## Relative Atomicity

(Based on joint work with Giorgos Spathas)

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## Two views of atomicity

- ABSOLUTE ATOMICITY (Link 1983, Schwarzshcild 1996, Chierchia 1998a,b, Landman 1989a,b, Winter 2001, among others)
- Atomic and non-atomic entities are distinguished at the model level
- Atomic entities are minimal elements of $\left(D, \leq_{i}\right)$
- RelAtive AtOMicity (Rothstein 2010, Landman 2011, 2016, Sutton \& Filip 2016, Rothstein 2017, Sutton \& Filip 2017)
- Atomicity is only defined relative to nouns


## Goals

- I will propose an 'intensional' theory of Relative Atomicity
- Main empirical argument comes from DP-external subatomic quantifiers
(1) a. The chair is partly yellow.
b. The chairs are partly yellow.
b. The furniture is partly yellow.

ABSOLUTE ATOMICITY

## Absolute atoms

Absolute atoms are often used in analyses of

- Number marking: singular, plural, mass, etc.
- Counting modifiers/quantifiers
- Distributivity

If we give up on Absolute Atomicity, we will have to reanalyse these phenomena
Notation

- Domain: $\left(D, \leq_{i}\right)$ is a join-semilattice
- Absolute atoms: $A:=\min _{\leq_{i}} D=\left\{x \in D \mid \neg \exists y\left[y \leq_{i} x \wedge y \neq x\right]\right\}$
- Complex entities: $C:=D-A$


## Number marking

Analysis with absolute atoms

- Singular count nouns denote sets of absolute atoms
- Plural count nouns denote the closure of their singular counterpart under $\sqcup_{i}$ (= Link's 1983 ネ-operator)
(2) a. $\llbracket \mathbf{c a t} \rrbracket=\{x \in A \mid x$ is a cat $\}$
b. $\llbracket \mathbf{c a t s} \rrbracket=\left\{\bigsqcup_{i} S \mid S \subseteq \llbracket \mathbf{c a t} \rrbracket \wedge S \neq \emptyset\right\}$
- Different views on mass nouns (Chierchia 1998a,b, 2010, Landman 2011, 2016, Link 1983, Rothstein 2010, 2017, Sutton \& Filip 2016, 2017, among others); often extensional


## Counting modifiers/quantifiers

Counting modififers/quantifiers count absolute atoms, and never complex entities

- "I saw three cats" won't be true if I saw $c_{1}, c_{2}$ and $c_{1} \sqcup c_{2}$.

Counting and non-counting quantifiers

- "Most of the suitcases are yellow."
- $\approx$ The number of atomic suitcases that are yellow is greater than the number of atomic suitcases that are not yellow
- $\not \approx$ The total surface area of the suitcases is yellow is greateer thatn the total surface area of the suitcases that is not yellow
- "I drank most of the coffee"


## Distributivity

(3) The children made a snowman.
a. Distributive: Each child made a snowman.
b. Collective: The children made a snowman together

The cover distributivity operator $\Delta$ quantifies over absolute atoms

$$
\llbracket \boldsymbol{\Delta} \rrbracket=\lambda P_{(e, t)} \cdot \lambda x_{e} . \forall y \in A\left[y \leq_{i} x \rightarrow P(x)=1\right]
$$

- Reciprocals
- Cumulative/Co-distributive readings
- Non-atomic distributive quantification


## Sub-atomic quantification

## Sub-atomic phenomena

Certain expressions access parts of absolute atoms (Link 1983, Krifka 1990, Wągiel 2018, 2019)

- Sub-atomic quantifiers
(4) a. Part of the flag is red.
b. The flag is partly red.
- Cumulative/co-distributive readings
(5) a. The flag is red and white.
b. The kids ate my hamburger.
- (Pluralia tantum)


## Two partial orders

To account for sub－atomic phenomena，we need another order，$\left(D, \leq_{i}, \leq_{p}\right)$
－Whenever $x \leq_{i} y, x \leq_{p} y$
－But $x \leq_{p} y$ doesn＇t imply $x \leq_{i} y$

- 【Tim＇s face】 $=f$
- 【Tim＇s nose】 $=n$
－$n \leq_{p} f$ but $n \not \mathbb{K}_{i} f$ ，because $f$ is atomic with respect to $\leq_{i}$


## Individuated domain?

- $\leq_{p}$ is not well-founded given atom-less nouns like reason, space, time, advice, line segment
- Alternatively, Cheirchia 1998a,b, 2010 maintains that all such cases are also built on atomic entities
- If $D$ contains atom-less entities, $\left(D, \leq_{p}\right)$ can be assumed to be a join-semilattice i.e., for each $x, y \in D, x \sqcup_{p} y \in D$, but ( $D, \leq_{i}$ ) won't be, because for two atom-less entities $x$ and $y, x \sqcup_{i} y \notin D$.
- We can then postulate $I$, the maximal subset of $D$ that forms a join semilattice with $\leq_{i}$.

- $A: \leq_{i}$-atoms, $C: \leq_{i}$ complex, $I:$ 'individuals'
- If you are Chierchia, $D=I$


## Atomic and sub-atomic quantifiers

(6) a. Part/*some of the suitcase is yellow.
b. *Part/some of the suitcases are yellow.
c. Part/some of the luggage is yellow.

- Assuming $\llbracket$ suitcases $\rrbracket=\llbracket$ luggage $\rrbracket$
- We could assume that part is morphosyntactically incompatible with plural nouns, and some has mass and plural versions (like much and many)
- $\llbracket$ part (of) $\rrbracket=\lambda x_{e} . \lambda P_{(e, t)} . \exists y \in D\left[y \leq_{p} x \wedge P(x)\right]$
- $\llbracket \mathbf{s o m e}_{\text {mass }} \mathbf{( 0 f )} \rrbracket=\lambda x_{e} \cdot \lambda P_{(e, t)} . \exists y \in D\left[y \leq_{p} x \wedge P(x)\right]$
- $\llbracket$ some $_{\text {plural }}(\mathbf{o f}) \rrbracket=\lambda x_{e} \cdot \lambda P_{(e, t)} . \exists y \in A\left[y \leq_{i} x \wedge P(x)\right]$


## Some more thoughts

- Part in part of the NP is probably a mass noun, with existential force coming from something else
- Some of the NP might be underlyingly Some NP of the NP


## DP external sub-atomic quantifiers

## Partly + singular count

(7) The flag is partly red.

- $\llbracket$ The flag is partly red $\rrbracket=1$ iff $\exists y \in D\left[y \leq_{p} \llbracket\right.$ the flag $\left.\rrbracket \rightarrow \operatorname{red}(y)\right]$
- This suggests: $\llbracket$ partly $\rrbracket=\lambda P_{(e, t)} . \lambda x_{e} . \exists y \in D\left[y \leq_{p} x \rightarrow P(y)\right]$



## Partly + plural count

(8) The flags are partly red.

- 【The flags are partly red】=1 iff $\exists y \in D\left[y \leq_{p} \llbracket\right.$ the flags $\left.\rrbracket \rightarrow \operatorname{red}(y)\right]$
- This should be true in both scenarios, but is only true in the
 second case
- This suggests:

$$
\llbracket \text { partly } \rrbracket=\lambda P_{(e, t)} \cdot \lambda x_{e} \notin C . \exists y \in D\left[y \leq_{p} x \rightarrow P(y)\right]
$$

- $\llbracket$ The flags are $\Delta$ partly red $\rrbracket=1 \mathrm{iff}$
 $\forall z \in A\left[z \leq_{i} \llbracket\right.$ the flags $\left.\rrbracket \rightarrow \exists y \in D\left[y \leq_{p} z \rightarrow \operatorname{red}(z)\right]\right]$


## Partly + mass

(9) a. Their suitcases are partly yellow.
b. Their luggage is partly yellow.

If $\llbracket$ suitcases $\rrbracket=\llbracket$ luggage $\rrbracket$, the two sentences should mean the same thing


## Partly + group noun

(10) a. These letters are partly red.
b. This logo is partly red.
(11) a. These playing cards are partly transparent.
b. This deck of playing cards is partly transparent.
$20 / 45$

## The puzzle of partly

- Partly needs to know if the number property of the head noun of its 'associate'
- If singular count or mass, direct quantification over parts via $\leq_{p}$
- If plural, decompose via $\Delta$, then quantify over parts via $\leq_{p}$
- This is unlikely to be syntactic agreement, given that none of the sentences are unacceptable
- How does partly access the necessary information compositionally?
- To save Absolute Atomicity, we could deny $\llbracket$ suitcases $\rrbracket=\llbracket$ luggage $\rrbracket$
- But I think that'll have unwelcome consequences, at least conceptually

Excursus: More on DP-external quantifiers

## Two positions

(12) a. The flag is partly red.
b. The flags are $\underline{\Delta}$ partly red.
(13) a. The flags are all partly red.
b. The flags are $\Delta \underline{\Delta}$ red.
(14) a. The flag is $\Delta$ red.
b. The flag is entirely red.
$\Delta$ : Homogeneous distributivity operator (details omitted here)

## Two positions

- Atomic quantifiers: $C \rightarrow A$
- Sub-atomic quantifiers: $A \rightarrow D \backslash C$

(cf. Aldridge \& Neeleman 2015)


## Lexical restrictions

|  | Atomic | Sub-atomic |
| :---: | :---: | :---: |
| partly | $\times$ | $\nabla$ |
| all | $\nabla$ | $\nabla$ |
| mostly | $\nabla$ | $\nabla$ |
| half | $\nabla$ | $\nabla$ |
| mostly | $\nabla$ | $\times$ |
| $\Delta$ | $\nabla$ | $\nabla$ |

## More uses and positions

- Quality readings and a position below sub-atomic (Aldridge \& Neeleman 2015)
(15) The door is (entirely) half transparent
- Occasion readings
(16) I mostly danced.


# 'Intensional' theory of ReLATIVE ATOMICITY 

## New model assumptions

- Only $\leq_{p}$, the intuitive part-whole relation. We'll just write $\leq$ from now on
- No absolute atoms in the model (not necessary, but intuitive)
- $(D, \leq)$ is a join semi-lattice
- A bit of history:
- Link 1983 postulated two domains $\left(\left(E, \leq_{i}\right)\right.$ and $(D, \leq)$ ), two partial orders
- One domain eliminated later
- One partial order eliminated now


## Illustration of the proposal

If there are no absolute atoms, how do DP-external quantifiers know what to quantify over?
My answer: Count nouns have the intensional effect of making $\llbracket \cdot \rrbracket$ 'blind' to certain parts of the model. Such a restricted domain may have minimal elements, or relative atoms


## Illustration of the proposal (cont.)

- Partly presupposes that the current domain doesn't have relative atoms, so cannot be in the direct scope of a plural count noun
- Operators like $\Delta$ reset the current domain



## Illustration of the proposal (cont.)

- Intensional effects
- A plural noun's intensional effect is to temporarily remove the 'sub-atomic' parts of its extensions, introducing relative atoms
- Singular count nouns keep all the sub-parts, so no relative atoms
- Mass nouns bring the domain back to ( $D, \leq$ ), so no relative atoms
- $\therefore$ partly can directly operate on the extensions of singular count and mass nouns
- A mass and a plural may be co-extensional, but differ intensionally


## $\sim$ Mass/count is partly INTENSIONAL

## More details

Caveat: I will be formally sloppy in certain compositional details. See the Appendix of the handout version of this talk for more formally precise details

## Reanalysing number morphology

Extensions of number morphology don't require reference to absolute atoms

- Singular nouns refer to sets of things describable by them, but they have non-trivial parts, e.g. $\llbracket \mathbf{c a t} \rrbracket=\{x \in D \mid x$ is a cat $\}$
- Plural counterparts denote any combinations of them, e.g.

$$
\llbracket \mathbf{c a t s} \rrbracket=\{\sqcup S \mid S \subseteq \llbracket \mathbf{c a t} \rrbracket \wedge S \neq \emptyset\}
$$



## Intension of number morphology

- Key assumption: number morphology, including mass/count, has intensional effects
- New aspect of intensionality: restricted domains
- Normally, evaluation relative to the domain of the entire model, $(D, \leq)$
- Count nouns temporarily introduce restricted domains in semantic derivation
- Such restricted domains may have minimal elements = relative atoms
- Notation: $S^{\leq}:=\left(S, \leq \upharpoonright_{S}\right)$
- $S \subseteq D$
- $\leq r_{s}:=\{(x, y) \mid x \leq y \wedge x, y \in S\}$


## Ex: Singular count nouns

- The interpretation of cat is insensitive to the domain parameter
- But it introduces the set of all individual cats and their parts as the new restricted domain, ${ }^{\downarrow} \mathrm{CAT}^{\leq}$, where for any $S \subseteq D,{ }^{\downarrow} S=\{x \in D \mid \exists y \in S[x \leq y]\}$ )
The VP is interpreted relative to this new restricted domain, ${ }^{\downarrow} \mathrm{CAT}^{\leq}$.

$$
\llbracket \text { the cat VP } \rrbracket^{S \leq} \Leftrightarrow \llbracket \mathbf{V P} \rrbracket^{\downarrow \mathrm{CAT}^{\leq}}\left(\llbracket \text { the cat } \rrbracket^{\downarrow \mathrm{CAT}^{\leq}}\right)
$$

- Partly in the VP will be happy, because there are no atoms in this case
- $\ulcorner$ partly $\urcorner \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{S^{\leq}}\right)$iff $\min \left(S^{\leq}\right)=\emptyset$
- If so, $\llbracket$ partly $\rrbracket^{S \leq}=\lambda P_{(e, t)}$. $\lambda x_{e}: x \in S . \exists y \in D[y \leq x \wedge P(y)]$


## Ex: Plural and mass nouns

- Cats introduces a restricted domain with relative atoms, ${ }^{\uparrow} \mathrm{CAT}{ }^{\leq}$, where for any $S \subseteq D$, $\left.{ }^{\uparrow} S=\{x \in D \mid \exists y \in S[y \leq x]\}\right)$

$$
\llbracket \text { the cats } \mathbf{V P} \rrbracket^{S \leq} \Leftrightarrow \llbracket \mathbf{V P} \rrbracket^{\uparrow} \mathrm{CAT}^{\leq}\left(\llbracket \text { the cats } \rrbracket^{\uparrow \mathrm{CAT}^{\leq}}\right)
$$

- Mass nouns use the entire domain

$$
\llbracket \text { the luggage } \mathbf{V P} \rrbracket^{S \leq} \Leftrightarrow \llbracket \mathbf{V P} \rrbracket^{(D, \leq)}\left(\llbracket \text { the luggage } \rrbracket^{(D, \leq)}\right)
$$

- Each requires the current domain to contain relative atoms (and it resets the intensional parameter to $(D, \leq))$
- $\ulcorner$ each $\urcorner \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{S \leq}\right)$ iff $\min \left(S^{\leq}\right) \neq \emptyset$
- If so, $\llbracket \mathbf{e a c h} \rrbracket^{S \leq}=\lambda P_{(e, t)} . \lambda x_{e}: x \in S . \forall y \in \min \left(S^{\leq}\right)[y \leq x \wedge P(y)]$


## DP-internal matters

Assumption: Nouns take scope at the DP edge and intensionally affect both NP and VP. Extensionally they simply reconstruct (cf. Charlow 2014, 2020)


## Reanalysing counting modifiers/quantifiers

Counting modifiers/quantifiers like numerals require relative atoms

- $\ulcorner$ three $\urcorner \in \operatorname{dom}\left(\llbracket \cdot \rrbracket^{S \leq}\right)$ iff $\min \left(S^{\leq}\right) \neq \emptyset$
- If so,

$$
\llbracket \text { three } \rrbracket^{S \leq}=\lambda P_{(e, t)} \cdot \lambda Q_{(e, t) \cdot} \cdot \exists X \subseteq \min \left(S^{\leq}\right)[|X|=3 \wedge P(\bigsqcup X) \wedge Q(\bigsqcup X)]
$$

This accounts for why it's bad to say "three furniture", "six logo"

- In other accounts, this restriction is often analyzed as morphosyntactic in nature.
- In the present account, it has to do with the presence of relative atoms
- Cf. three letters vs. three logos

Complication: 'How many triangles are there?', von Neumann universe


Conclusions

## ReLative Atomicity

- Without absolute atoms, we reanalysed
- Number morphology, mass/count
- Counting modifier/quantifier
- Distributivity
- Empirical motivation: Semantic restrictions on DP-external quantifiers like partly
- Key idea: Plural count nouns introduce relative atoms; some expressions need them
- Consequence: One domain and one partial order $(D, \leq)$
- (Skipped: Comparisons with other similar theories: Rothstein 2010, Landman 2011, 2016, Sutton \& Filip 2016, Rothstein 2017, Sutton \& Filip 2017)


## Open issue 1: Classifiers

- Assumption: Classifier languages (e.g. Japanese) only have mass nouns.
- Then nouns never introduce relative atoms.
- Idea (to be worked out): Classifiers introduce relative atoms, indicating what to count

```
(17) pan san-ko / san-hon / san-mai / san-kin /
bread three-cLround / three-CLlong / three-CLflat / three-cLloaf /
san-hukuro
three-clbag
```


## Open issue 2: Cross-sentential relative atoms?

(18) We have cows and sheep here. They live in different barns.

Cf. Schwarzschild 1995
Relatedly, how to account for relative atoms of conjunction and disjunction

## Open issue 3: Group nouns and intensionality

- In the present system, all nouns can be seen as group nouns
- Co-extensional nouns may differ in intensional aspects (this logo, these letters)
- A cat is more than the collection of its parts; so is a committee
- Plural to group shift might be reponsible for the not so clear judgments in some cases
(19) a. The homework was partly interesting.
b. The homework assignemnts were partly interesting.

Cf. Erbach \& Sudo 2023 on potatoes

## Nāgasena and Menander I (Mévavסoo̧) in Milinda Panha (2nd BCE)

'How then did you come, on foot, or in a chariot?'
'I did not come, Sir, on foot. I came in a carriage.'
'Then if you came, Sire, in a carriage, explain to me what that is. Is it the pole that is the chariot?'
'I did not say that.'
'Is it the axle that is the chariot?'
'Certainly not.'
'Is it the wheels, or the framework, or the ropes, or the yoke, or the spokes of the wheels, or the goad, that are the chariot?'

And to all these he still answered no.
'Then is it all these parts of it that are the chariot?'

## 'No, Sir.'

'But is there anything outside them that is the chariot?'
And still he answered no.
'Then thus, ask as I may, I can discover no chariot. [...]'
And Milinda the king replied to Nâgasena, and said: 'I have spoken no untruth, reverend Sir. It is on account of its having all these things-the pole, and the axle, the wheels, and the framework, the ropes, the yoke, the spokes, and the goad-that it comes under the generally understood term, the designation in common use, of "chariot."'
'Very good! Your Majesty has rightly grasped the meaning of "chariot."'
(Translation by Davids 1890: pp.43-44)

