assertion is $\begin{cases} 1 & \text{if each member of } g(X) \text{ passed} \\ 0 & \text{if some member of } g(X) \text{ didn't pass} \end{cases}$

Such a trivalent presupposition is satisfied with respect to a context $c$ if in every possible world in $c$, the presupposition denotes $1$. Consequently, we perceive a universal presupposition for (47). Since that fixes the value of $X$, we get a doubly universal reading.

The trivalent presupposition is predicted to have non-maximal readings in certain contexts, but we'll put that aside for today (but this prediction seems to be on the right track).

### 4.3 Other non-monotonic presuppositional expressions

In other non-monotonic environments, a definite plural triggers a homogeneity effect in both dimensions of meaning.

(48) Alice is <u>unaware</u> that Ben solved **the three problems**.

a. Presupposition is $\begin{cases} 1 & \text{if Ben solved all three problems} \\ 0 & \text{if Ben solved none of the three problems} \\ \# & \text{otherwise} \end{cases}$

b. Assertion is $\begin{cases} 1 & \text{if for each } w \in \text{Dox}_a \text{ Ben solved all three problems in } w \\ 0 & \text{if for some } w \in \text{Dox}_a \text{ Ben solved at least one of the three problems in } w \\ \# & \text{otherwise} \end{cases}$

(49) John <u>forgot</u> to call **his sisters**.

a. Presupposition is $\begin{cases} 1 & \text{if John has sisters and was supposed to call all his sisters} \\ 0 & \text{if John doesn't have sisters or was not supposed to call any of his sisters} \\ \# & \text{otherwise} \end{cases}$

b. Assertion is $\begin{cases} 1 & \text{if John didn't call any of his sisters} \\ 0 & \text{if John called all of his sisters} \\ \# & \text{otherwise} \end{cases}$

(50) I only know a BRITISH professor who taught **these three students**.

a. Presupposition is $\begin{cases} 1 & \text{if I know a British professor who taught all three students} \\ 0 & \text{if I know no British professor who taught any of the three students} \\ \# & \text{otherwise} \end{cases}$

b. Assertion is $\begin{cases} 1 & \text{if for all nationalities X distinct from British, I know no X professor who taught any of the three students} \\ 0 & \text{if for some nationality X distinct from British, I know a X professor who taught all three students} \\ \# & \text{otherwise} \end{cases}$

The crucial difference from the quantificational determiners is that there's no common domain/variable that the two dimensions of meaning are both about. Consequently, the definite plural is evaluated twice, once in the presupposition and once in the assertion, giving rise to a mixed reading.

**Open questions**: Why are quantificational determiners special in this particular way? Are there other items that trigger similar meanings (see Appendix for conditionals)?

## 5 NPI licensing

### 5.1 Strawson monotonicity

*Every* and *no* license NPIs in their restrictors.

(51) a. Everyone who ever lived in London hates it.
b. No one who ever lived in London loves it.
c. *Most people who ever lived in London hate it.

They even license minimizers.

(52) a. Everyone who lifted a finger to help me was a woman.
b. No one who lifted a finger to help me was a man.
c. *Most people who lifted a finger to help me were men.

Due to the existence presupposition, *every* is not downward monotonic.

(53) a. Every cat is sleeping.
b. Every cold-blooded cat is sleeping.

(54) a. I lost everything.
b. I lost every Ferrari I had.

Strawson monotonicity (von Fintel 1999):

(55) Generalized Strawson entailment
Let $\tau$ be a conjoinable type. For any $x, y \in D_\tau$, $x \overset{\text{S}}{\Rightarrow} y$ iff
a. $\tau = t$ and $x = 0$ or $y = 1$ or;
b. $\tau = \langle \sigma_1, \sigma_2 \rangle$ and for each $z \in$ dom$(x) \cap$ dom$(y)$, $x(z) \overset{\text{S}}{\Rightarrow} y(z)$

(56) Strawson UE and DE
Let $f$ be a function of type $\langle \sigma, \tau \rangle$ where both $\sigma$ and $\tau$ are conjoinable types.
a. $f$ is Strawson upward entailing (SUE) iff for each $x, y \in D_\sigma$ such that $x \overset{\text{S}}{\Rightarrow} y$, $f(x) \overset{\text{S}}{\Rightarrow} f(y)$.
b. $f$ is Strawson downward entailing (SDE) iff for each $x, y \in D_\sigma$ such that $x \overset{\text{S}}{\Rightarrow} y$, $f(y) \overset{\text{S}}{\Rightarrow} f(x)$.

### 5.2 Problem

Our determiners are technically both SUE and SDE.

(57) $[\![\textbf{every}_X]\!]^g = \lambda P_{\langle e,t \rangle}. \dfrac{X \in \text{dom}(g) \text{ and } g(X) \neq \varnothing \text{ and for each } x \in g(X), P(x) = 1}{\lambda Q_{\langle e,t \rangle}. \text{ for each } x \in g(X), Q(x) = 1}$

Consider, ‘Every$_X$ $P_1$ $Q$’ and ‘Every$_X$ $P_2$ $Q$’.

- Whenever the presuppositions of both sentences are satisfied, $g(X) \neq \varnothing$ and for each $x \in g(X)$, $P_1(x) = 1$ and $P_2(x) = 1$.
- Since the assertions of these sentences will be identical (‘for each $x \in X$, $Q(x) = 1$’), they Strawson entail each other.

Furthermore, other strong determiners will similarly be both SUE and SDE, e.g.

(58) $$[\![\textbf{most}_X]\!]^g = \lambda P_{\langle e,t\rangle}.\ \frac{X \in \text{dom}(g) \text{ and } g(X) \neq \varnothing \text{ and for each } x \in g(X), P(x) = 1}{\lambda Q_{\langle e,t\rangle}.\ |g(X) \cap \{\, x \mid Q(x) = 1 \,\}| \div |g(X)| \gg 0.5}$$

But not all determiners license NPIs. So we fail to capture the distribution of NPIs.

## 6 Going dynamic

Our account makes crucial use of anaphora: The presupposition and assertion are about the same domain variable $X$ whose value is constant thanks to the assignment function, which is the standard formal mechanism used to account for anaphora in static semantics.

In the static account we assumed above, this variable $X$ needed to be represented syntactically, because that's the only place that both dimensions of meaning commonly have access to, so to speak. But this created the issue of NPI licensing.

We solve this dilemma by moving the variable in the semantics, and letting the two dimensions of meaning directly communicate with each other via dynamic binding.

### 6.1 A rough illustration

We propose to get rid of the domain variable from the syntax, and put it in the semantics.

- The presupposition is existential, introducing the domain $X$.
- The assertion dynamically refers to $X$.

Informally:

(59) Every student passed.
  a. Presupposition: $\underline{\exists} X[X \neq \varnothing \text{ and } X = \texttt{STUDENT}]$
  b. Assertion: each member of $X$ passed

  where $\underline{\exists}$ is the dynamic existential quantifier binding $X$ in the assertion.

As we will see, *every* will be SDE but not SUE, because the two statements 'Every $P_1$ $Q$' and 'Every $P_2$ $Q$' will now be about (potentially) different domains.

### 6.2 Dynamic semantics with generalised quantifiers and presupposition

In order to formally implement the above idea, we have to turn everything dynamic. We'll use Dynamic Predicate Logic (Groenendijk & Stokhof 1991) as our intermediate language, except:

- Variables may denote plural entities (used for domain variables).
- Heimian presuppositions (Heim 1982): presupposition failure is represented by $\star$. Note that we distinguish $\star$ from homogenetiy failure $\#$.
- To properly deal with presuppositions, we 'lift' the relational semantics of DPL to context change potentials over world-assignments pairs.
- Presuppositions have dynamic effects (Beaver 1992, Elliott & Sudo 2020).

(60) $c +$ **Every**$^X$ **student passed**

$$= \begin{cases} \star & \text{if for some } (w, g) \in c, \texttt{STUDENT}_w = \varnothing \\ c[\neg\neg\exists x(\mathsf{STUDENT}(x)) \wedge \exists X(X = \sigma x(\mathsf{STUDENT}(x)))][\forall x[x \sqsubseteq_a X \rightarrow \mathsf{PASSED}(x)]] & \text{otherwise} \end{cases}$$

We will abbreviate the presupposition as '$\exists X(X = \sigma^+ x(\mathsf{STUDENT}(x)))$' below.

We have to make this analysis (even) more complex in order to account for homogeneity.

### 6.3 Bidimensional trilateral dynamic semantics

Recall that our proposal was to make both presuppositions and assertions trivalent. We now dynamicise our static bidimensional trivalent system. When the VP contains a plural definite.[4]

(61) Presupposition

a. $c(\!(\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems})\!)^1 = c[\exists X(X = \sigma^+ x(\mathsf{STUDENT}(x))) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y))]$

b. $c(\!(\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems})\!)^0 = c[\neg\exists X(X = \sigma^+ x(\mathsf{STUDENT}(x))) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y))]$

[4]We analyse the plural definite here as a 'familiar definite'. We could alternatively analyse it as a 'unique/maximal definite' by letting it introduce a new discourse referent, and familiar definites as a special case with an indentity predicate that refers to an old discourse referent. Obviously details of this need to be worked out (YS is currently working on such a system to account for wide scope indefinites).

c. $c(\!(\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems})\!)^{\#} = \varnothing$

(62) Assertion

a. $c[\![\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}]\!]^1 = c[\forall x(x \sqsubseteq_a X \to \forall y(y \sqsubseteq_a Y \to \text{SOLVED}(x,y)))]$

b. $c[\![\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}]\!]^0 = c[\neg\forall x(x \sqsubseteq_a X \to \exists y(y \sqsubseteq_a Y \wedge \text{SOLVED}(x,y)))]$

c. $c[\![\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}]\!]^{\#}$
$= c[\neg\forall x(x \sqsubseteq_a X \to \forall y(y \sqsubseteq_a Y \to \text{SOLVED}(x,y))) \wedge \forall x(x \sqsubseteq_a X \to \exists y(y \sqsubseteq_a Y \to \text{SOLVED}(x,y)))]$

(63) Update rule

$$c + \textbf{S} = \begin{cases} c(\!(\textbf{S})\!)^1[\![\textbf{S}]\!]^1 & \text{if for each } (w,g) \in c\text{, for some } (w,g') \in c(\!(\textbf{S})\!)^1,\ g \leqslant g' \\ \star_0 & \text{if for some } (w,g) \in c\text{, for some } (w,g') \in c(\!(\textbf{S})\!)^0,\ g \leqslant g' \\ \star_{\#} & \text{otherwise} \end{cases}$$

(64) $g \leqslant g' :\Leftrightarrow$ for each $x \in \mathrm{dom}(g), g(x) = g'(x)$.

Here, we don't really use the $0$ denotation: The $1$ denotation is obviously referenced in (63), and the $\#$ denotation will be used for non-maximal readings. The $0$ denotation will be crucial in account for operators like negation, which switches the $0$ and $1$ denotations in the assertion (but not in the presupposition).

(65)
$$\begin{array}{lcllcl} (\!(\textbf{not } \phi)\!)^1 & = & (\!(\phi)\!)^1 & \quad [\![\textbf{not } \phi]\!]^1 & = & [\![\phi]\!]^0 \\ (\!(\textbf{not } \phi)\!)^0 & = & (\!(\phi)\!)^0 & \quad [\![\textbf{not } \phi]\!]^0 & = & [\![\phi]\!]^1 \\ (\!(\textbf{not } \phi)\!)^{\#} & = & (\!(\phi)\!)^{\#} & \quad [\![\textbf{not } \phi]\!]^{\#} & = & [\![\phi]\!]^{\#} \end{array}$$

In this system presuppositions play two roles:

- A presupposition puts a Heim-Stalnaker satisfaction condition. But this is only about its propositional aspect.[5]
- With respect to anaphora, a presupposition may convey new information.

Elliott & Sudo 2020 argued for such a theory based on independent evidence, e.g., (66).

[5]In Stalnakerian Pragmatics, assertion also puts a felicity condition that it be non-trivial. Note that this is also computed with respect to the truth-conditional meaning only, e.g., *#Something isn't both inside and outside*.

(66) a. None of the authors is aware/noticed that there is a mistake in this manuscript. But it is major.
b. None of the authors is certain/suspects that there is a mistake in this manuscript. #But it is major.

Shorthand for propositional truth:

(67) Let $S$ be a set of world-assignment pairs.
$(w, g) \Vdash S :\Leftrightarrow$ for some $(w, g') \in S, g \leqslant g'$.

(68) Update rule

$$c + \textbf{S} = \begin{cases} c(\!(\textbf{S})\!)^1[\![\textbf{S}]\!]^1 & \text{if for each } (w,g) \in c,\ (w,g) \Vdash c(\!(\textbf{S})\!)^1 \\ \star_0 & \text{if for some } (w,g) \in c,\ (w,g) \Vdash c(\!(\textbf{S})\!)^0 \\ \star_\# & \text{otherwise} \end{cases}$$

### 6.4 Homogeneity effects

First, when the VP contains a plural definite:

(69) $c +$ **Every$^X$ student solved the$_Y$ problems**

$$= \begin{cases} c\big(\!\big(\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}\big)\!\big)^1 \big[\!\big[\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}\big]\!\big]^1 \\ \qquad \text{if for each } (w,g) \in c,\ (w,g) \Vdash c\big(\!\big(\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}\big)\!\big)^1 \\ \star_0 \qquad \text{if for some } (w,g) \in c,\ (w,g) \Vdash c\big(\!\big(\textbf{Every}^X \textbf{ student solved the}_Y \textbf{ problems}\big)\!\big)^0 \end{cases}$$

The denotations are in (61) and (62).

The presupposition is bivalent here, but the assertion is non-trivially trivalent and when it's true, the plural definite is read universally.

When the restrictor contains a definite plural:

(70) Presupposition

a. $c\big(\!\big(\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}\big)\!\big)^1$
$= c[Y = \sigma^+ y(\text{PROBLEM}(y)) \wedge \exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \text{STUDENT}(x) \wedge \forall y(y \sqsubseteq_a Y \to \text{SOLVED}(x, y)))))]$

b. $c\big(\!\big(\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}\big)\!\big)^0$
$= c[Y \neq \sigma^+ y(\text{PROBLEM}(y)) \vee \neg\exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \text{STUDENT}(x) \wedge \exists y(y \sqsubseteq_a Y \wedge \text{SOLVED}(x, y)))))]$

c. $c\big(\!\big(\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}\big)\!\big)^\#$
$= c[Y = \sigma^+ y(\text{PROBLEM}(y)) \wedge \neg\exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \text{STUDENT}(x) \wedge \forall y(y \sqsubseteq_a Y \to \text{SOLVED}(x, y)))) \wedge \exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \text{STUDENT}(x) \wedge \exists y(y \sqsubseteq_a Y \to \text{SOLVED}(x, y)))))]$

(71) Assertion

a. $c[\![\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}]\!]^1 = c[\forall x(x \sqsubseteq_a X \to \text{PASSED}(x))]$

b. $c[\![\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}]\!]^0 = c[\neg\forall x(x \sqsubseteq_a X \to \text{PASSED}(x))]$

c. $c[\![\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}]\!]^\# = \varnothing$

(72) $c + \textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}$

$$= \begin{cases} c\big(\!\big(\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}\big)\!\big)^1 [\![\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}]\!]^1 \\ \qquad \text{if for each } (w, g) \in c,\ (w, g) \Vdash c\big(\!\big(\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}\big)\!\big)^1 \\ \star_0 \qquad \text{if for some } (w, g) \in c \text{ and } (w, g') \in c\big(\!\big(\textbf{Every}^X \textbf{ student that solved the}_Y \textbf{ problems passed}\big)\!\big)^0,\ g \leqslant g' \\ \star_\# \qquad \text{otherwise} \end{cases}$$

We get the doubly universal reading, as before.

### 6.5 Dynamic Strawson Monotonicity

Now we can define a dynamic version of Strawson Monotonicity. We'll put aside the details of compositional semantics (essentially, we need to type-generalise (73), which could be done), but we make use of dynamic generalised quantifiers whose arguments are dynamic statements.

(73) $\phi$ dynamically Strawson entails $\psi$ ($\phi \overset{\text{DS}}{\Rightarrow} \psi$) iff for each $c$ such that neither $c + \phi$ nor $c + \psi$ is $\star$., if $c + \phi \neq \varnothing$, then $c + \psi \neq \varnothing$. [6]

[6]Note that there are other notions of entailment, e.g., we could compare $c+\phi$ and $c+\phi+\psi$, or $c+\psi$ and $c+\psi+\phi$. These different notions differ with respect to anaphora. For our purposes the definition in (73) will do.

(74) Let ⌜**NP**$_1$⌝ $\overset{\text{DS}}{\Rightarrow}$ ⌜**NP**$_2$⌝.

a. ⌜**Q**⌝ is dynamically Strawson upward entailing (DSUE) if ⌜**Q NP**$_1$ **VP**⌝ $\overset{\text{DS}}{\Rightarrow}$ ⌜**Q NP**$_2$ **VP**⌝.

b. ⌜**Q**⌝ is dynamically Strawson downward entailing (DSDE) if ⌜**Q NP**$_2$ **VP**⌝ $\overset{\text{DS}}{\Rightarrow}$ ⌜**Q NP**$_1$ **VP**⌝.

*Every* and *no* are DSDE, but not DSUE.

(75) Suppose that there are semanticists in each $(w, g) \in c$:

a. $c +$ **Every**$^X$ **linguist is funny** $= c[\exists X(X = \sigma^+(\text{LINGUIST}))][\forall x(x \sqsubseteq_a X \rightarrow \text{FUNNY}(x))]$

b. $c +$ **Every**$^X$ **semanticist is funny** $= c[\exists X(X = \sigma^+(\text{SEMANTICIST}))][\forall x(x \sqsubseteq_a X \rightarrow \text{FUNNY}(x))]$

So far NPI licensing and homogeneity are sensitive to different flavours of entailment. We will claim below that homogeneity can actually be computed with respect to dynamic Strawson entailment as well.

## 7 Doubly existential readings and homogeneity projection

With the domain variable, we have accounted for why definite plurals in the restrictors of *every* and *no* lack mixed readings and how they give rise to doubly universal readings (when the existence presupposition is UE with respect to the position of the definite plural).

We are now ready to come back to the ambiguity between doubly universal and doubly existential readings.

Our strategy is inspired by Križ 2016, but his theory has no place for ambiguity (except in the QuD-related pragmatics that has to do with non-maximality). We propose to have a systematic ambiguity using the two notions of entailment we discussed above.

### 7.1 Projection through quantifiers

So far we have not been explicit about how homogeneity projects through a quantifier.

Križ proposes a Strong Kleene recipe.

- Each quantifier comes with two versions: one that turns the trivalent meaning of its argument into a binary one by collapsing the $\#$ and $0$ conditions, and one that do so by collapsing the $\#$ and $1$ conditions.

- The overall meaning of a quantified sentence will become (potentially) trivalent via supervaluationism: It's true if both versions of the quantifier are true, it's false if both versions of the quantifier are false, $\#$ otherwise.

For the sake of illustration, let's momentarily go back to the static extensional system.

(76) a. $[\![\textbf{some student}]\!]_{\#\approx 0} = \lambda P_{\langle e,t\rangle}. \begin{cases} 1 & \text{if for some } x \in \text{STUDENT}, P(x) = 1 \\ 0 & \text{otherwise (i.e., for every } x \in \text{STUDENT}, P(x) = 0 \text{ or } P(x) = \#) \end{cases}$

b. $[\![\textbf{some student}]\!]_{\#\approx 1} = \lambda P_{\langle e,t\rangle}. \begin{cases} 1 & \text{if for some } x \in \text{STUDENT}, P(x) = 1 \text{ or } P(x) = \# \\ 0 & \text{otherwise (i.e., for every } x \in \text{STUDENT}, P(x) = 0) \end{cases}$

(77) $[\![\textbf{some student}]\!] = \lambda P_{\langle e,t\rangle}. \begin{cases} 1 & \text{if } [\![\textbf{some student}]\!]_{\#\approx 0}(P) = [\![\textbf{some student}]\!]_{\#\approx 1}(P) = 1 \\ 0 & \text{if } [\![\textbf{some student}]\!]_{\#\approx 0}(P) = [\![\textbf{some student}]\!]_{\#\approx 1}(P) = 0 \\ \# & \text{otherwise} \end{cases}$

Thanks to the upward monotonicity of *some student*, this is equivalent to:

(78) $[\![\textbf{some student}]\!] = \lambda P_{\langle e,t\rangle}. \begin{cases} 1 & \text{if } [\![\textbf{some student}]\!]_{\#\approx 0}(P) = 1 \\ 0 & \text{if } [\![\textbf{some student}]\!]_{\#\approx 1}(P) = 0 \\ \# & \text{otherwise (i.e., for no } x \in \text{STUDENT}, P(x) = 1 \text{ and for some } x \in \text{STUDENT}, P(x) = \#) \end{cases}$

(79) $[\![\textbf{some student solved the three problems}]\!]$

$= \begin{cases} 1 & \text{if } [\![\textbf{some student}]\!]_{\#\approx 0}([\![\textbf{solved the three problems}]\!]) = 1 \\ 0 & \text{if } [\![\textbf{some student}]\!]_{\#\approx 1}([\![\textbf{solved the three problems}]\!]) = 0 \\ \# & \text{otherwise} \end{cases}$

$= \begin{cases} 1 & \text{if for some } x \in \text{STUDENT}, x \text{ solved all three problems} \\ 0 & \text{if for each } x \in \text{STUDENT}, x \text{ solved none of the three problems} \\ \# & \text{otherwise (i.e., no student solved all three problems and some student solved at least one} \end{cases}$

The same recipe applies to the restrictor:

(80) a. $[\![\textbf{some}]\!]_{\#\approx 0} = \lambda Q_{(e,t)}.\lambda P_{(e,t)}.\begin{cases} 1 & \text{for some } x \in D_e,\ Q(x) = P(x) = 1 \\ 0 & \text{otherwise} \end{cases}$

b. $[\![\textbf{some}]\!]_{\#\approx 1} = \lambda Q_{(e,t)}.\lambda P_{(e,t)}.\begin{cases} 1 & \text{for some } x \in D_e,\ Q(x) \neq 0 \text{ and } P(x) \neq 0 \\ 0 & \text{otherwise} \end{cases}$

(81)
$$[\![\textbf{some}]\!] = \lambda Q_{(e.t)}.\lambda P_{(e,t)}.\begin{cases} 1 & \text{if } [\![\textbf{some}]\!]_{\#\approx 0}(Q)(P) = [\![\textbf{some}]\!]_{\#\approx 1}(Q)(P) = 1 \\ 0 & \text{if } [\![\textbf{some}]\!]_{\#\approx 0}(Q)(P) = [\![\textbf{some}]\!]_{\#\approx 1}(Q)(P) = 0 \\ \# & \text{otherwise} \end{cases}$$
$$= \lambda Q_{(e.t)}.\lambda P_{(e,t)}.\begin{cases} 1 & \text{if } [\![\textbf{some}]\!]_{\#\approx 0}(Q)(P) = 1 \\ 0 & \text{if } [\![\textbf{some}]\!]_{\#\approx 1}(Q)(P) = 0 \\ \# & \text{otherwise} \end{cases}$$

For downward monotonic quantifiers, we get the following pattern.

(82) a. $[\![\textbf{10 or fewer students}]\!]_{\#\approx 0} = \lambda P_{(e,t)}.\begin{cases} 1 & \text{if } |\text{STUDENT} \cap \{\, x \in D_e \mid P(x) = 1 \,\}| \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$

b. $[\![\textbf{10 or fewer students}]\!]_{\#\approx 1} = \lambda P_{(e,t)}.\begin{cases} 1 & \text{if } |\text{STUDENT} \cap \{\, x \in D_e \mid P(x) = 1 \text{ or } P(x) = \# \,\}| \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$

(83)
$$[\![\textbf{10 or fewer students}]\!] = \lambda P_{(e,t)}.\begin{cases} 1 & \text{if } [\![\textbf{10 or fewer students}]\!]_{\#\approx 1}(P) = 1 \\ 0 & \text{if } [\![\textbf{10 or fewer students}]\!]_{\#\approx 0}(P) = 0 \\ \# & \text{otherwise} \end{cases}$$

For non-monotonic quantifiers, the two versions give rise to different readings, so we need to keep them both.

(84) a. $[\![\textbf{exactly 10 students}]\!]_{\#\approx 0} = \lambda P_{(e,t)}.\begin{cases} 1 & \text{if } |\text{STUDENT} \cap \{\, x \in D_e \mid P(x) = 1 \,\}| = 10 \\ 0 & \text{otherwise} \end{cases}$

b. $[\![\textbf{exactly 10 students}]\!]_{\#\approx 1} = \lambda P_{(e,t)}. \begin{cases} 1 & \text{if } |\text{STUDENT} \cap \{\, x \in D_e \mid P(x) = 1 \text{ or } P(x) = \# \,\}| = 10 \\ 0 & \text{otherwise} \end{cases}$

(85) $$[\![\textbf{exactly 10 students}]\!] = \lambda P_{(e,t)}. \begin{cases} 1 & \text{if } [\![\textbf{exactly 10 students}]\!]_{\#\approx 0}(P) = [\![\textbf{exactly 10 students}]\!]_{\#\approx 1}(P) = 1 \\ 0 & \text{if } [\![\textbf{exactly 10 students}]\!]_{\#\approx 0}(P) = [\![\textbf{exactly 10 students}]\!]_{\#\approx 1}(P) = 0 \\ \# & \text{otherwise} \end{cases}$$
$$= \lambda P_{(e,t)}. \begin{cases} 1 & \text{if } |\text{STUDENT} \cap \{\, x \mid P(x) = 1 \,\}| = 10 \text{ and } |\text{STUDENT} \cap \{\, x \mid P(x) = 1 \text{ or } P(x) = \# \,\}| = 10 \\ 0 & \text{if } |\text{STUDENT} \cap \{\, x \mid P(x) = 1 \,\}| \neq 10 \text{ and } |\text{STUDENT} \cap \{\, x \mid P(x) = 1 \text{ or } P(x) = \# \,\}| \neq 10 \\ \# & \text{otherwise} \end{cases}$$

Križ 2016 doesn't talk about presuppositions (he only has three truth-values), but we can generalise the rule to presuppositions by seeing presuppositions as trivalent propositions themselves.[7]

(86) a. $(\!(\textbf{either NP VP})\!)^{\#\approx 0} = \begin{cases} 1 & \text{if } |\{\, x \in D_e \mid (\!(\textbf{NP})\!)(x) = [\![\textbf{NP}]\!](x) = 1 \,\}| = 2 \\ 0 & \text{if } |\{\, x \in D_e \mid (\!(\textbf{NP})\!)(x) = [\![\textbf{NP}]\!](x) = 1 \,\}| \neq 2 \end{cases}$

b. $(\!(\textbf{either NP VP})\!)^{\#\approx 1} = \begin{cases} 1 & \text{if } |\{\, x \in D_e \mid (\!(\textbf{NP})\!)(x) \neq 0 \text{ and } [\![\textbf{NP}]\!](x) \neq 0 \,\}| = 2 \\ 0 & \text{if } |\{\, x \in D_e \mid (\!(\textbf{NP})\!)(x) \neq 0 \text{ and } [\![\textbf{NP}]\!](x) \neq 0 \,\}| \neq 2 \end{cases}$

(87) $$(\!(\textbf{either NP VP})\!) = \begin{cases} 1 & \text{if } (\!(\textbf{either NP VP})\!)_{\#\approx 0} = (\!(\textbf{either NP VP})\!)_{\#\approx 1} = 1 \\ 0 & \text{if } (\!(\textbf{either NP VP})\!)_{\#\approx 0} = (\!(\textbf{either NP VP})\!)_{\#\approx 1} = 0 \\ \# & \text{otherwise} \end{cases}$$

In the assertion, *either* is just an existential quantifier.

Since the duality presupposition is non-monotonic, we should get the bipolar homogeneity inference for (88) that there are two lists that contain all five keywords and no other lists contain any of the five keywords.

(88) Either list that contains the five keywords is written on the paper.

[7] (86) runs into the so-called Binding Problem, but we can fix it once we go dynamic.

### 7.2 Only maximal/minimal ones

**Idea**: Instead of always using both candidate meanings for a given quantifier, we order them via entailment. If $Q^1$ asymmetrically entails $Q^2$, $Q^1$ is maximal and $Q^2$ is minimal. If they are not ordered by entailment, then they are both maximal and minimal. In such a case we take their grand conjunction or grand disjunction. To unify these, we pick out the supremum and infimum of the two candidate meanings (in the space of quantifiers, static for now, dynamic later).

This system will give rise to ambiguity by using different notions of entailment, entailment *simpliciter* vs. dynamic Strawson entailment, to order the candidate meanings.

To illustrate, let's go back to the one-dimensional static extensional setting:

(89) a. $$[\![\textbf{some}]\!]_{\#\approx 0} = \lambda Q_{(e,t)}.\lambda P_{(e,t)}. \begin{cases} 1 & \text{for some } x \in D_e,\ Q(x) = P(x) = 1 \\ 0 & \text{otherwise} \end{cases}$$

b. $$[\![\textbf{some}]\!]_{\#\approx 1} = \lambda Q_{(e,t)}.\lambda P_{(e,t)}. \begin{cases} 1 & \text{for some } x \in D_e,\ Q(x) \neq 0 \text{ and } P(x) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

(90) $$\begin{aligned} [\![\textbf{some}]\!] &= \lambda Q_{(e.t)}.\lambda P_{(e,t)}. \begin{cases} 1 & \text{if } \sup\{[\![\textbf{some}]\!]_{\#\approx 0}, [\![\textbf{some}]\!]_{\#\approx 1}\}(Q)(P) = 1 \\ 0 & \text{if } \inf\{[\![\textbf{some}]\!]_{\#\approx 0}, [\![\textbf{some}]\!]_{\#\approx 1}\}(Q)(P) = 0 \\ \# & \text{otherwise} \end{cases} \\ &= \lambda Q_{(e.t)}.\lambda P_{(e,t)}. \begin{cases} 1 & \text{if } [\![\textbf{some}]\!]_{\#\approx 0}(Q)(P) = 1 \\ 0 & \text{if } [\![\textbf{some}]\!]_{\#\approx 1}(Q)(P) = 0 \\ \# & \text{otherwise} \end{cases} \end{aligned}$$

We get the same results as before, provided that $\sup$ and $\inf$ are computed in terms of standard entailment.

### 7.3 Back to dynamic semantics

We want to allow $\sup()$ and $\inf()$ to be able to be based on dynamic Strawson entailment as well. In order to do this, we have to dynamicise the above system. For now, we ignore presuppositions.

Also, since we haven't presented full compositional details, we will be ordering whole dynamic statements, rather than the candidate denotations of quantificational determiners themselves.

(91) Some$^X$ students solved the problems.

(92) a. $c[\![(91)]\!]^1_{\#\approx 0} = c[\exists X \forall x(x \sqsubseteq_a X \to \forall y(y \sqsubseteq_a \sigma^+ y(\mathsf{PROBLEM}(y)) \to \mathsf{SOLVED}(x,y)))]$
b. $c[\![(91)]\!]^0_{\#\approx 0} = c[\neg\exists X \forall x(x \sqsubseteq_a X \to \forall y(y \sqsubseteq_a \sigma^+ y(\mathsf{PROBLEM}(y)) \to \mathsf{SOLVED}(x,y)))]$

(93) a. $c[\![(91)]\!]^1_{\#\approx 1} = c[\exists X \forall x(x \sqsubseteq_a X \to \exists y(y \sqsubseteq_a \sigma^+ y(\mathsf{PROBLEM}(y)) \wedge \mathsf{SOLVED}(x,y)))]$
b. $c[\![(91)]\!]^0_{\#\approx 1} = c[\neg\exists X \forall x(x \sqsubseteq_a X \to \exists y(y \sqsubseteq_a \sigma^+ y(\mathsf{PROBLEM}(y)) \wedge \mathsf{SOLVED}(x,y)))]$

Based on the two versions of *some*, the dynamic meaning of the entire sentence is computed as in (94).

(94) $c + (91) = \sup\{\,[\![(91)]\!]^1_{\#\approx 0}, [\![(91)]\!]^1_{\#\approx 1}\,\}(c)$
$= c[\![(91)]\!]^1_{\#\approx 0}$

Here, we are assuming the usual notation of entailment, but which notion we use doesn't matter here anyway, because we are ignoring presuppositions for now.

**A note on non-maximality** To deal with non-maximal readings, we need a way to identify the $\#$ denotation at the update level (note that it might contain a dynamic existential quantifier). It will be the conjunction of the $0$ denotation of the stronger denotation, $[\![(91)]\!]^0_{\#\approx 0}$ and the $1$ denotation of the weaker denotation, $[\![(91)]\!]^1_{\#\approx 1}$.

Of course, this won't be the case for non-monotonic quantifiers.

(95) Exactly one$^X$ student solved the$_Y$ problems.

In this case the update will be based on the conjunction of both versions of the meaning.

The $\#$ denotation of a given sentence $S$ is generally identified with. Let $\mathrm{nmax} S \coloneqq S - \max S$ (recall $\max S$ is the set of maximal elements in $S$ with respect to entailment).

(96) $\bigsqcap \mathrm{nmax}\{\,[\![S]\!]^1_{\#\approx 0}, [\![S]\!]^1_{\#\approx 1}\,\} \sqcap \bigsqcap \mathrm{nmax}\{\,[\![S]\!]^0_{\#\approx 0}, [\![S]\!]^0_{\#\approx 1}\,\}$

### 7.4 Adding presuppositions

Now, let's add presuppositions.

(97) Every$^{X}$ student solved the$_{Y}$ problems.

For this example, there is no homogeneity in the presupposition, the two versions of *every* give rise to the same presupposition.

(98) a. $c(\!((97))\!)^{1}_{\#\approx 0} = c(\!((97))\!)^{1}_{\#\approx 1} = c[\exists X(X = \sigma^{+}x(\text{STUDENT}(x)) \wedge Y = \sigma^{+}y(\text{PROBLEM}(y)))]$
b. $c(\!((97))\!)^{0}_{\#\approx 0} = c(\!((97))\!)^{0}_{\#\approx 1} = c[\neg\exists X(X = \sigma^{+}x(\text{STUDENT}(x)) \wedge Y = \sigma^{+}y(\text{PROBLEM}(y)))]$

(99) a. $c[\![(97)]\!]^{1}_{\#\approx 0} = c[\forall x(x \sqsubseteq_{a} X \to \forall y(y \sqsubseteq_{a} Y \to \text{SOLVED}(x,y)))]$
b. $c[\![(97)]\!]^{0}_{\#\approx 0} = c[\neg\forall x(x \sqsubseteq_{a} X \to \forall y(y \sqsubseteq_{a} Y \to \text{SOLVED}(x,y)))]$

(100) a. $c[\![(97)]\!]^{1}_{\#\approx 1} = c[\forall x(x \sqsubseteq_{a} X \to \exists y(y \sqsubseteq_{a} Y \wedge \text{SOLVED}(x,y)))]$
b. $c[\![(97)]\!]^{0}_{\#\approx 1} = c[\neg\forall x(x \sqsubseteq_{a} X \to \exists y(y \sqsubseteq_{a} Y \wedge \text{SOLVED}(x,y)))]$

We'll be talking about entailment between two dimensional meanings under different ways of dealing with $\#$. We define two supremum operators, $\sup^{\Rightarrow}$ and $\sup^{\overset{\text{DS}}{\Rightarrow}}$.

(101) Standard entailment
a. $\phi$ is true with respect to $c$ iff $c + \phi \neq \star_{0}$, $c + \phi \neq \star_{\#}$, and $c + \phi \neq \varnothing$.
b. $\phi$ entails $\psi$ (written $\phi \Rightarrow \psi$) iff whenever $\phi$ is true with respect to $c$, $\psi$ is also true with respect to $c$.

(102) Dynamic Strawson entailment
$\langle \pi_1, \alpha_1 \rangle$ dynamically Strawson entails $\langle \pi_2, \alpha_2 \rangle$ (written $\langle \pi_1, \alpha_1 \rangle \overset{\text{DS}}{\Rightarrow} \langle \pi_2, \alpha_2 \rangle$) iff for every context $c$ such that for each $(w,g) \in c$, $(w,g) \Vdash c[\pi_1]$ and $(w,g) \Vdash c[\pi_2]$, if $c[\pi_1][\alpha_1] \neq \varnothing$, then $c[\pi_2][\alpha_2] \neq \varnothing$.

(103) a. $\sup^{\Rightarrow}\{\langle \pi_1, \alpha_1 \rangle, \ldots, \langle \pi_n, \alpha_n \rangle\} = \langle \sup\{\pi_1, \ldots, \pi_n\}, \sup\{\alpha_1, \ldots, \alpha_n\}\rangle$.
b. $\sup^{\overset{\text{DS}}{\Rightarrow}}\{\langle \pi_1, \alpha_1 \rangle, \ldots, \langle \pi_n, \alpha_n \rangle\} = \bigsqcap\{\langle \pi_i, \alpha_i \rangle \mid \langle \pi_i, \alpha_i \rangle \overset{\text{DS}}{\Rightarrow} \langle \pi_j, \alpha_j \rangle$ for each $j$ $(1 \leqslant j \leqslant n)\}$

Generalised conjunction $\sqcap$ applies point-wise:

(104) a. $\langle\pi_1, \alpha_1\rangle \sqcap \langle\pi_2, \alpha_2\rangle = \langle\pi_1 \sqcap \pi_2, \alpha_1 \sqcap \alpha_2\rangle$
b. $\bigsqcap\{\langle\pi_i, \alpha_i\rangle\}_{1\leqslant i\leqslant n} = \langle\pi_1 \sqcap \cdots \sqcap \pi_n, \alpha_1 \sqcap \cdots \sqcap \alpha_n\rangle$

Intuition:

- If the presupposition and assertion are equally important, the two dimensions of meaning are ordered separately = standard entailment
- If the assertion is somehow more important, dynamic Strawson entailment is used to order the candidate assertive meanings.

**Examples** Update with (97) will look as follows. As before, it doesn't matter which notion of entailment we use, so we won't make it explicit. The following shorthands will be handy:

(105) a. $c + \langle\pi, \alpha\rangle = c[\pi][\alpha]$
b. $c \circ \langle\pi, \alpha\rangle = c[\pi]$

(97) Every$^X$ student solved the$_Y$ problems.

(106) $$c + \text{(97)} = \begin{cases} c + \sup\left\{\begin{array}{l} \langle(\!(\text{(97)})\!)^1_{\#\approx 0}, [\![\text{(97)}]\!]^1_{\#\approx 0}\rangle, \langle(\!(\text{(97)})\!)^1_{\#\approx 0}, [\![\text{(97)}]\!]^1_{\#\approx 1}\rangle, \\ \langle(\!(\text{(97)})\!)^1_{\#\approx 1}, [\![\text{(97)}]\!]^1_{\#\approx 0}\rangle, \langle(\!(\text{(97)})\!)^1_{\#\approx 1}, [\![\text{(97)}]\!]^1_{\#\approx 1}\rangle \end{array}\right\} \\ \qquad \text{if for each } (w,g) \in c,\ (w,g) \Vdash c \circ \sup\left\{\begin{array}{l} \langle(\!(\text{(97)})\!)^1_{\#\approx 0}, [\![\text{(97)}]\!]^1_{\#\approx 0}\rangle, \langle(\!(\text{(97)})\!)^1_{\#\approx 0}, [\![\text{(97)}]\!]^1_{\#\approx 1}\rangle, \\ \langle(\!(\text{(97)})\!)^1_{\#\approx 1}, [\![\text{(97)}]\!]^1_{\#\approx 0}\rangle, \langle(\!(\text{(97)})\!)^1_{\#\approx 1}, [\![\text{(97)}]\!]^1_{\#\approx 1}\rangle \end{array}\right\} \\ \star_0 \qquad \text{otherwise} \end{cases}$$

$$= \begin{cases} c[\exists X(X = \sigma^+ x(\mathsf{STUDENT}(x)) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y))][\![\text{(97)}]\!]^1_{\#\approx 0} \\ \qquad \text{if for each } (w,g) \in c,\ (w,g) \in c[\exists X(X = \sigma^+ x(\mathsf{STUDENT}(x)) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y)))] \\ \star_0 \qquad \text{otherwise} \end{cases}$$

The non-maximal reading is dealt with in the same way as before.

The difference between $\sup^{\Rightarrow}$ and $\sup^{\overset{\text{DS}}{\Rightarrow}}$ will matter for the following example.

(107) Every$^X$ student that solved the$_Y$ problems passed.

(108) a. $c(\!((107))\!)^1_{\#\approx 0} = c[\exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \mathsf{STUDENT}(x) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y)) \wedge \forall y(y \sqsubseteq_a Y \rightarrow \mathsf{SOLVED}(x,y)))))]$

b. $c(\!((107))\!)^0_{\#\approx 0} = c[\neg\exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \mathsf{STUDENT}(x) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y)) \wedge \forall y(y \sqsubseteq_a Y \rightarrow \mathsf{SOLVED}(x,y)))))]$

(109) a. $c(\!((107))\!)^1_{\#\approx 1} = c[\exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \mathsf{STUDENT}(x) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y)) \wedge \exists y(y \sqsubseteq_a Y \rightarrow \mathsf{SOLVED}(x,y)))))]$

b. $c(\!((107))\!)^0_{\#\approx 1} = c[\neg\exists X(X = \sigma^+ x(x \sqsubseteq_a X \wedge \mathsf{STUDENT}(x) \wedge Y = \sigma^+ y(\mathsf{PROBLEM}(y)) \wedge \exists y(y \sqsubseteq_a Y \rightarrow \mathsf{SOLVED}(x,y)))))]$

(110) a. $c[\![(107)]\!]^1_{\#\approx 0} = [\![(107)]\!]^1_{\#\approx 1} = c[\forall x(x \sqsubseteq_a X \rightarrow \mathsf{PASSED}(x))]$

b. $c[\![(107)]\!]^0_{\#\approx 0} = [\![(107)]\!]^0_{\#\approx 1} = c[\neg\forall x(x \sqsubseteq_a X \rightarrow \mathsf{PASSED}(x))]$

With $\sup^{\Rightarrow}$, we get the doubly universal reading, as before.

(111)
$$c +^{\Rightarrow} (107) = \begin{cases} c + \sup^{\Rightarrow}\left\{ \begin{array}{l} \langle (\!((107))\!)^1_{\#\approx 0}, [\![(107)]\!]^1_{\#\approx 0}\rangle, \langle (\!((107))\!)^1_{\#\approx 0}, [\![(107)]\!]^1_{\#\approx 1}\rangle, \\ \langle (\!((107))\!)^1_{\#\approx 1}, [\![(107)]\!]^1_{\#\approx 0}\rangle, \langle (\!((107))\!)^1_{\#\approx 1}, [\![(107)]\!]^1_{\#\approx 1}\rangle \end{array} \right\} \\ \qquad \text{if for each } (w,g) \in c, (w,g) \Vdash c \circ \sup^{\Rightarrow}\left\{ \begin{array}{l} \langle (\!((107))\!)^1_{\#\approx 0}, [\![(107)]\!]^1_{\#\approx 0}\rangle, \langle (\!((107))\!)^1_{\#\approx 0}, [\![(107)]\!]^1_{\#\approx 1}\rangle, \\ \langle (\!((107))\!)^1_{\#\approx 1}, [\![(107)]\!]^1_{\#\approx 0}\rangle, \langle (\!((107))\!)^1_{\#\approx 1}, [\![(107)]\!]^1_{\#\approx 1}\rangle \end{array} \right\} \\ \star_0 \qquad \text{if for some } (w,g) \in c, (w,g) \Vdash c \circ \sup^{\Rightarrow}\left\{ \begin{array}{l} \langle (\!((107))\!)^0_{\#\approx 0}, [\![(107)]\!]^0_{\#\approx 0}\rangle, \langle (\!((107))\!)^0_{\#\approx 0}, [\![(107)]\!]^0_{\#\approx 1}\rangle, \\ \langle (\!((107))\!)^0_{\#\approx 1}, [\![(107)]\!]^0_{\#\approx 0}\rangle, \langle (\!((107))\!)^0_{\#\approx 1}, [\![(107)]\!]^0_{\#\approx 1}\rangle \end{array} \right\} \\ \star_\# \qquad \text{otherwise} \end{cases}$$

$$
= \begin{cases}
c + \langle (\!(\text{(107)})\!)^{1}_{\#\approx 0}, \forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))\rangle \\
\qquad\qquad \text{if for each } (w,g) \in c,\ (w,g) \Vdash c \circ \langle (\!(\text{(107)})\!)^{1}_{\#\approx 0}, \forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))\rangle \\
\\
\star_0 \qquad\quad \text{if for some } (w,g) \in c,\ (w,g) \Vdash c \circ \langle (\!(\text{(107)})\!)^{0}_{\#\approx 0}, \forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))\rangle \\
\\
\star_\# \qquad\quad \text{otherwise}
\end{cases}
$$

$$
= \begin{cases}
c(\!(\text{(107)})\!)^{1}_{\#\approx 0}[\forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))] & \text{if for each } (w,g) \in c,\ (w,g) \Vdash c(\!(\text{(107)})\!)^{1}_{\#\approx 0} \\
\star_0 & \text{if for some } (w,g) \in c,\ (w,g) \Vdash c(\!(\text{(107)})\!)^{0}_{\#\approx 1} \\
\star_\# & \text{otherwise}
\end{cases}
$$

With $\sup^{\overset{\mathsf{DS}}{\Rightarrow}}$, we get the doubly existential reading.

(112) $$
c \,+^{\overset{\mathsf{DS}}{\Rightarrow}} \text{(107)} = \begin{cases}
c + \sup^{\overset{\mathsf{DS}}{\Rightarrow}} \left\{ \begin{array}{l} \langle (\!(\text{(107)})\!)^{1}_{\#\approx 0}, [\![\text{(107)}]\!]^{1}_{\#\approx 0}\rangle, \langle (\!(\text{(107)})\!)^{1}_{\#\approx 0}, [\![\text{(107)}]\!]^{1}_{\#\approx 1}\rangle, \\ \langle (\!(\text{(107)})\!)^{1}_{\#\approx 1}, [\![\text{(107)}]\!]^{1}_{\#\approx 0}\rangle, \langle (\!(\text{(107)})\!)^{1}_{\#\approx 1}, [\![\text{(107)}]\!]^{1}_{\#\approx 1}\rangle \end{array} \right\} \\
\qquad\qquad \text{if for each } (w,g) \in c,\ (w,g) \Vdash c \circ \sup^{\overset{\mathsf{DS}}{\Rightarrow}} \left\{ \begin{array}{l} \langle (\!(\text{(107)})\!)^{1}_{\#\approx 0}, [\![\text{(107)}]\!]^{1}_{\#\approx 0}\rangle, \langle (\!(\text{(107)})\!)^{1}_{\#\approx 0}, [\![\text{(107)}]\!]^{1}_{\#\approx 1}\rangle, \\ \langle (\!(\text{(107)})\!)^{1}_{\#\approx 1}, [\![\text{(107)}]\!]^{1}_{\#\approx 0}\rangle, \langle (\!(\text{(107)})\!)^{1}_{\#\approx 1}, [\![\text{(107)}]\!]^{1}_{\#\approx 1}\rangle \end{array} \right\} \\
\\
\star_0 \qquad\quad \text{if for some } (w,g) \in c,\ (w,g) \in c \circ \sup^{\overset{\mathsf{DS}}{\Rightarrow}} \left\{ \begin{array}{l} \langle (\!(\text{(107)})\!)^{0}_{\#\approx 0}, [\![\text{(107)}]\!]^{0}_{\#\approx 0}\rangle, \langle (\!(\text{(107)})\!)^{0}_{\#\approx 0}, [\![\text{(107)}]\!]^{0}_{\#\approx 1}\rangle, \\ \langle (\!(\text{(107)})\!)^{0}_{\#\approx 1}, [\![\text{(107)}]\!]^{0}_{\#\approx 0}\rangle, \langle (\!(\text{(107)})\!)^{0}_{\#\approx 1}, [\![\text{(107)}]\!]^{0}_{\#\approx 1}\rangle \end{array} \right\} \\
\\
\star_\# \qquad\quad \text{otherwise}
\end{cases}
$$

$$= \begin{cases} c + \langle (((107)))^1_{\#\approx 1}, \forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))\rangle \\ \qquad \text{if for each } (w,g) \in c\text{, } (w,g) \Vdash c \circ \langle (((107)))^1_{\#\approx 1}, \forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))\rangle \\ \\ \star_0 \qquad \text{if for some } (w,g) \in c\text{, for some } (w,g) \Vdash c \circ \langle (((107)))^1_{\#\approx 0}, \forall x(x \sqsubseteq_a X \rightarrow \text{PASSED}(x))\rangle \\ \\ \star_\# \qquad \text{otherwise} \end{cases}$$

Either way, a plural definite in the assertion, if any, is always strengthened.

# 8 Open issues and further directions

### Quick summary

- **Observation**: Definite plurals in the restrictors of *every* and *no* are ambiguous between doubly universal and doubly existential readings, and lack mixed readings.
- *Every* and *no* refer to domain variables, which are dynamically introduced in their existence presuppositions, and referred back to in their assertions. Consequently, the restrictor is only interpreted once in the presupposition.
- This accounts for NPI licensing facts and the lack of mixed readings for plural definites in their restrictors.
- The ambiguity between doubly universal and doubly existential readings is account for by ambiguity in homogeneity projection in quantified sentences with respect to how candidate meanings are ordered (standard entailment vs. Strawson entailment).

## 8.1 Selective ambiguity

What if there are two definite plurals in the same restrictor? (Thanks to an anonymous reviewer for HNM1).

(113) a. Every immigrant who lives in the five Nordic countries is worried.
b. Every immigrant who lives in the five Nordic countries and watched the three documentaries about Ukraine on Netflix is worried.

(114) a. Every time I meet with my three colleagues, I get depressed.
b. Every time I meet with my three colleagues first and my PhD students afterwards, I get depressed.

We could make the system selective by using indices on plural definites and defining composite entailments based on either notion of entailment for each index (details to be worked out).

### 8.2 Universality of the doubly existential reading

The doubly existential reading is not pure existential.

(115) Every resident who speaks these three Tungusic languages is over 70.

This seems to presuppose that each of the three Tungusic languages has a speaker or speakers. But this isn't derived in our account.

There are two potential routes:

- Derive it as a separate scalar inference, akin to 'distributivity inferences':

  (116) a. Every resident who speaks Manchu or Nanai is over 70.
  b. Every redisdent speaks Manchu or Nanai.

- Use a version of Križ & Spector 2021 where candidate meanings are different denotations (rather than just universal vs. existential).

## 9 Comparisons with SIs

We used Križ's theory of homogeneity. Another popular approach to homogeneity analyses it as a case of SI (Bar-Lev 2018).

(117) John correctly answered the three difficult problems.
a. Literal: John correctly answered at least one of the three difficult problems.
b. Strengthened by Exh: John correctly answered all of the three difficult problems.

One might think that it's an advantage of this theory that it can derive the doubly universal reading (with Exh) and the doubly existential readings (without Exh) straightforwardly.

But this is not so straightforward: Exh needs to be able to give rise to mixed readings in other non-monotonic environments.

(118) a. The linguist is unaware that most of her colleagues are depressed.
b. The linguist forgot to invite some of her students.

(119) a. The linguist is unaware that her colleagues are depressed.
b. The linguist forgot to invite her students.

There are different ways of accounting for (118), and we expect them to be applicable to (119) as well.

But there are differences between definite plurals and SIs in the restrictors of *every* and *no*. Firstly, SIs give rise to mixed readings.

(120) Every student who correctly answered most of the difficult problems passed.

This has the following mixed reading:

(121) a. Presupposition (+exh): Some students correctly answered most but not all of the difficult problems.
b. Assertion (−exh): Every student who correctly answered most or all of the difficult problems passed.

Compare this to (122), which doesn't have a mixed reading.

(122) Every student who correctly answered the five difficult problems passed.

A related issue is that homogeneity doesn't seem to give rise to a 'non-local' reading.

(123) a. Every student correctly answered the first or second problem.
b. Every student correctly answered the first two problems.

(123a) has a reading that is derived with global Exh:

(124) Every student correctly answered either of the two problems, and not every student correctly answered both.

(123b) doesn't have a comparable (exclusion) reading:

(125) Every student correctly answered either of the two problems, and not every student correctly answered the first problem and not every student correctly answered the second problem.

## Appendix: Conditionals

Plural definites in conditionals give rise to doubly universal readings.

(126) If John took **the two intro courses** last year, he will take my seminar this year.

(127) a. Presupposition: It's possible that John took both intro courses last year.
b. Assertion: If John took both intro courses last year, he will take my seminar this year.

It seems that mixed readings are not possible.

(128) a. Presupposition: It's possible that John took both intro courses last year.
b. Assertion: If John took one or both of the intro courses last year, he will take my seminar this year.

Doubly existential readings?

(129) a. If John speaks **these two Tungusic languages** in my talk, we'll hire him.
b. ??If John lives in **the five Nordic countries**, he is definitely happy.

But:

(130) a. If **these two people show up**, I'll be nervous.
b. If you invited **these three guys**, I'm definitely not coming.

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