The Existential Problem of Scalar Implicatures and Anaphora Across Alternatives

Yasutada Sudo

Abstract It is argued that sentences of the form '$\exists x \ldots \sigma \ldots$', which contain both an existential quantifier $\exists x$ and a scalar item σ like *some*, *most*, etc. pose a challenge for standard views of scalar implicature computation that make use of alternative sentences. Specifically, it is observed that the predicted scalar implicature that $\neg(\exists x \ldots \sigma^+ \ldots)$ for a stronger scalar item σ^+ is systematically unavailable. In order to solve this problem, it is proposed that the existential quantifier contained in the alternative sentence behaves as an anaphoric term. This idea is formalized in File Change Semantics augmented with the exhaustivity operator exh. It is furthermore observed that there are two types of existential quantifiers in the modal domain in this respect: While the behavior of epistemic modals in alternative sentences is similar to that of indefinite DPs, root modals always behave as existential quantifiers, giving rise to scalar implicatures that $\neg(\Diamond \ldots \sigma^+ \ldots)$. This difference is captured in File Change Semantics by assuming that indefinite DPs and epistemic possibility modals denote variables, while root possibility modals always perform random assignment.

Keywords Scalar Implicature · Alternatives · Anaphora · Dynamic Semantics

Y. Sudo, University College London, y.sudo@ucl.ac.uk

In Christopher Piñón (ed.), Empirical Issues in Syntax and Semantics 11, 00–00. Paris: CSSP.
http://www.cssp.cnrs.fr/eiss11/

1 Introduction

Sentences containing so-called *scalar items* like *some* and *most* typically give rise to scalar implicatures (SIs), as illustrated by (1).

(1) John read some/most of the books.
$\rightsquigarrow$ ¬(John read all of the books)

It is widely held that SI computation makes reference to alternative sentences.[1] For the sake of concreteness, I will adopt here the “grammatical view” of SI computation (Chierchia 2006, Chierchia et al. 2012, Fox 2007). It should be stressed, however, that the same problem arises in all views of SI computation that resort to alternative sentences.

According to the grammatical view of SIs, SIs arise via a phonologically silent operator exh, which is often defined as follows.

(2) $[\![\mathsf{exh}(S)]\!]^w = [\![S]\!]^w \wedge \forall S' \in \mathrm{Alt}(S)((S \not\rightarrow S') \rightarrow \neg[\![S']\!]^w)$

In words, exh strengthens the meaning of the sentence S with the negation of alternatives S' that are not entailed by S. For instance, for (1), exh negates the alternative sentence in (3), which it does not entail.

(3) John read all of the books.

This accounts for the intuitively available SI that John did not read all of the books.

A crucial part of this theory of SIs is the theory of alternatives. In the above example, the SI in question is only explained under the assumption that the sentence in (3) counts as an alternative to (2). It is, however, beyond the scope of the present paper to thoroughly solve the vexing issue of how exactly alternatives are constructed. Here, we simply assume with Horn (1972) that alternatives are constructed by replacing scalar items with their lexically specified alternatives called “scale-mates” (e.g. *some*, *most*, and *all* are scale mates).[2] This theoretical choice, however, is only tentative, and the assumptions I will make about alternatives will not rely crucially on lexically specified scale mates.

---

[1] This of course does not mean that there are no theories without alternatives. See Van Rooij & Schulz (2004), Van Rooij & Schulz (2006) and Van Rooij (2014), for example. The problem I will discuss in this paper does not arise in these theories, but detailed comparisons between these theories and alternative-based theories are beyond the scope of the present paper.

[2] See Katzir (2007) and Fox & Katzir (2011) for problems of this view and an alternative view, but see Breheny et al. (2016) and references therein for potential problems for their theory.

2 The Existential Problem

A problem of the standard view of SIs arises when the sentence has the following schematic form:[3]

(4) $\exists x \ldots \text{some/most} \ldots$

If we construct an alternative by replacing the scalar item *some/most* with *all*, it will look like (5).

(5) $\exists x \ldots \text{all} \ldots$

Since this alternative is not entailed by (5), exh negates it, deriving the SI (6).

(6) $\neg(\exists x \ldots \text{all} \ldots)$

The problem, which I call the *existential problem* here, is that this SI is too strong. Rather, the SI that is actually available seems to be about the same individual that $\exists x$ in the asserted sentence introduces. Thus, the sentence with the SI seems to mean (7a), rather than (7b).

(7) a. $\exists x(\ldots \text{some/most} \cdots \wedge \neg(\ldots \text{all} \ldots))$
b. $\exists x(\ldots \text{some/most} \ldots) \wedge \neg \exists x(\ldots \text{all} \ldots)$

Here is an example illustrating this problem. As *some* allows an exceptional wide scope reading, which is not of our interest here, we will use *most* in the examples below.

(8) There are one or more students who read most of the books.

The problematic existential quantifier $\exists$ comes from *one or more students* (which is assumed to have no relevant SI of its own). The alternative sentence will look like (9).

(9) There are one or more students who read all of the books.

[3]Geurts (2008, 2009) independently notices the same problem, and makes suggestions that are closely related to what is proposed here, but he does not present a concrete implementation or discuss the differences among quantifiers in different domains.

By negating this, we obtain the SI that no student read all of the books. Intuitively, however, this is not the SI of (9). Rather, one tends to infer that the students who read most of the books did not read all of the books.

The same problem arises with other forms of $\exists$, as in (10).

(10) a. There is a student who read most of the books.
b. There are students who read most of the books.
c. There is at least one student who read most of the books.

These seem to mean (7a), rather than (7b).[4]

At this point one might wonder if this is really a problem, especially given that under the approach to SIs that postulates exh, (8) is predicted to have two possible SIs. One is what we have just derived by applying exh to the entire sentence, but there is also another possibility where exh takes scope within the relative clause. Since this reading corresponds to (7a), one might think that it is fine to also derive the other reading (7b), which is stronger, as a possible interpretation. Contrary to this, I argue that the reading predicted with wide scope exh in fact is absent and needs to be blocked. I make this point concrete by using two tests.

2.1 Hurford Disjunction Test

Our first test is the "Hurford Disjunction Test" (HDT).[5] It is known that disjunction is infelicitous if one of the disjuncts entails the other, as in (11) (Hurford 1974, Chierchia et al. 2012, Singh 2008).

(11) #Either John lives in London or he lives in the UK.

Interestingly, scalar items are seemingly exempt from this constraint, as illustrated by (12).

[4]However, one needs to be cautious about other pragmatic inferences than the SI triggered by the scalar item in question, which might lead one to conclude (7b), at least in certain contexts. For instance, upon asked if they have good students in their class this year, a professor could say (10a). This utterance typically triggers an inference that the student in question is the best student they have, or perhaps the only student that is worth mentioning in this context, from which one could conclude that the other students didn't do better, so none of the students read all of the books. Arguably, this is due to other pragmatic considerations than SIs triggered by scalar items *per se*.

[5]I thank Jacopo Romoli (p.c.) for suggesting this to me.

(12) Either John read some of the books or he read all of them.

As suggested by Chierchia et al. (2012), one way to understand this state of affairs is that the first disjunct of (12) can have an SI within the disjunct it is contained in, which breaks the entailment to the other disjunct and makes the disjunction acceptable. In fact, the following sentence is synonymous with (12) and as acceptable.

(13) Either John read some but not all of the books or he read all of them.

This can be used as a test for potential SIs.[6] The logic is that if a sentence S can have $\neg S'$ as an SI and mean $(S \wedge \neg S')$, then a disjunction of the form 'Either S or S'' should be felicitous. If the disjunction turns out to be infelicitous, it suggests that $\neg S'$ is unavailable as a potential SI of S.

Let us apply this test to one of the examples we are after, (10b) (the same point can be made with the other examples mentioned above). The relevant sentence will look like (14).

(14) ??Either there are students who read most of the books, or there are ones who read most but not all and ones who read all.

Notice, importantly, that the other potential reading that there are students who read most but not all of the books doesn't not break the entailment here, as the same thing is asserted in the second disjunct. If the first disjunct here could mean (7b), there wouldn't be an entailment from the first disjunct to the second. Thus, the infelicity indicates that the SI that no student read all of the books is unavailable.

One important obstacle here, however, is that sentences like (14) are a mouthful and might not be easy for native speakers to judge, as the difficulty associated with the length of the sentence might make it sound already less than perfect. Nevertheless, the contrast with (15) is suggestive, which does have the inference (7b) as an entailment due to *only*.

[6]The validity of this test is not theoretically uncontroversial and should be independently defended. However, I do not think this paper is the right place to do so, and leave it for future work.

(15) Either there are only students who read most_F of the books, or there are ones who read most but not all and ones who read all.

It seems that (14) is comparatively worse than (15), suggesting that the reading (7b) is unavailable.

The difference between (14) and (15) also indicates an interesting difference between exh and *only*, which are often said to have very similar semantic functions, if not completely identical (Groenendijk & Stokhof 1984, Fox & Hackl 2006, Chierchia et al. 2012). The above contrast indicates that *only* does negate the stronger alternative and give rise to the reading (7b), while exh does not seem to do the same.

2.2 Question-Answer Pairs

Another test we could use to make the same point is question-answer pairs. As illustrated by (16), SIs can be used to provide justifications for negative answers to polar questions.

(16) Q: Did John read all of these books?
A: He read some of them, so no.

(16A) has an SI that John did not read all of the relevant books, which justifies the negative answer to the question.

The idea here is to set up a pragmatic context that requires the presence of the target SI. If it is available at all, the sentence is expected to be felicitous.

Let us apply this test to the sentence under consideration. Here, too, it is instructive to compare (17A) with the version of the sentence with *only*, as in (17A′).

(17) Q: Did any of your students read all of the books?
A: ??There are ones who read most of them, so no.
A′: There are only ones who read most_F of them, so no.

As indicated by the question marks, (17A) does not seem to be a reasonable justification for the negative answer, suggesting that it cannot have the SI that there is no one who read all of the books. On the other hand, (17A′), which does have this inference as an entailment, can successfully justify the negative answer.

3 Alternatives in File Change Semantics

The existential problem pointed out in the previous section is a general overgeneration problem for alternative-based theories of SI computation. I now offer a solution whose underlying idea can be implemented in most, possibly all, of these theories, although I will stick to the framework with exh here for expository purposes. In particular, it should be pointed out that according to the idea proposed here, wide scope exh results in the same reading as narrow scope exh for the problematic sentences, which means that it does not rely on the embeddability of the SI computation mechanism (which is unavailable under some theories).

The main ideas are the following: we do not observe the predicted SI in sentences of the form $\exists x(\ldots\text{some/most}\ldots)$, because there is an anaphoric term in the alternative instead of the existential quantifier.[7] Thus, the only reading we can derive looks like (18a), rather than (18b). In these representations, the second conjunct is meant to be the SI, and x in the second conjunct of (18a) is dynamically bound by $\exists x$.

(18) a. $\exists x(x$ read most of the books$) \wedge \neg(x$ read all of the books$))$
b. $\exists x(x$ read most of the books$) \wedge$
$\neg\exists x(x$ read all of the books$)$

3.1 File Change Semantics

To flesh out this idea more concretely, I adopt File Change Semantics (Heim 1982). A file F is a set of assignments, which are partial functions from variables $\mathcal{V}$ to objects $\mathcal{O}$ in the model. I assume that the assignments in a file have the same domain, and write $\text{dom}(F)$ for the common domain of the assignments in F.

Variables carry two roles in this system. If a variable x is an "old variable" at F (i.e. $x \in \text{dom}(F)$), it functions as an anaphor, while if it is a "new variable" at F (i.e. $x \notin \text{dom}(F)$), it effectively acts as an existential quantifier by triggering *random assignment*. This is ensured by the rule for updating the file with simple sentences. $\mathcal{I}$ here is the interpretation

[7] See Bumford (2015) and Elliott & Sudo (to appear) for related ideas.

function assigning the usual extensions to constants.

$$F[P^1(x)] := \begin{cases} \{ g \in F \mid g(x) \in \mathfrak{I}(P^1) \} & \text{if } x \in \mathrm{dom}(F) \\ \{ g' \mid \exists g \in F (g[x]g' \wedge g'(x) \in \mathfrak{I}(P^1)) \} & \text{if } x \notin \mathrm{dom}(F) \end{cases}$$

Here, $g[x]g'$ is a *random assignment* of a value to the variable x:

$$g[x]g' \text{iff } g \text{ and } g' \text{ differ at most in that } x \notin \mathrm{dom}(g) \text{ and } x \in \mathrm{dom}(g')$$

More generally, for n-place predicates:

$$F[P^n(x_1,\ldots,x_n)] := \left\{ g' \;\middle|\; \exists g \in F \left(\begin{array}{c} g[x_{i_1}]g' \wedge \ldots g[x_{i_m}]g' \\ \wedge \langle g'(x_1),\ldots,g'(x_n) \rangle \in \mathfrak{I}(P^n) \end{array} \right) \right\}$$
$$\text{for each } x_{i_j} \text{ such that } x_{i_j} \notin \mathrm{dom}(F)$$

Both indefinites and pronouns introduce variables but indefinites are associated with a felicity condition requiring that they introduce a new variable, which is known as the "Novelty Condition". Crucially, this condition is understood to be a condition on speech acts, rather than a presupposition (cf. Heim 1991, Elliott & Sudo to appear). This assumption will become crucial when we compute alternatives.

The connectives are as in standard dynamic systems.

$$F[\phi \wedge \psi] := F[\phi][\psi]$$
$$F[\neg\phi] := \{ g \in F \mid \neg\exists g' \in F[\phi](g \leq g') \}$$

Here $g \leq g'$ iff for all $x \in \mathrm{dom}(g)$, $g(x) = g'(x)$.

As the scalar items in the examples in question are generalized quantifiers, I postulate dynamic selective generalized quantifiers (van Eijck & de Vries 1992, Kanazawa 1993, 1994, Chierchia 1995). For the purposes of the present paper, this is only for the sake of completeness, and the particular way of implementing dynamic generalized quantifiers here is largely inconsequential. To keep the discussion as simple as possible, I assume that generalized quantifiers are not externally dynamic, meaning they do not introduce new discourse referents (see van den Berg 1996, Nouwen 2003, 2007, Brasoveanu 2007, 2010a,b for externally dynamic generalized quantifiers).

Here are the details. Determiners are assumed to be associated with variables (indicated by superscripts), which are assumed to be subject to the Novelty Condition on a par with indefinites. For example, all has the following meaning. Here $g[x/o]$ is that assignment that differs from g at most in that $g[x/o](x) = o$.

$$(19)\qquad F[\mathsf{all}^x(\phi)(\psi)] := \left\{ g \in F \;\middle|\; \begin{array}{l} \{o \in \mathcal{O} \mid \{g[x/o]\}[\phi] \neq \emptyset\} \\ \subseteq \{o \in \mathcal{O} \mid \{g[x/o]\}[\phi][\psi] \neq \emptyset\} \end{array} \right\}$$

More generally, the interpretation of a determiner Q in the present system can be defined in terms of its classical counterpart $\mathbb{Q}$ of type $((et)((et)t))$ as follows:

$$(20)\qquad F[\mathsf{Q}^x(\phi)(\psi)] = \left\{ g \in F \;\middle|\; \begin{array}{l} \mathbb{Q}(\{o \in \mathcal{O} \mid \{g[x/o]\}[\phi] \neq \emptyset\}) \\ (\{o \in \mathcal{O} \mid \{g[x/o]\}[\phi][\psi] \neq \emptyset\}) \end{array} \right\}$$

The idea here is that the dynamic generalized quantifier Q collects the objects that make its restrictor ϕ true (the "maxset") and the objects that make both its restrictor ϕ and nuclear scope ψ true (the "refset"), and applies the classical generalized quantifier $\mathcal{Q}$ to these two sets. The fact that the refset refers both to the restrictor ϕ and nuclear scope ψ. See the works cited above for more on this.

3.2 Anaphora in Alternatives

Coming back to SIs, I propose that exh dynamically conjoins S and its negated alternatives. If the only relevant alternative of S is S', then we have:

$$F[\mathsf{exh}(S)] = F[S \wedge \neg S'] = F[S][\neg S']$$

More generally:

$$F[\mathsf{exh}(S)] := \left\{ g \in F[S] \;\middle|\; \forall S' \in \mathrm{Alt}(S) \left(\begin{array}{l} (F[S][S'] \subset F[S]) \\ \to g \in F[S][\neg S'] \end{array} \right) \right\}$$

That is, when there are multiple alternatives to S that could have strengthened S at F, that is, $F[S][S'] \subset F[S]$, then their SIs are computed in parallel.[8]

[8]Alternatively, the alternatives could be ordered and used to perform sequential updates, but I don't see any empirical reasons to favor or disfavor this possibility. I leave

To see how this works concretely, let us take the sentence (8) with the indexing as in (21a). The alternative sentence looks like (21b).

(21) a. There is one or more^x students who read most^y of the books.
b. There is one or more^x students who read all^z of the books.

It is crucial that the same variable x is used on *one or more* in (21a) and (21b), which is taken to be an indefinite determiner here, while the variables on the scalar items are distinct. I assume that this is ensured by how alternatives are constructed syntactically. That is, from the sentence (21a), the alternative (21b) is constructed by keeping everything, including the indices constant, except for the scalar item and the index on it.[9]

The sentences in (21a) and (21b) (without exh) are translated as (22a) and (22b), respectively.

(22) a. $\mathsf{student}(x) \wedge \mathsf{most}^y(\mathsf{book}(y))(\mathsf{read}(x, y))$
b. $\mathsf{student}(x) \wedge \mathsf{all}^z(\mathsf{book}(z))(\mathsf{read}(x, z))$

When exh is applied to (22a), the negation of the alternative (22b) is processed after (22a). Then, the variable x in it acts as an anaphoric term, because x is an old variable at $F[(21a)]$ (whenever $F[(21a)] \neq \emptyset$), although it is new at F. Notice here that the Novelty Condition (qua condition on speech acts) is not violated, because x is novel at F. The resulting file, then, is the following:

$$
\begin{aligned}
&F[(22a)][\neg(22b)] \\
&= \left\{ g' \;\middle|\; \exists g \in F \left(\begin{array}{l} g[x]g' \wedge g'(x) \in \mathcal{I}(\mathsf{student}) \\ \wedge\, \mathbb{MOST}(\{ o \in \mathcal{O} \mid o \in \mathcal{I}(\mathsf{book}) \}) \\ \quad (\{ o \in \mathcal{O} \mid o \in \mathcal{I}(\mathsf{book}) \wedge \langle g'(x), o \rangle \in \mathcal{I}(\mathsf{read}) \}) \\ \wedge\, \neg\mathbb{ALL}(\{ o \in \mathcal{O} \mid o \in \mathcal{I}(\mathsf{book}) \}) \\ \quad (\{ o \in \mathcal{O} \mid o \in \mathcal{I}(\mathsf{book}) \wedge \langle g'(x), o \rangle \in \mathcal{I}(\mathsf{read}) \}) \end{array} \right) \right\}
\end{aligned}
$$

this as a theoretical choice.

[9] Note that this does not hinge on the theory of alternatives we are tentatively adopting here. Rather, it is an additional constraint that I am unable to derive from independent principles. I believe other theories of alternatives similarly do not necessarily predict this constraint. In fact, the structural theory of alternatives (Katzir 2007, Fox & Katzir 2011) might allow alternatives to have different indices and overgenerate here.

This amounts to the following reading: there is a student x who read at least most of the books, and it's not the case that *the same student x* read all of the books). Importantly in the present account this is the only possible reading of the sentence, as the same SI is predicted if exh applies within the relative clause. Thus, the problematic SI cannot be generated here, and the existential problem does not arise.

4 Modals

So far so good. Interestingly, however, not all existential quantifiers give rise to the existential problem. Specifically, we observe that among existential quantifiers in the modal domain, that is, possibility modals, there are ones that behave differently from indefinite DPs. That is, sentences of the form $\Diamond(\ldots\text{some/most}\ldots)$ where $\Diamond$ is a root possibility modal do give rise to SIs such that $\neg\Diamond(\ldots\text{all}\ldots)$ holds. To see this more concretely, consider (23), where the relevant existential modal is the deontic possibility modal *allowed*.[10]

(23) John is allowed to read most of the books.
$\rightsquigarrow$ ¬(John is allowed to read all of the books)

One can infer from this sentence that John is not allowed to read all of the books. A HDT confirms this observation.

(24) a. Either John is allowed to read most of the books, or he can choose whether to read most of them or all of them.
b. Either John is only allowed to read most$_F$ of the books, or he can choose whether to read most of them or all of them.

Both of these sentences seem to be acceptable. Similarly, the following question-answer pair points to the same conclusion:

(25) A: Is John allowed to read all of these books?
B: He's allowed to read most of them, so no.
B′: He's only allowed to read most$_F$ of them, so no.

[10] (23) has more SIs, which is due to the fact that *allowed* is also a scalar item. We will not discuss these SIs to simplify the discussion. See Fox (2007), Chemla (2009) and Romoli (2012).

Other root possibility modals also give rise to SIs that $\neg\Diamond(\ldots\text{all}\ldots)$, for example, other deontic possibility modals, as in (26), and ability modals, as in (27). As above, we are only interested in the narrow scope reading of the scalar item.

(26) a. You may eat most of the cookies.
$\rightsquigarrow$ $\neg$(You may eat all of the cookies)
b. You can keep most of this money.
$\rightsquigarrow$ $\neg$(You can keep all of this money)

(27) a. I can read most of these papers by tomorrow.
$\rightsquigarrow$ $\neg$(I can read all of these papers by tomorrow)
b. John is able to finish some of the work.
$\rightsquigarrow$ $\neg$(John is able to finish all of the work)

It is furthermore observed that there is variation among modals. Specifically, epistemic possibility modals differ from root possibility modals in this respect, and pattern with indefinite DPs. For example, the sentences in (28) do not seem to have the SI $\neg\Diamond(\ldots\text{all}\ldots)$, that is, they do not have SIs to the effect that the speaker considers it impossible that John read all of these books. Rather, the intuitively available reading is one where the SI seemingly takes scope below the modal.

(28) a. John might have read most of these books.
b. It is possible that John read most of these books.

This is confirmed with a HDT as in (29), and a question-answer pair test as in (30).[11]

(29) #Either John might have read most of these books, or all we know is that he read at least most, possibly all of them.

(30) Q: Do you think it's possible that John read all of the books?
A: #He might have read most of them, so no.

To summarize the observations, indefinite DPs and epistemic possibil-

[11]Unfortunately, the corresponding sentences with overt *only* cannot be constructed, due to restrictions on the scope of *only* relative to epistemic modals. Yet, the judgments seem to be reasonably robust.

ity modals do not give rise to SIs that involve negated existential quantifiers, while root possibility modals do. In order to account for this difference, I claim that indefinite DPs and epistemic possibility modals denote variables, while root possibility modals are always interpreted as existential quantifiers, even in alternatives. I will demonstrate below that File Change Semantics offers a way to model this difference.

4.1 Epistemic Modals

Let us first tackle epistemic possibility modals, which work like indefinite DPs with respect to the existential problem of SIs.

It is known that modals have anaphoric properties, just like quantificational DPs. Anaphora in the modal domain is often discussed under the rubric of *modal subordination* (Roberts 1987, Geurts 1999, Stone 1999, Brasoveanu 2007, 2010a,b, Sudo 2014).[12] The phenomenon of modal subordination itself is not of particular interest here, but to illustrate, consider (31).

(31) John might come. John would bring a bottle of sake with him.

The meaning of the second sentence of (31) depends on *might* in the first sentence: *might* introduces the possibility that John will come, and the second sentence elaborates on this possibility by saying that if he comes he will bring a bottle of sake with him.

There are several theories of modal subordination in the literature, but I adopt here the idea of Stone (1999) and Brasoveanu (2010a) and postulate variables over sets of possible worlds (see also Sudo 2014). The theoretical choice here is largely arbitrary, however, and as far as I can see, nothing in the idea below crucially relies on this theory of modal subordination. That is, the only crucial part of the idea is that *might* is an indefinite in the modal domain.[13]

Let us see a concrete example. The first conjunct of (31), for example, can be given the following meaning, where ω is a variable over sets of

[12] The term *modal subordination* is often used to refer also to pronominal anaphora mediated by modal anaphora, for example, *John might bring a bottle of sake with him. But he wouldn't share it with us*. Such complex cases are not of our concern here.

[13] This, however, means that we could not use the theory of epistemic modals as tests due to Veltman (1996).

possible worlds, functioning as the modal base for the epistemic modal. We assume *John* is a rigid designator denoting j, and assignments are functions from variables over objects $\mathcal{V}_o$ and variables over sets of possible worlds $\mathcal{V}_w$ to objects $\mathcal{O}$ and possible worlds $\mathcal{W}$. In addition, $\mathcal{I}$ is parametrized to a possible world. Dox_s is the set of doxastic alternatives for the speaker. As with indefinite DPs, I assume that *might* is subject to the Novelty Condition, requiring at the speech act level that it introduce a new discourse referent.

(32) $$F[\mathsf{might}^{\omega}(\mathsf{come}(j))] = \begin{cases} \{\, g \in F \mid \exists w \in g(\omega)(j \in \mathcal{I}_w(\mathsf{come}))\,\} & \text{if } \omega \in \mathrm{dom}(F) \\ \left\{ g' \;\middle|\; \begin{array}{l} \exists g \in F(g[\omega]g' \wedge g'(\omega) \subseteq \mathsf{Dox}_s \\ \wedge \forall w \in g(\omega)(j \in \mathcal{I}_w(\mathsf{come}))) \end{array} \right\} & \text{if } \omega \notin \mathrm{dom}(F) \end{cases}$$

In order for this analysis to be complete, some presuppositions need to be added. For instance, whenever old, ω should be a set of epistemic possibilities, rather than any set of possible worlds. One could also state the meaning of mood as a presupposition, for example, indicative mood presupposes that the actual world might be in ω. To avoid unnecessary clutter, I will omit these presuppositions here.

The important aspect of this analysis is that *might* introduces a new set of epistemic possibilities, which later sentences can anaphorically refer back to. In the above example, *would* refers to the possibilities that John will come and discard all the assignments but g such that in all $w \in g(\omega)$ John will bring a bottle of sake in w.

(33) $$\begin{aligned} &F[\mathsf{would}_{\omega}(\mathsf{sake}(x) \wedge \mathsf{bring}(j,x))] \\ &= \{\, g \in F \mid \forall w \in g(\omega)(g(x) \in \mathcal{I}_w(\mathsf{sake}) \wedge \langle j, g(x)\rangle \in \mathcal{I}_w(\mathsf{bring}))\,\} \end{aligned}$$

Let us now combine this analysis of epistemic modals and our analysis of SIs developed in the previous section. Sentences like (28) above are, then, predicted to lack SIs of the form (7b), on the assumption that the variable associated with the epistemic possibility modal stays the same in the alternative. This is shown more concretely in (34).

(34) $$\begin{aligned} &F[\mathsf{exh}(\mathsf{might}^{\omega}(\mathsf{most}^{x}(\mathsf{book}(x))(\mathsf{read}(x,j))))] \\ &= F[\mathsf{might}^{\omega}(\mathsf{most}^{x}(\mathsf{book}(x))(\mathsf{read}(x,j)))] \end{aligned}$$

$$[\neg(\text{might}^{\omega}(\text{all}^{y}(\text{books})(\lambda x.\text{read}(j)(x))))]$$

As in the case of indefinite DPs, it is crucial that *might* in the asserted sentence has a new variable ω, and the same variable ω appears on *might* in the alternative. This is again assumed to be ensured by the syntax of alternatives. (34) will result in the following set of assignments.

$$\left\{ g' \;\middle|\; \exists g \in F \left(\begin{array}{l} g[\omega]g' \wedge g'(\omega) \subseteq \mathsf{Dox}_s \\ \wedge \forall w \in g'(\omega)(\mathbb{MOST}(\{\, o \in \mathcal{O} \mid o \in \mathcal{I}_w(\text{book}) \,\}) \\ \quad (\{\, o \in \mathcal{O} \mid o \in \mathcal{I}_w(\text{book}) \wedge \langle j, o \rangle \in \mathcal{I}_w(\text{read}) \,\})) \\ \wedge \neg \forall w \in g'(\omega)(\mathbb{ALL}(\{\, o \in \mathcal{O} \mid o \in \mathcal{I}_w(\text{book}) \,\}) \\ \quad (\{\, o \in \mathcal{O} \mid o \in \mathcal{I}_w(\text{book}) \wedge \langle j, o \rangle \in \mathcal{I}_w(\text{read}) \,\})) \end{array} \right) \right\}$$

In words, there is an epistemic possibility that John read most of the books where it is not the case that John read all of the books.[14] The SI here therefore corresponds to (7a), and the SI of the form (7b) cannot be derived in the present system.

4.2 Root Modals

Now, what about root possibility modals, which do give rise to SIs that $\neg\Diamond(\ldots\text{all}\ldots)$, unlike epistemic modals and indefinite DPs? Although I do not have a satisfactory answer at the moment as to why root modals are different in this particular way from epistemic modals, the framework we are assuming at least offers a way to capture their behavior. Specifically, I propose that unlike epistemic possibility modals, root possibility modals always perform random assignment. This is shown by (35) for deontic

[14] In the above representation, $\neg$ is taking scope over the universal quantification over possible worlds, which is arguably too weak. However, assuming that this universal quantification is associated with a homogeneity requirement, the SI becomes adequately strong. That is, the homogeneity requirement says that either of the following is the case: (i) in all the worlds in $g'(\omega)$ John reads all of the books; or (ii) in none of the worlds does John read all of the books. Since the SI here is only compatible with (ii), and one concludes that, as desired. This assumption about homogeneity is not at all far-fetched, as the universal quantifier here is due to plural predication of possible worlds and plural predication in natural language generally gives rise to such homogeneity effects. For instance, *It is not the case that John read the books* seems to entail that John read none of the books.

possibility, where DEON is the set of deontically ideal worlds.[15]

(35) $F[\mathsf{allowed}^{\omega}(\mathsf{come}(j))]$
$$= \left\{ g' \;\middle|\; \begin{array}{l} \exists g \in F(g[\omega]g' \wedge g'(\omega) \subseteq \text{DEON} \\ \wedge \forall w \in g(\omega)(j \in \mathcal{I}_w(\mathsf{come}))) \end{array} \right\}$$

This meaning will derive the desired reading for (23) that John is allowed to read most of the books and he is not allowed to read all of the books, which involves an SI of the form (7b). More specifically, after processing the asserted sentence *John is allowed*$^{\omega}$ *to read most of the books* against file F, we obtain the following file F':

$$F' = \left\{ g' \;\middle|\; \exists g \in F \left(\begin{array}{l} g[\omega]g' \wedge g'(\omega) \subseteq \text{DEON} \\ \wedge \forall w \in g'(\omega)(\mathcal{MOST}(\{ o \in \mathcal{O} \mid o \in \mathcal{I}_w(\mathsf{book}) \}) \\ \quad (\{ o \in \mathcal{O} \mid o \in \mathcal{I}_w(\mathsf{book}) \wedge \langle j, o \rangle \in \mathcal{I}_w(\mathsf{read}) \})) \end{array} \right) \right\}$$

Now we process the negation of the alternative sentence *John is allowed*$^{\omega}$ *to read all of the books*, and obtain the following file:

$$\left\{ g' \in F' \;\middle|\; \neg\exists g'' \left(\begin{array}{l} g'[\omega]g'' \wedge g''(\omega) \subseteq \text{DEON} \\ \wedge \forall w \in g''(\omega)(\mathcal{ALL}(\{ o \in \mathcal{O} \mid o \in \mathcal{I}_w(\mathsf{book}) \}) \\ \quad (\{ o \in \mathcal{O} \mid o \in \mathcal{I}_w(\mathsf{book}) \wedge \langle j, o \rangle \in \mathcal{I}_w(\mathsf{read}) \})) \end{array} \right) \right\}$$

Importantly, ω here is again used to perform random assignment. Consequently, each g' in this file maps ω to a set of deontically ideal worlds in which John reads most of the books, and additionally, it is ensured that there's no way to assign ω a set of deontically ideal worlds in which John reads all of the books, because such assignments are culled out by the SI.[16]

[15] This, of course, is a gross oversimplification of the meaning of deontic modals. See Kratzer (1981, 1991) in particular. Although I do not see any obstacle in adopting Kratzer's ideas in our current framework, I will assume the simplistic semantics here too keep the exposition simple. Also, this analysis does not predict that root modals cannot participate in modal subordination, as modal subordination is about anaphora about the domain of quantification. I do not represent the domain of quantification here explicitly, which would require a different variable and complicate the exposition. In a complete theory of modals and quantifiers, such domain variables need to be represented. See, for example, Brasoveanu (2007, 2010b,a). I thank Christopher Piñon for related discussion.

[16] If one believes that the embedded SI is also available for sentences like (23), one could resort to one's favorite way of accounting for embedded SIs. In the current set

5 Conclusions and Further Issues

In the present paper I have made two main observations: (i) sentences of the form $\exists \ldots$some/most$\ldots$ where $\exists$ is an indefinite DP or an epistemic possibility modal lack the negation of $\exists \ldots$all$\ldots$ as a (potential) SI, but (ii) this SI is observed when $\exists$ is a root modal. To account for (i), I pursued the following idea: indefinites DPs and epistemic modals introduce variables to the discourse, which denote new discourse referents in the asserted sentence but behave as anaphoric terms in the negated alternative sentences. As for the issue (ii), I proposed that root possibility modals do not introduce variables to the discourse. Rather, they are existential quantifiers, and always perform random assignment.

Admittedly, this account of (ii) is still preliminary, as it is essentially a lexical stipulation made just to account for what is observed and lacks independent justification. Differences between epistemic vs. root modals are a very well discussed topic (cf. Ross 1969, Perlmutter 1971, Jackendoff 1972, Brennan 1993, von Fintel & Iatridou 2003, Hacquard 2006, 2011), and I hope the present analysis will eventually relate to the insights offered by this body of literature, and lead to a deeper explanation of their syntax and semantics. This issue is left for future research.

Another remaining issue that is set aside in the present paper is the interactions between SIs and other types of quantifiers than indefinites. That is, it is natural to extend the ideas of the present paper to sentences like the following.

(36) a. 20% of the students read most of the books.
b. Most of the students read most of the books.

(37) a. John is likely to have read most of the books.
b. John has probably read most of the books.

The reason why I am not discuss these cases here is because those quantifiers come with their own SIs, giving rise to independent problems of multiple scalar items discussed by Fox (2007), Chemla (2009) and Romoli (2012) (as mentioned already in footnote 10). It is expected that

up, exh can simply take scope in the infinitival clause to yield this reading. Since the availability of embedded SIs is not the main concern of the present paper, I will remain uncommitted to this issue here.

the perspective of the present dynamic semantic account provides new insights into this issue, which is left for another occasion.

Acknowledgments I would like to thank Sam Alxatib, Richard Breheny, Nathan Klinedinst, Andreea Nicolae, Rick Nouwen, Jacopo Romoli, and Daniel Rothschild, and the audience at the London Semantics Day in May 2015 for helpful discussion and criticisms on the material presented here. I also thank Christopher Piñon and an anonymous reviewer for helpful feedback on an earlier version of this paper. All remaining errors are my own.

References

van den Berg, Martin. 1996. *Some Aspects of the Internal Structure of Discourse: The Dynamics of Nominal Anaphora*: Universiteit van Amsterdam Ph.D. dissertation.

Brasoveanu, Adrian. 2007. *Structured Nominal and Modal Reference*: Rutgers University Ph.D. dissertation.

Brasoveanu, Adrian. 2010a. Decomposing modal quantification. *Journal of Semantics* 27(4). 1–91. doi:10.1093/jos/ffq008.

Brasoveanu, Adrian. 2010b. Structured anaphora to quantifier domains. *Information and Computation* 208(12). 450–473. doi:10.1016/j.ic.2008.10.007.

Breheny, Richard, Nathan Klinedinst, Jacopo Romoli & Yasutada Sudo. 2016. Does the structural approach to alternatives give us just enough alternatives to solve the symmetry problem? Ms., University College London and Ulster University.

Brennan, Virginia. 1993. *Root and Epistemic Modal Auxiliary Verbs*: University of Massachusetts, Amherst dissertation.

Bumford, Dylan. 2015. Incremental quantification and the dynamics of pair-list phenomena. *Semantics & Pragmatics* 8(9). 1–70.

Chemla, Emmanuel. 2009. Similarity: towards a unified account of scalar implicatures, free choice permission and presupposition projection. Ms.

Chierchia, Gennaro. 1995. *Dynamics of Meaning: Anaphora, Presupposition, and the Theory of Grammar*. Chicago: University of Chicago Press.

Chierchia, Gennaro. 2006. Broaden your views: Implicatures of domain widening and the "Logicality" of language. *Linguistic Inquiry* 37(4). 535–590.

Chierchia, Gennaro, Danny Fox & Benjamin Spector. 2012. Scalar implicature as a grammatical phenomenon. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An International Handbook of Natural Language Meaning*, 2297–2331. Berlin: de Gruyter.

Elliott, Patrick D. & Yasutada Sudo. to appear. E-type readings of quantifiers under ellipsis. In *Proceedings of Sinn und Bedeutung 20*, .

von Fintel, Kai & Sabine Iatridou. 2003. Epistemic containment. *Linguistic Inquiry* 34(2). 173–198. doi:10.1162/002438903321663370.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 71–112. Palgrave Macmillan.

Fox, Danny & Martin Hackl. 2006. The universal density of measurement. *Linguistics and Philosophy* 29(5). 537–586. doi:10.1007/s10988-006-9004-4.

Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19(1). 87–107. doi:10.1007/s11050-010-9065-3.

Geurts, Bart. 1999. *Presuppositions and Pronouns*. Amsterdam: Elsevier.

Geurts, Bart. 2008. Implicture as a discourse phenomenon. In *Proceedings of Sinn und Bedeutung 11*, 261–275.

Geurts, Bart. 2009. Scalar implicature and local pragmatics. *Mind & Language* 24(1). 51–79. doi:10.1111/j.1468-0017.2008.01353.x.

Groenendijk, Jeroen & Martin Stokhof. 1984. *Studies on the Semantics of Questions and the Pragmatics of Answers*: University of Amsterdam dissertation.

Hacquard, Valentine. 2006. *Aspects of Modality*: Massachusetts Institute of Technology dissertation.

Hacquard, Valentine. 2011. Modality. In Klaus von Heusinger, Claudia Maienborn & Paul Portner (eds.), *Semantics: An International Handbook of Natural Language Meaning*, vol. 2, 1484–1515. Berlin: de Gruyter.

Heim, Irene. 1982. *The Semantics of Definite and Indefinite Noun Phrases*: University of Massachusetts, Amherst Ph.D. dissertation.

Heim, Irene. 1991. Artikel und Definitheit. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung/Semantics: An International Handbook of Contemporary Research*, 487–535. Berlin: de Gruyter.

Horn, Laurence. 1972. *On the Semantic Properties of the Logical Operators*: University of California Los Angeles dissertation.

Hurford, James R. 1974. Exclusive or inclusive disjunction. *Foundations of Language* 11(3). 409–411.

Jackendoff, Ray. 1972. *Semantic Interpretation in Generative Grammar*. Cambridge, MA: MIT Press.

Kanazawa, Makoto. 1993. Dynamic generalized quantifiers and monotonicity. Tech. rep. ILLC.

Kanazawa, Makoto. 1994. Weak vs. strong readings of donkey sentences and monotonicity inferences in a dynamic setting. *Linguistics and Philosophy* 17(2). 109–158. doi:10.1007/BF00984775.

Katzir, Roni. 2007. Structurally-defined alternatives. *Linguistics and Philosophy* 30(6). 669–690. doi:10.1007/s10988-008-9029-y.

Kratzer, Angelika. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, Worlds, and Contexts*, 38–74. Berlin: de Gruyter.

Kratzer, Angelika. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung/Semantics: An International Handbook of Contemporary Research*, 639–650. Berlin: de Gruyter.

Nouwen, Rick. 2003. Complement anaphora and interpretation. *Journal of Semantics* 20(1). 73–113.

Nouwen, Rick. 2007. On dependent pronouns and dynamic semantics. *Journal of Philosophical Logic* 36(2). 123–154. doi:10.1007/s10992-006-9029-8.

Perlmutter, David. 1971. *Deep and Surface Structure Constraints in Syntax*. New York: Holt, Rinehart, and Winston.

Roberts, Craige. 1987. *Modal Subordination, Anaphora and Distributivity*: University of Massachusetts, Amherst Ph.D. dissertation.

Romoli, Jacopo. 2012. *Soft But Strong: Neg-Raising, Soft Triggers, and Exhaustification*: Harvard University dissertation.

van Rooij, Robert. 2014. A fine-grained global analysis of implicatures. Ms., Universiteit van Amsterdam.

van Rooij, Robert & Katrin Schulz. 2004. Exhaustive interpretation of complex sentences. *Journal of Logic, Language and Information* 13(4). 491–519.

van Rooij, Robert & Katrin Schulz. 2006. Pragmatic meaning and non-monotonic reasoning: The case of exhaustive interpretation. *Linguistics and Philosophy* 29(2). 205–250. doi:10.1007/s10988-005-3760-4.

Ross, John. 1969. Auxiliaries as main verbs. In William Todd (ed.), *Studies in Philosophical Linguistics* I, 77–102. Evanston, IL: Great Expectations Press.

Singh, Raj. 2008. On the interpretation of disjunction: asymmetric, incremental, and eager for inconsistency. *Linguistics and Philosophy* 31(2). 245–260. doi: 10.1007/s10988-008-9038-x.

Stone, Matthew. 1999. Reference to possible worlds. RuCCS Report 49 Rutgers University.

Sudo, Yasutada. 2014. Presupposition satisfaction in attitude contexts. In Luka Crnič & Uli Sauerland (eds.), *The Art and Craft of Semantics: A Festschrift for Irene Heim*, vol. 2, 175–199. MITWPL.

van Eijck, Jan & Fer-Jan de Vries. 1992. Dynamic interpretation and Hoare deduction. *Journal of Logic, Language and Information* 1(1). 1–44. doi:10.1007/BF00203385.

Veltman, Frank. 1996. Defaults in update semantics. *Journal of Philosophical Logic* 25(3). 221–261. doi:10.1007/BF00248150.