The Plurality Inference as a Global Quantity Implicature

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1 Introduction

1.1 Plurality Inferences and Unmarked Plurals

Plural noun phrases typically give rise to **plurality inferences** (alt.: multiplicity inferences).

(1) Andrew wrote papers last year.
= Andrew wrote more than one paper.

(2) Andrew's papers have been published in journals.
= Andrew has more than one paper, and they have been published in journals.

Plurality inferences disppear in 'negative contexts' (Sauerland 2003, Sauerland, Anderssen & Yatsushiro 2005, among others):

(3) a. Andrew didn't write papers last year.
≠ Andrew didn't write more than one paper
b. If Andrew wrote papers last year, they have already been published.
≠ If Andrew wrote more than one paper last year, they have already been published.
c. Andrew arrived before semanticists arrived.
≠ Andrew arrived before more than one semanticist arrived.

Current theories of the plurality inference postulate number-neutral meaning for the plural.

- Scalar implicature theories (Spector 2007, Zweig 2009, Ivlieva 2013, Mayr 2015)
  The plurality inference is a scalar implicature.
- Ambiguity theories (Farkas & de Swart 2010, Grimm 2013, Martí 2018)
  The plural is ambiguous between plural and number-neutral.
- Anti-presupposition theories (Sauerland 2003, Sauerland et al. 2005)
  The plurality inference is an anti-presupposition.

We will focus on the first group of theories today.

1.2 Scalar Implicature Theories

The main idea of the scalar implicature theories is that the plural is semantically number-neutral, and the plurality inference arises as a scalar implicature in competition with the singular.
But the literal meanings of (4) are truth-conditionally equivalent on the assumption that the plural is semantically unmarked.

(4) a. Andrew wrote papers last year. ⇔ b. Andrew wrote a paper last year.

In order to generate a scalar implicature, there needs to be some semantic asymmetry between the alternatives. Some ideas for the truth-conditional asymmetry:

- Local scalar implicatures (Zweig 2009, Ivlieva 2013, Mayr 2015):
  The two sentences differ in truth-conditions at some non-global level.

- Higher-order scalar implicatures (Spector 2007):
  The plural competes with the singular enriched with its own scalar implicature.

I propose a new scalar implicature account without them (without necessarily denying them).

1.3 The Proposal in a Nutshell

- Scalar implicatures can arise due to non-truth-conditional aspects of meaning.
- (4a) and (4b) have the same truth-conditions but different anaphoric potentials.
  - (4a) introduces a discourse referent ranging over both singular and plural entities.
  - (4b) introduces a discourse referent ranging only over singular entities.

  Since (4b) yields fewer possibilities, it's stronger than (4a).
- This semantic asymmetry gives rise to a (secondary) scalar implicature that what (4b) would have meant is not what the speaker intended to mean. So, the discourse referent should range only over plural entities.

The Gricean Maxim of Quantity is about informativity in general, and should apply to non-truth-conditional aspects of meaning as well.

(5) a. Make your contribution as informative as is required (for the current purposes of the exchange).
    b. Do not make your contribution more informative than is required.

1.4 Embedding

This analysis accounts for the behaviour under negation:

(6) a. Andrew didn't write papers. b. Andrew didn't write a paper.

Neither of these sentences introduce a discourse referent, so their meanings are completely identical. As a consequence, there's no semantic asymmetry, and no scalar implicature.

One advantage of the scalar implicature theories is that it accounts for quantified sentences, especially those with non-monotonic quantifiers (Spector 2007, Ivlieva 2013).

(7) a. Exactly one linguist wrote papers. b. Exactly one linguist wrote a paper.

The present account can deal with this straightforwardly.

- (7a) introduces two discourse referents, one ranging over singular linguists, and one ranging over singular or plural papers.
- (7b) introduces two discourse referents, one ranging over singular linguists, and one ranging over singular papers.

The quantity implicature amounts to that the second discourse referent does not range over singular papers.

2 Update Semantics

2.1 Basic Update Semantics

Definition 1. (Assignments and contexts)

- An *assignment* is a total function from variables to the domain D of the model $\mathcal{M}$.[1]
- A *context* is a set of pairs consisting of a possible world w and an assignment a.
- The possible worlds of a context $\mathbf{W}_c$ is defined as $\{\, w \mid \text{for some } \langle w, a\rangle \in c \,\}$.
- The assignments of a context $\mathbf{A}_c$ is defined as $\{\, a \mid \text{for some } \langle w, a\rangle \in c \,\}$.

Definition 2. (Heimian Update Semantics) For any model $\mathcal{M} = \langle D, W, I\rangle$:[2]

$$
\begin{aligned}
[\mathtt{t}]^a_{\mathcal{M}} &\coloneqq \begin{cases} I(\mathtt{t}) & \text{if } \mathtt{t} \text{ is a constant} \\ a(\mathtt{t}) & \text{if } \mathtt{t} \text{ is a variable} \end{cases} \\
c[\mathrm{P}(\mathtt{t}_1, \dots, \mathtt{t}_n)]_{\mathcal{M}} &\coloneqq \{\, \langle w, a\rangle \in c \mid \langle [\mathtt{t}_1]^a, \dots, [\mathtt{t}_n]^a)\rangle \in I(w, \mathrm{P}) \,\} \\
c[\neg\varphi]_{\mathcal{M}} &\coloneqq \{\, \langle w, a\rangle \in c \mid \{\, \langle w, a\rangle \,\}\,[\varphi] = \varnothing \,\} \\
c[(\varphi \wedge \psi)]_{\mathcal{M}} &\coloneqq c[\varphi][\psi] \\
c[(\varphi \vee \psi)]_{\mathcal{M}} &\coloneqq c[\varphi] \cup c[\psi] \\
c[(\mathtt{t}_1 = \mathtt{t}_2)]_{\mathcal{M}} &\coloneqq \{\, \langle w, a\rangle \in c \mid [\mathtt{t}_1]^a = [\mathtt{t}_2]^a \,\}
\end{aligned}
$$

The model parameter will be omitted from now on.

(8) Andrew sat down. $\rightsquigarrow \mathtt{SatDown}(\mathtt{andrew})$

(9) Suppose there are three worlds:

- w_1: Only Andrew sat down.
- w_2: Only Ben sat down.
- w_3: Andrew and Chris sat down. No one else sat down.

$$
\left\{ \begin{array}{l} \langle w_1, a\rangle, \langle w_1, b\rangle, \\ \langle w_2, a\rangle, \langle w_2, c\rangle, \langle w_2, d\rangle, \\ \langle w_3, b\rangle \end{array} \right\} [\mathtt{SatDown}(\mathtt{andrew})] = \left\{ \begin{array}{l} \langle w_1, a\rangle, \langle w_1, b\rangle, \\ \\ \langle w_3, b\rangle \end{array} \right\}
$$

2.2 Indefinites and Random Assignment

Indefinites trigger *random assignment* (note that $\exists\mathtt{x}$ is taken to be a formula):

Definition 3. (Random Assignment) We'll write '$a[\mathtt{x} \mapsto e]$' to mean that assignment that differs from a at most in that it maps variable $\mathtt{x}$ to entity e.

$$c[\exists\mathtt{x}] \coloneqq \{\, \langle w, a[\mathtt{x} \mapsto e]\rangle \mid e \in D \text{ and } \langle w, a\rangle \in c \,\}$$

We adopt the Barwise notation where new variables are represented as superscripts and old variables are represented by subscripts.

(10) A$^{\mathtt{x}}$ farmer walked in. $\rightsquigarrow \exists\mathtt{x} \wedge \mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})$

(11) Let's consider a model with three entities e, f, g and the following three possible worlds:

w_1: e and f are farmers, f walked in.

w_2: e, f and g are farmers, e and f walked in.

w_3: no one is a farmer.

[1] We could use partial functions instead of total functions.

[2] I'll adopt a simple symmetric disjunction for convenience (it might not be entirely adequate as a translation of natural language disjunction). Parentheses are omitted by association to the right. I assume an intermediate language for the sake of clarity, but as usual, it is dispensable.

$$\left\{\begin{array}{l}
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & g & \dots \\ \hline\end{array}\right\rangle
\end{array}\right\} [\exists \mathtt{x} \wedge \mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})]$$

$$= \left\{\begin{array}{l}
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_3, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle
\end{array}\right\} [\mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})]$$

$$= \left\{\begin{array}{l}
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline g & g & \dots \\ \hline\end{array}\right\rangle
\end{array}\right\} [\mathtt{WalkedIn}(\mathtt{x})]$$

$$= \left\{\begin{array}{l}
\left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & e & \dots \\ \hline\end{array}\right\rangle, \left\langle w_1, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline f & g & \dots \\ \hline\end{array}\right\rangle, \\
\left\langle w_2, \begin{array}{|c|c|c|}\hline \mathtt{x} & \mathtt{y} & \dots \\ \hline e & g & \dots \\ \hline\end{array}\right\rangle
\end{array}\right\}$$

This semantics accounts for cross-sentential anaphora with an indefinite antecedent:

(12) A$^{\mathtt{x}}$ farmer walked in. He$_{\mathtt{x}}$ sat down.
$\rightsquigarrow (\exists \mathtt{x} \wedge \mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})) \wedge \mathtt{SatDown}(\mathtt{x})$

$\exists \mathtt{x}$ randomly introduces new values for $\mathtt{x}$, and $\mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})$ discards those world-assignment pairs that do not assign $\mathtt{x}$ a farmer who walked in in the respective possible worlds, as in (11). Then, $\mathtt{SatDown}(\mathtt{x})$ will operate on the resulting set of world-assignment pairs, and eliminate those pairs that assign $\mathtt{x}$ an entity that did not sit down in the respective possible worlds.

2.3 Plural Entities

Variables are now allowed to range over plural entities in addition to atomic entities.[3]

We assume that from any two entities e and f in the domain of the model, a new entity $e \oplus f$ can be formed that has e and f as parts, and all of these entities (and only they) are members of the domain D of the model (Link 1983).[4]

Predicates are also specified for plurality in the standard way. Crucially, we assume that plurals are semantically unmarked and number neutral. A predicate like `Farmers` is inherently distributive:

(13) a. $e \in I(w, \texttt{Farmer}) \Leftrightarrow e$ is an atomic entity and e is a farmer in w
b. $e \in I(w, \texttt{Farmers}) \Leftrightarrow$ each atomic part of e is a farmer in w

(We won't talk about non-distributive predicates, but the semantics is compatible with them.)

Recall that under the assumption that the plural is semantically unmarked, the following two sentences have the same truth-conditions.

(14) a. Andrew wrote a$^{\mathtt{x}}$ paper. $\rightsquigarrow \exists \mathtt{x} \wedge \texttt{Wrote}(\texttt{andrew}, \mathtt{x}) \wedge \texttt{Paper}(\mathtt{x})$
b. Andrew wrote papers$^{\mathtt{x}}$. $\rightsquigarrow \exists \mathtt{x} \wedge \texttt{Wrote}(\texttt{andrew}, \mathtt{x}) \wedge \texttt{Papers}(\mathtt{x})$

However, their dynamic meanings are different. We'll write $a \approx_{\mathtt{x}} b$ to mean that the two assignments a and b differ at most in the value for $\mathtt{x}$.

(15) a. $$c[\exists \mathtt{x} \wedge \texttt{Wrote}(\texttt{andrew}, \mathtt{x}) \wedge \texttt{Paper}(\mathtt{x})] = \left\{ \langle w, b \rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, a \rangle \in c,\ a \approx_{\mathtt{x}} b \text{ and} \\ \text{Andrew wrote } b(\mathtt{x}) \text{ in } w \text{ and} \\ b(\mathtt{x}) \text{ is atomic and } b(\mathtt{x}) \text{ is a paper in } w \end{array} \right\}$$

b. $$c[\exists \mathtt{x} \wedge \texttt{Wrote}(\texttt{andrew}, \mathtt{x}) \wedge \texttt{Papers}(\mathtt{x})] = \left\{ \langle w, b \rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, a \rangle \in c,\ a \approx_{\mathtt{x}} b \text{ and} \\ \text{Andrew wrote each atomic part of } b(\mathtt{x}) \text{ in } w \text{ and} \\ \text{each atomic part of } b(\mathtt{x}) \text{ is a paper in } w \end{array} \right\}$$

In (15b), $b(\mathtt{x})$ can be a plural entity, but not in (15a). We will make use of this semantic asymmetry to derive the plurality inference.

Here's a concrete example.

(16) Suppose the following worlds:

- w_1: Andrew wrote p_1 and no other papers.
- w_2: Andrew wrote p_1 and p_2 and no other papers.
- w_3: Andrew wrote no papers.

and $c = \{ \langle w_1, a \rangle, \langle w_1, b \rangle, \langle w_2, a \rangle, \langle w_2, d \rangle, \langle w_3, d \rangle \}$.

$$c[\exists \mathtt{x} \wedge \texttt{Wrote}(\texttt{andrew}, \mathtt{x}) \wedge \texttt{Paper}(\mathtt{x})] = \left\{ \begin{array}{l} \langle w_1, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_1, b[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_2] \rangle \end{array} \right\}$$

$$c[\exists \mathtt{x} \wedge \texttt{Wrote}(\texttt{andrew}, \mathtt{x}) \wedge \texttt{Papers}(\mathtt{x})] = \left\{ \begin{array}{l} \langle w_1, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_1, b[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_2] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_1 \oplus p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1 \oplus p_2] \rangle \end{array} \right\}$$

[3] See van den Berg (1996:Ch.3) for a similar idea. He eventually proposes a different way to deal with pluralities where they are encoded in a set of assignments. See §4.

[4] Nothing crucial hinges on this. You can use sets instead (Landman 1989a,b, Schwarzschild 1996, Winter 2000).

Note that according this semantics, *Andrew wrote a paper* is compatible with him having written more than one paper (as in w_2 above). This is fine as *a*-indefinites are generally non-maximal. Its maximal reading, if it's available, needs to be derived by some other means, possibly as a scalar implicature (see Spector 2007). We will not deal with this here.

3 Scalar Implicature in Update Semantics

3.1 How to Derive the Plurality Inference

Let $c_{\text{sg}} = c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x})]$ and $c_{\text{pl}} = c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x})]$.
Whenever c_{sg} and c_{pl} are non-empty and non-equivalent (that is, there are worlds in $\mathbf{W}_c$ in which Andrew wrote more than one paper), there is an asymmetric semantic relation, namely:

$$c_{\text{sg}} \subset c_{\text{pl}}$$

Importantly, $\mathbf{W}_{c_{\text{sg}}} = \mathbf{W}_{c_{\text{pl}}}$, because the two sentences are truth-conditionally equivalent. The crucial difference is coming from the assignment functions, i.e. $\mathbf{A}_{c_{\text{sg}}} \subset \mathbf{A}_{c_{\text{pl}}}$.
I propose that the plurality inference is derived by subtracting c_{sg} from c_{pl}. For example, in the case of (15), we get:

$$\{ \ \langle w_2, a[\mathrm{x} \mapsto p_1 \oplus p_2] \rangle, \langle w_2, d[\mathrm{x} \mapsto p_1 \oplus p_2] \rangle \ \}$$

In w_2, Andrew wrote two papers and all the values for $\mathtt{x}$ are pluralities consisting of multiple papers.
More generally, let c' be the resulting context after the above subtraction operation. Then, whenever $c' \neq \emptyset$, Andrew wrote multiple papers in $\mathbf{W}_{c'}$ and for each $a \in \mathbf{A}_{c'}$, $a(\mathrm{x})$ is a plural entity. This is the plurality inference.
We also correctly account for the fact that the plural papers can be referred back to later in the discourse by a plural pronoun, but not by a singular pronoun.

(17) They$_{\mathrm{x}}$ were about Slovenian duals.

3.2 Scalar Implicature and Informativity in Dynamic Semantics

The Gricean Maxim of Quantity says:

(18) a. Make your contribution as informative as is required (for the current purposes of the exchange).
b. Do not make your contribution more informative than is required.

The notation of 'informativity' is often understood in terms of truth-conditional entailment:

(19) φ is truth-conditionally more informative than ψ iff φ entails ψ but ψ does not entail φ (alt.: φ asymmetrically entails ψ).

In our update semantics, this can be paraphrased as follows:

Definition 4. (Truth-Conditional Informativity) φ is *truth-conditionally more informative* than ψ iff for each context c, $\mathbf{W}_{c[\varphi]} \subseteq \mathbf{W}_{c[\psi]}$ but in some context c', $\mathbf{W}_{c'[\psi]} \nsubseteq \mathbf{W}_{c'[\varphi]}$.

Note that the smaller set is more informative, because the bigger set contains more live possibilities, so less information.
In update semantics we can have a different notation of informativity:

Definition 5. (Dynamic Informativity) φ is *dynamically more informative* than ψ iff for each context c, $c[\varphi] \subseteq c[\psi]$ but in some context c', $c'[\psi] \nsubseteq c'[\varphi]$.

This is distinct from truth-conditional informativity, because the asymmetry may come from the anaphoric potentials encoded in the assignments. And this is exactly what we use to derive the plurality inference.

I formulate the scalar implicature computation as follows.[5]

(20) If φ has an alternative ψ that is dynamically more informative, then an assertion of φ in c by a cooperative speaker is interpreted as $c[\varphi] - c[\psi]$.

We have already seen above that (20) gives rise to the plurality inference, as in (16).

3.2.1 Negation

Recall that in negative sentences plurality inferences are not observed. This is explained as follows: under negation, the singular and plural sentences have the exact same dynamic meaning. So neither of them is more informative under either notion of informativity:

(21) a. $c[\neg(\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x}))] = \{\, \langle w, a\rangle \in c \mid \text{Andrew wrote no paper in } w \,\}$

b. $c[\neg(\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x}))] = \{\, \langle w, a\rangle \in c \mid \text{Andrew wrote no paper in } w \,\}$

3.2.2 *Most*

The above mechanism of scalar implicature computation works for other types of scalar implicatures more generally, e.g. *most*, which by assumption competes with *all*.

In order to account for these generalized quantifiers, we need to introduce the maximality operator (van den Berg 1996).

Definition 6. (Maximality Operator)

$$c[\mathtt{M}^{\mathtt{x}}(\varphi)] = \{\, \langle w, a\rangle \in c[\exists x \wedge \varphi] \mid \text{for no } \langle w, a'\rangle \in c[\exists x \wedge \varphi],\ a(\mathtt{x}) \sqsubset a'(\mathtt{x}) \,\}$$

In words, for each $\langle w, a\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi)]$, a assigns a maximal value to $\mathtt{x}$ that satisfies φ in w.

This operator is useful in defining (selective) generalized quantifiers, because they can be seen as expressing relations between two maximal entities (which stand for sets in the classical setting). $\#(e)$ is the number of atomic entities in e.[6]

(22) a. $$c[\mathtt{Most}^{\mathtt{x}}(\varphi)(\psi)] = \left\{ \langle w, a\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi \wedge \psi)] \;\middle|\; \begin{array}{l} \text{for some } \langle w, a'\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi)] \\ \dfrac{\#(a(\mathtt{x}))}{\#(a'(\mathtt{x}))} \gg \dfrac{1}{2} \end{array} \right\}$$

b. $$c[\mathtt{All}^{\mathtt{x}}(\varphi)(\psi)] = \left\{ \langle w, a\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi \wedge \psi)] \;\middle|\; \begin{array}{l} \text{for some } \langle w, a'\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi)] \\ \#(a(\mathtt{x})) = \#(a'(\mathtt{x})) \end{array} \right\}$$

(23) Let us assume that in w_1, w_2, w_3 and w_4, there are exactly 10 linguists, ℓ_1, ..., ℓ_{10}.

- w_1: All linguists smoke.
- w_2: Only ℓ_1, ..., ℓ_8 smoke.
- w_3 Only ℓ_1, ..., ℓ_3 smoke.
- w_4: No linguists smoke.

[5] Note that I'm only dealing with *secondary implicatures* in the sense of Sauerland (2004) here. See §5

[6] In order to deal with mass nouns, which *most* and *all* are compatible with, we need a more general definition of #.

a. $\{\langle w_1, a_1\rangle, \langle w_2, a_2\rangle, \langle w_3, a_3\rangle, \langle w_4, a_4\rangle\}\,[\mathtt{Most}^{\mathtt{x}}(\mathtt{Linguists}(\mathtt{x}))(\mathtt{Smoke}(\mathtt{x}))]$
$= \{\langle w_1, a_1[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_{10}]\rangle, \langle w_2, a_2[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_8]\rangle\}$

b. $\{\langle w_1, a_1\rangle, \langle w_2, a_2\rangle, \langle w_3, a_3\rangle, \langle w_4, a_4\rangle\}\,[\mathtt{All}^{\mathtt{x}}(\mathtt{Linguists}(\mathtt{x}))(\mathtt{Smoke}(\mathtt{x}))]$
$= \{\langle w_1, a_1[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_{10}]\rangle\}$

The *all*-alternative is dynamically more informative. Consequently, we derive the scalar implicature, and end up with the following singleton set, as desired.

$$\{\langle w_2, a_2[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_8]\rangle\}$$

Notice also that this accounts for so-called *refset anaphora* (van den Berg 1996, Nouwen 2003). Refset anaphora is anaphora with respect to the entities that satisfy both the restrictor and nuclear scope as in (24).

(24) Most$^{\mathtt{x}}$ linguists smoke. They$_{\mathtt{x}}$ (= the linguists who smoke) also drink.

4 Plurality Inferences in Quantificational Contexts

One of the advantages of scalar implicature approaches to plurality inferences is that they account for plurality inferences in non-monotonic contexts:

(25) a. Exactly one$^{\mathtt{x}}$ linguist wrote papers$^{\mathtt{y}}$ last year.
b. Exactly one$^{\mathtt{x}}$ linguist wrote a$^{\mathtt{y}}$ paper last year.

(25a) implies that the only linguist who wrote papers last year wrote more than one paper.

Our analysis derives this inference without resorting to local computation of scalar implicatures or higher-order scalar implicatures (unlike other scalar implicature theories, e.g. Spector 2007, Ivlieva 2013).

4.1 Plural Information States

In order to deal with (25), we need to be able to encode the dependency between two variables, $\mathtt{x}$ and $\mathtt{y}$. We will adopt Van den Berg's (1996) idea of *plural information states* (see also Nouwen 2003, 2007, Brasoveanu 2007, 2008, 2010).

The idea is to model contexts as a pair consisting of a possible world and a *set* of assignments. Following Brasoveanu (2008) and Dotlačil (2013) we allow assignments to return plural entities.

Definition 7. (Assignments and contexts)

- The domain D of the model $\mathcal{M}$ is closed under sum-formation $\oplus$.
- An *assignment* is a total function from variables to D.
- A *context* is a set of pairs consisting of a possible world w and a set A of assignments.
- The possible worlds of a context $\mathbf{W}_c$ is defined as $\{\, w \mid \text{for some } \langle w, A\rangle \in c \,\}$.
- The assignment sets of a context $\mathbf{A}_c$ is defined as $\{\, A \mid \text{for some } \langle w, A\rangle \in c \,\}$.

Definition 8. (Plural Dynamic Semantics)

$$
\begin{aligned}
[\mathtt{t}]^{A}_{\mathcal{M}} &\coloneqq \begin{cases} I(\mathtt{t}) & \text{if } \mathtt{t} \text{ is a constant} \\ \bigoplus \{\, a(\mathtt{t}) \mid a \in A \,\} & \text{if } \mathtt{t} \text{ is a variable} \end{cases} \\
c[\mathtt{P}(\mathtt{t_1}, \dots, \mathtt{t_n})]_{\mathcal{M}} &\coloneqq \{\, \langle w, A\rangle \in c \mid \langle [\mathtt{t_1}]^{A}, \dots, [\mathtt{t_n}]^{A}\rangle \in I_w(\mathtt{P}) \,\} \\
c[\neg\varphi]_{\mathcal{M}} &\coloneqq \{\, \langle w, A\rangle \in c \mid \{\, \langle w, A\rangle \,\}\,[\varphi] = \varnothing \,\} \\
c[(\varphi \wedge \psi)]_{\mathcal{M}} &\coloneqq c[\varphi][\psi] \\
c[(\varphi \vee \psi)]_{\mathcal{M}} &\coloneqq c[\varphi] \cup c[\psi] \\
c[(\mathtt{t_1} \wedge \mathtt{t_2})]_{\mathcal{M}} &\coloneqq \{\, \langle w, A\rangle \in c \mid [\mathtt{t_1}]^{A} = [\mathtt{t_2}]^{A} \,\}
\end{aligned}
$$

From now on, we will write $A(\mathtt{x})$ instead of $\bigoplus \{\, a(\mathtt{x}) \mid a \in A \,\}$.

Definition 9. (Random Assignment)

$$
c[\exists \mathtt{x}] \coloneqq \left\{ \langle w, B\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, A\rangle \in c, \\ \text{for each } a \in A, \text{ there is } b \in B \text{ such that } a \approx_{\mathtt{x}} b, \text{ and} \\ \text{for each } b \in B, \text{ there is } a \in A \text{ such that } a \approx_{\mathtt{x}} b \end{array} \right\}
$$

We keep the same assumptions about the semantics of nouns as before:

(26) a. $e \in I(w, \mathtt{Farmer}) \Leftrightarrow e$ is an atomic entity and is a farmer in w
b. $e \in I(w, \mathtt{Farmers}) \Leftrightarrow$ each atomic part of e is a farmer in w

$$
\begin{aligned}
c[\mathtt{Farmer}(\mathtt{x})] &= \{\, \langle w, A\rangle \in c \mid A(\mathtt{x}) \in I_w(\mathtt{Farmer}) \,\} \\
&= \left\{ \langle w, A\rangle \in c \;\middle|\; \begin{array}{l} \text{for any } a, a' \in A,\ a(\mathtt{x}) = a'(\mathtt{x}) \text{ and} \\ \text{the unique } e = a(\mathtt{x}) \text{ for any } a \in A \text{ is a farmer in } w \end{array} \right\} \\
c[\mathtt{Farmers}(\mathtt{x})] &= \{\, \langle w, A\rangle \in c \mid A(\mathtt{x}) \in I_w(\mathtt{Farmers}) \,\}
\end{aligned}
$$

Definition 10. (Maximality Operator)

$$
c[\mathtt{M}^{\mathtt{x}}(\varphi)] \coloneqq \{\, \langle w, A\rangle \in c[\exists \mathtt{x} \wedge \varphi] \mid \text{for no } \langle w, A'\rangle \in c[\exists \mathtt{x} \wedge \varphi],\ A(\mathtt{x}) \sqsubset A'(\mathtt{x}) \,\}
$$

4.2 'Exactly One'

(27) $c[\mathtt{ExactlyOne}^{\mathtt{x}}(\varphi)(\psi)] = \{\, \langle w, A\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi \wedge \psi)] \mid \#(A(\mathtt{x})) = 1 \,\}$

(28) Exactly one$^{\mathtt{x}}$ linguist smokes. $\rightsquigarrow \mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}))(\mathtt{Smokes}(\mathtt{x}))$

(29) Consider the following possible worlds.

- w_1: Andrew, a linguist, smokes. No other linguist smokes.
- w_2: Benjamin, a linguist, smokes. No other linguist smokes.
- w_3: Andrew and Benjamin, both linguists, smoke. No other linguist smokes.
- w_4: No linguist smokes.

Firstly, let:

$$
\left\{ \begin{array}{l} \langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle \end{array} \right\} [\mathtt{M}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}) \wedge \mathtt{Smokes}(\mathtt{x}))] = c'
$$

For each $\langle w, A\rangle \in c'$, and for each $a \in A$, a must be an atomic entity that is a linguist in w and smokes in w. So w_4 will not be in $\mathrm{W}_{c'}$.

But we will still have pairs like $\langle w_3, A\rangle$, as long as each $a \in A$ assigns $\mathtt{x}$ either Andrew or Benjamin, and but not Andrew$\oplus$ Benjamin, but at the same time, it's required that $A(\mathtt{x}) =$ Andrew $\oplus$ Benjamin, because of maximality.

The number restriction of *exactly one* filters out the pairs whose world is w_3.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle\end{array}\right\}[\mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}))(\mathtt{Smokes}(\mathtt{x}))]$$
$$= \{\langle w, A\rangle \in c' \mid \#(A(\mathtt{x})) = 1\}$$
$$= \{\langle w_1, A_1'\rangle \in \{w_1, A_1\}[\exists x] \mid A_1'(\mathtt{x}) = \text{Andrew}\} \cup \{\langle w_2, A_2'\rangle \in \{w_2, A_2\}[\exists x] \mid A_2'(\mathtt{x}) = \text{Benjamin}\}$$

4.3 Plurality Inferences in Non-monotonic Contexts

(30) a. Exactly one$^{\mathtt{x}}$ linguist wrote papers$^{\mathtt{y}}$ (last year).
$\rightsquigarrow \mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}))(\exists \mathtt{y} \wedge \mathtt{Papers}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))$

b. Exactly one$^{\mathtt{x}}$ linguist wrote a$^{\mathtt{y}}$ paper (last year).
$\rightsquigarrow \mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}))(\exists \mathtt{y} \wedge \mathtt{Paper}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))$

The idea is the same as before. The singular version (30b) is dynamically more informative than the plural version (30a), although they are truth-conditionally equivalent. This gives rise to an implicature that $\mathtt{y}$ is assigned a plural entity as its value.

(31) Consider the following worlds.

- w_1: Andrew, a linguist, wrote exactly one paper, p_1. No other linguist wrote any paper.
- w_2: Andrew, a linguist, wrote exactly two papers, p_1 and p_2. No other linguist wrote any paper.
- w_3: Benjamin, a linguist, wrote exactly one paper, p_3. No other linguist wrote any paper.
- w_4: Benjamin, a linguist, wrote exactly two papers, p_3 and p_4. No other linguist wrote any paper.
- w_5: Andrew, a linguist, wrote exactly one paper, p_1 and Benjamin, a linguist, wrote exactly one paper, p_3.
- w_6: No linguist wrote any paper.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle, \\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}) \wedge \exists \mathtt{y} \wedge \mathtt{Paper}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))] = c'_{\text{sg}}$$
$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle, \\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}) \wedge \exists \mathtt{y} \wedge \mathtt{Papers}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))] = c'_{\text{pl}}$$

Note that $\mathbf{W}_{c'_{\text{sg}}} = \mathbf{W}_{c'_{\text{pl}}} = \{w_1, w_2, w_3, w_4, w_5\}$.

- For each $\langle w, A\rangle \in c'_{\text{sg}}$, each $a \in A$, $a(\mathtt{y})$ is either an atomic entity that is a paper in w, and furthermore, $a(\mathtt{x})$ is the author of $a(\mathtt{y})$ in w.
- For each $\langle w, A\rangle \in c'_{\text{pl}}$, each $a \in A$, $a(\mathtt{y})$ is either an atomic entity that is a paper or a plural entity that is made up of multiple papers in w, and furthermore, $a(\mathtt{x})$ is the author of $a(\mathtt{y})$ in w.

The number restriction of *exactly one* will eliminate those pairs whose possible world is w_4.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle, \\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\} [\mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}))(\exists \mathtt{y} \wedge \mathtt{Paper}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))]$$

$$= \{\langle w, A\rangle \in c'_{\mathrm{pl}} \mid \#(A(\mathtt{x})) = 1\}$$

$$= \bigcup \left\{\begin{array}{l} \{\langle w_1, A'_1\rangle \in \{w_1, A_1\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \mid A'(\mathtt{x}) = \text{Andrew and } A'(\mathtt{y}) = p_1\}, \\ \left\{\langle w_2, A'_2\rangle \in \{w_2, A_2\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \;\middle|\; \begin{array}{l} A'_2(\mathtt{x}) = \text{Andrew and} \\ \text{either } A'_2(\mathtt{y}) = p_1 \text{ or } A'_2(\mathtt{y}) = p_2 \end{array}\right\}, \\ \{\langle w_3, A'_3\rangle \in \{w_3, A_3\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \mid A'_3(\mathtt{x}) = \text{Benjamin and } A'_3(\mathtt{y}) = p_3\}, \\ \left\{\langle w_4, A'_4\rangle \in \{w_4, A_4\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \;\middle|\; \begin{array}{l} A'_4(\mathtt{x}) = \text{Benjmain and} \\ \text{either } A'_4(\mathtt{y}) = p_3 \text{ or } A'_4(\mathtt{y}) = p_4 \end{array}\right\} \end{array}\right\}$$

$$= c''_{\mathrm{sg}}$$

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle, \\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\} [\mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist}(\mathtt{x}))(\exists \mathtt{y} \wedge \mathtt{Papers}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))]$$

$$= \{\langle w, A\rangle \in c'_{\mathrm{pl}} \mid \#(A(\mathtt{x})) = 1\}$$

$$= \bigcup \left\{\begin{array}{l} \{\langle w_1, A'_1\rangle \in \{w_1, A_1\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \mid A'_1(\mathtt{x}) = \text{Andrew and } A'_1(\mathtt{y}) = p_1\}, \\ \left\{\langle w_2, A'_2\rangle \in \{w_2, A_2\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \;\middle|\; \begin{array}{l} A'_2(\mathtt{x}) = \text{Andrew and} \\ \text{either } A'_2(\mathtt{y}) = p_1 \text{ or } A'_2(\mathtt{y}) = p_2 \text{ or } A'_2(\mathtt{y}) = p_1 \oplus p_2 \end{array}\right\}, \\ \{\langle w_3, A'_3\rangle \in \{w_3, A_3\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \mid A'_3(\mathtt{x}) = \text{Benjamin and } A'_3(\mathtt{y}) = p_3\}, \\ \left\{\langle w_4, A'_4\rangle \in \{w_4, A_4\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \;\middle|\; \begin{array}{l} A'_4(\mathtt{x}) = \text{Benjamin and} \\ \text{either } A'_4(\mathtt{y}) = p_3 \text{ or } A'_4(\mathtt{y}) = p_3 \text{ or } A'_4(\mathtt{y}) = p_3 \oplus p_4 \end{array}\right\} \end{array}\right\}$$

$$= c''_{\mathrm{pl}}$$

Since $c''_{\mathrm{sg}} \subset, c''_{\mathrm{pl}}$, the scalar implicature is generated, yielding the following set:

$$\{\langle w_2, A'_2\rangle \in \{w_2, A_2\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \mid A'_2(\mathtt{x}) = \text{Andrew and } A'_2(\mathtt{y}) = p_1 \oplus p_2\}$$
$$\cup$$
$$\{\langle w_4, A'_4\rangle \in \{w_4, A_4\}[\exists \mathtt{x} \wedge \exists \mathtt{y}] \mid A'_4(\mathtt{x}) = \text{Benjmain and } A'_4(\mathtt{y}) = p_3 \oplus p_4\}$$

So in the end, only w_2 and w_4 survived, as desired. Furthermore, this accounts for cross-sentential anaphora naturally:

(32) He$_{\mathrm{x}}$ submitted them$_{\mathrm{y}}$ to journals.

4.4 'Everyone'

The same mechanism makes good predictions for other quantificational contexts.

(33) a. Everyone^{x} wrote papers^{y}. $\rightsquigarrow \mathtt{Everyone}^{\mathtt{x}}(\exists \mathtt{y} \wedge \mathtt{Papers}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))$
b. Everyone^{x} wrote a^{y} paper. $\rightsquigarrow \mathtt{Everyone}^{\mathtt{x}}(\exists \mathtt{y} \wedge \mathtt{Paper}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y}))$

The predicted scalar inference is that at least one person wrote multiple papers.

This is because the singular version (33b) will produce a set of pairs $\langle w, A\rangle$ such that for each $a \in A$, $a(\mathtt{y})$ is an atomic entity that is a paper in w, and $a(\mathtt{x})$ is its author in w, and $A(\mathtt{x})$ is the plurality consisting of all the people.

The pairs resulting from (33a) will contain in addition to these pairs those pairs $\langle w, A\rangle$ such that for some $a \in A$, $a(\mathtt{y})$ is a plural entity that is a paper in w. And only these pairs remain after computing the scalar implicature.

Here are the details. First, we need the distributivity operator, which creates quantificational dependency in the current setting.[7]

Definition 11. (Distributivity Operator)

$$c[\mathtt{D}_{\mathtt{x}}(\varphi)] \coloneqq \left\{ \langle w, A' \rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, A \rangle \in c,\ A(\mathtt{x}) = A'(\mathtt{x}) \text{ and} \\ \text{for each } e \sqsubseteq_a A(\mathtt{x}), \langle w, A' \restriction_{\mathtt{x} \mapsto e} \rangle \in \{ \langle w, A \restriction_{\mathtt{x} \mapsto e} \rangle \} [\varphi] \end{array} \right\}$$

$$e \sqsubseteq_a E :\Leftrightarrow e \sqsubseteq E \text{ and } e \text{ is atomic}$$

$$A \restriction_{x \mapsto e} \coloneqq \{ a \in A \mid a(\mathtt{x}) = e \}$$

(34) $c[\mathtt{Everyone}^{\mathtt{x}}(\varphi)] = \{ \langle w, A \rangle \in c[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human}(\mathtt{x}) \wedge \varphi))] \mid A(\mathtt{x}) = \bigoplus \{ e \in D \mid e \text{ is a human in } w \} \}$

(35) Let's assume that there are three humans Andrew, Benjamin, and Chris. Consider the following worlds:

- w_1: Andrew wrote exactly one paper, p_1, Benjamin wrote exactly one paper, p_2, and Chris wrote exactly one paper, p_3.
- w_2: Andrew wrote exactly two papers, p_1 and q_1, Benjamin wrote exactly two papers, p_2 and q_2, and Chris wrote exactly two papers, p_3 and q_3.
- w_3: Andrew wrote exactly two papers, p_1 and q_1, Benjamin wrote exactly one paper p_2 and Chris wrote exactly one paper p_3.
- w_4: Andrew wrote exactly two papers, p_1 and q_1, Benjamin and Chris wrote no papers.
- w_5: No one wrote any paper.

$$\left\{ \begin{array}{l} \langle w_1, A_1 \rangle, \langle w_2, A_2 \rangle, \langle w_3, A_3 \rangle \\ \langle w_4, A_4 \rangle, \langle w_5, A_5 \rangle \end{array} \right\} [\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human}(\mathtt{x}) \wedge \mathtt{Paper}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y})))] = c'_{\mathrm{sg}}$$

$$\left\{ \begin{array}{l} \langle w_1, A_1 \rangle, \langle w_2, A_2 \rangle, \langle w_3, A_3 \rangle \\ \langle w_4, A_4 \rangle, \langle w_5, A_5 \rangle \end{array} \right\} [\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human}(\mathtt{x}) \wedge \mathtt{Paper}(\mathtt{y}) \wedge \mathtt{Wrote}(\mathtt{x}, \mathtt{y})))] = c'_{\mathrm{pl}}$$

$\mathbf{W}_{c'_{\mathrm{sg}}} = \mathbf{W}_{c'_{\mathrm{pl}}} = \{ w_1, w_2, w_3, w_4 \}$. Note that w_4 is not excluded at this point.

- For each $\langle w', A' \rangle \in c'_{\mathrm{sg}}$, for each $a' \in A'$, $a(\mathtt{y})$ is an atomic entity that is a paper in w', and $a(\mathtt{x})$ is its author in w'.
- For each $\langle w', A' \rangle \in c'_{\mathrm{pl}}$, for each $a' \in A'$, each atomic part of $a(\mathtt{y})$ is a paper in w', and $a(\mathtt{x})$ is the author of the paper or papers in w'.
- Fro each $\langle w', A' \rangle \in c'_{\mathrm{sg/pl}}$, $A'(\mathtt{x}) = \text{Chris}$ if $w' = w_4$ and $A'(\mathtt{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris}$, if otherwise.

[7] We actually have to deal with partiality more carefully in the general case, but to keep the exposition simple, I'll ignore it (this will do because we are only talking about cases involving random assignment). See Van den Berg (1996) and Nouwen (2003) in particular. See also Nouwen (2003) and Nouwen (2007) for an undergeneration problem of this system and a solution to it.

$$
\begin{aligned}
&\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle\end{array}\right\}[\texttt{Everyone}^{\mathrm{x}}(\exists \mathrm{y} \wedge \texttt{Paper}(\mathrm{y}) \wedge \texttt{Wrote}(\mathrm{x}, \mathrm{y}))] \\
=&\{\langle w', A'\rangle \in c'_{\mathrm{pl}} \mid A'(\mathrm{x}) = \bigoplus\{e \in D \mid e \text{ is a human in } w\}\} \\
=&\bigcup\left\{\begin{array}{l}
\left\{\begin{array}{l|l}
\langle w_1, A'_1\rangle \in \langle w_1, A_1\rangle[\exists x \wedge \exists y] & \begin{array}{l}A'_1(\mathrm{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris and} \\ A'_1(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \text{ and for each } a' \in A'_1, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1, \text{ and} \\ a'(\mathrm{x}) = \text{Benjamin iff } a'(\mathrm{y}) = p_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3\end{array}
\end{array}\right\}, \\
\left\{\begin{array}{l|l}
\langle w_2, A'_2\rangle \in \langle w_2, A_2\rangle[\exists x \wedge \exists y] & \begin{array}{l}A'_2(\mathrm{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris and} \\ A'_2(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \oplus q_2 \oplus q_3 \text{ and} \\ \text{for each } a' \in A'_2, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1, \text{ and} \\ a'(\mathrm{x}) = \text{Benjamin iff } a'(\mathrm{y}) = p_2 \text{ or } a'(\mathrm{y}) = q_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3 \text{ or } a'(\mathrm{y}) = q_3\end{array}
\end{array}\right\}, \\
\left\{\begin{array}{l|l}
\langle w_3, A'_3\rangle \in \langle w_3, A_3\rangle[\exists x \wedge \exists y] & \begin{array}{l}A'_3(\mathrm{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris and} \\ A'_3(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \text{ and} \\ \text{for each } a' \in A'_3, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1, \text{ and} \\ a'(\mathrm{x}) = \text{Benjamin iff } a'(\mathrm{y}) = p_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3\end{array}
\end{array}\right\}
\end{array}\right\} \\
=&c''_{\mathrm{sg}}
\end{aligned}
$$

$$
\begin{aligned}
&\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle\end{array}\right\}[\texttt{Everyone}^{\mathrm{x}}(\exists \mathrm{y} \wedge \texttt{Paper}(\mathrm{y}) \wedge \texttt{Wrote}(\mathrm{x}, \mathrm{y}))] \\
=&\{\langle w', A'\rangle \in c'_{\mathrm{pl}} \mid A'(\mathrm{x}) = \bigoplus\{e \in D \mid e \text{ is a human in } w\}\} \\
=&\bigcup\left\{\begin{array}{l}
\left\{\begin{array}{l|l}
\langle w_1, A'_1\rangle \in \langle w_1, A_1\rangle[\exists x \wedge \exists y] & \begin{array}{l}A'_1(\mathrm{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris and} \\ A'_1(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \text{ and for each } a' \in A'_1, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1, \text{ and} \\ a'(\mathrm{x}) = \text{Benjamin iff } a'(\mathrm{y}) = p_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3\end{array}
\end{array}\right\}, \\
\left\{\begin{array}{l|l}
\langle w_2, A'_2\rangle \in \langle w_2, A_2\rangle[\exists x \wedge \exists y] & \begin{array}{l}A'_2(\mathrm{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris and} \\ A'_2(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \oplus q_2 \oplus q_3 \text{ and} \\ \text{for each } a' \in A'_2, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1 \\ \quad \text{or } a'(\mathrm{y}) = p_1 \oplus q_1, \text{ and} \\ a'(\mathrm{x}) = \text{Benjamin iff } a'(\mathrm{y}) = p_2 \text{ or } a'(\mathrm{y}) = q_2 \\ \quad \text{or } a'(\mathrm{y}) = p_2 \oplus q_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3 \text{ or } a'(\mathrm{y}) = q_3 \\ \quad \text{or } a'(\mathrm{y}) = p_3 \oplus q_3\end{array}
\end{array}\right\}, \\
\left\{\begin{array}{l|l}
\langle w_3, A'_3\rangle \in \langle w_3, A_3\rangle[\exists x \wedge \exists y] & \begin{array}{l}A'_3(\mathrm{x}) = \text{Andrew} \oplus \text{Benjamin} \oplus \text{Chris and} \\ A'_3(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \text{ and} \\ \text{for each } a' \in A'_3, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1 \\ \quad \text{or } a'(\mathrm{y}) = p_1 \oplus q_1, \text{ and} \\ a'(\mathrm{x}) = \text{Benjamin iff } a'(\mathrm{y}) = p_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3\end{array}
\end{array}\right\}
\end{array}\right\} \\
=&c''_{\mathrm{pl}}
\end{aligned}
$$

As before, $c''_{\mathrm{sg}} \subset c''_{\mathrm{pl}}$, and after computing the scalar implicature, we will get a subset of c''_{pl} such that for each $\langle w'', A''\rangle$ in this set, there is at least one $a'' \in A''$ such that $a''(\mathrm{y})$ is a plural entity. This means that w_1 is no longer in this set.

Again, we don't need local computation of scalar implicatures, or higher order implicatures.[8]

5 Primary Implicatures

5.1 Primary vs. Secondary Implicatures

Two types of conversational implicatures are often distinguished, *primary* and *secondary* (Sauerland 2004).

(36) Some of these movies are interesting.
  a. *¬the speaker believes all of these movies are interesting* (Primary Implicature)
  b. *the speaker believes ¬all of these movies are interesting* (Secondary Implicature)

Standard Gricean reasoning generates a primary implicature, which may be strengthened to a secondary implicature via additional assumptions, e.g. Opinionatedness.

(37) a. Suppose that the speaker obeys the Maxim of Quantity.
  b. If she is certain that the alternative sentence *All of these movies are interesting* is true, she should have uttered it.
  c. Because she didn't, she is not certain that it is true.

If there's reason to believe that the speaker is opinionated, i.e. she knows that all of the movies are interesting or that not all of the movies are interesting, then it follows from (37b) that the speaker is certain that not all of the movies are interesting.

Under our account of the plurality inference, the reasoning is not about the speaker's propositional beliefs, but about what referents the speaker believes a variable/discourse referent to vary across.

(38) a. Suppose that the speaker obeys the Maxim of Quantity.
  b. If she intends to restrict the referents of the variable only to atomic entities, she should have uttered *Andrew wrote a paper*.
  c. Because she didn't, she didn't intend to restrict the referents only to atomic entities.

The resulting primary implicature (38b) is weaker than the plurality inference.

To derive the plurality inference, we need an extra assumption similar to Opinionatedness, e.g. either the speaker believes that the variable should only vary across atomic entities, or the speaker believes that it should only vary across plural entities. This is not particularly a strange assumption. It would follow if the speaker is assumed to know how many papers Andrew wrote, for example.

5.2 Definite Plurals

Definite plurals also give rise to plurality inferences.

(39) a. Chris's student is smart.
  b. Chris's students are smart.

The two sentences have different presuppositions.

- If Chris is known to have exactly one student, then (39a) and (39b) will mean the same

[8] Furthermore, I think this accounts for Crnič, Chemla & Fox's (2015) observation about disjunction under a universal quantifier without nesting scalar implicatures, with the assumption that the disjuncts are alternatives for a disjunction. But this is left for another day.

thing.

- Otherwise, (39a) is not usable. So (39b) should be the only option, and should not have a plurality inference.

Sauerland (2003) derives the plurality inferences of plural definites as **anti-presuppositions**. I could adopt this analysis cases like (39).

Mayr (2015) observes, however, that the anti-presuppositional account wrongly predicts definite plurals to be felicitous in contexts where the exactly number is not known. His main data is the following:[9]

(40) *Context: It is common belief that Paul either wrote exactly one song or several songs.*
  a. #The song is good.
  b. #The songs are good. (Mayr 2015:p. 211)

Mayr (2015) proposes instead that they should be accounted for by NP-level embedded scalar implicatures, and always have plurality inferences.

But do definite plurals always have plurality inferences?

(41) I haven't met a Japanese philosopher or read his papers.

Speculation: The anomaly of (40) might be due to something about restrictions on when the opinionatedness assumption holds and when it does not?

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[9] His other data involve dynamic binding, and I'm not sure whether they should be understood in the same way, although they are certainly problematic for the simplest version of the anti-presupposition idea.

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