Relative Atomicity*

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https://www.ucl.ac.uk/~ucjtudo/pdf/RelativeAtomicity.pdf

23/09/2023
SinFonIJA 16 at Masaryk University

*This is part of an ongoing (but currently dormant) project with Giorgos Spathas. Prior to our collaboration, I discussed the main puzzle of DP-external quantifiers with Laura Aldridge around 2015, while she was a PhD student at UCL. She subsequently wrote a manuscript with Ad Neeleman, mainly about their syntax (Aldridge & Neeleman 2015). I have also had very stimulative and helpful discussion about related matters with Kurt Erbach and Marcin Wągiel.

1 Introduction

1.1 Background

The standard model-theoretic approach to plurality is **Absolute Atomicity**, which defines atomicity once and for all at the model level (Link 1983, Schwarzschild 1996, Chierchia 1998a,b, Landman 1989a,b, 2000, Winter 2001, Chierchia 2010, among many others).

- The domain D of entities is partially ordered by a part-whole relation $\leqslant_i$.
- The minimal elements of $(D, \leqslant_i)$ are called atomic entities. Everything else is a complex entity (I reserve 'singular' vs. 'plural' for morphosyntax).

(1) $A \coloneqq \min_{\leqslant_i} D = \{\, x \in D \mid \neg\exists y[y \leqslant_i x \wedge y \neq x] \,\}$

(2) a. $x \in D$ is atomic iff $x \in A$.
  b. $x \in D$ is complex iff x is not atomic.

(3) $[\![\textbf{cat}]\!] = \{\, x \in A \mid x \text{ is a cat} \,\}$

Relative Atomicity is an alternative approach, where atomicity is defined relative to nouns, and what counts as an atom changes in the course of semantic composition (Rothstein 2010, Landman 2011, 2016, Sutton & Filip 2016, Rothstein 2017, Sutton & Filip 2017).

1.2 Goals

1. Point out the puzzle of **DP-external sub-atomic quantification**
2. Develop a theory of **Relative Atomicity**

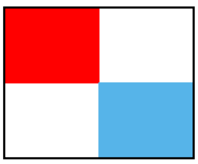
1.3 The puzzle in a nutshell

- Sub-atomic quantifiers quantify over parts of their associate. (Wągiel 2018, 2019):

(4) a. **Part of** this flag is red. DP-internal
  b. This flag is **partly** red. DP-external

- DP-external quantifiers are particularly interesting for compositional reasons.
- **Observation**: Singular and plural associates behave differently.
  - Singular count: *Partly* quantifies over parts of the denotation of the subject.

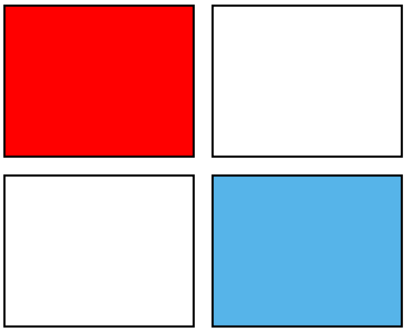
(5) This flag is partly red.



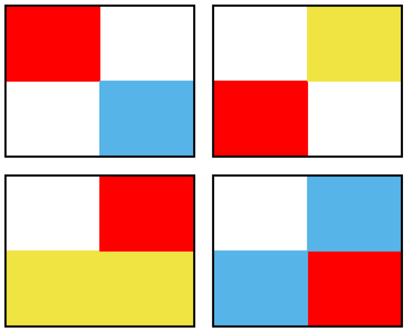
– Plural count: *Partly* cannot quantify directly over parts of the denotation of the subject *the flags*.

(6) These flags are partly red.

a.



b.



(7) These flags are all partly red.

- **Analysis**: *Partly* cannot directly quantify over parts of the complex entity that a plural DP denotes. But once some other operator (e.g. the phonologically null distributivity operator Δ, *all*) decomposes it into atoms, it can quantify over parts of these atoms.

(8) These flags are $\underbrace{\Delta/\text{all}}_{\text{for each flag}}$ $\overbrace{\text{partly}}^{\text{for some sub-atomic part of the flag}}$ red.

- **Puzzle**: One the same complex entity can be referred to in different ways, and how you refer to it matters for the interpretation of *partly*.

(9) a. The Google logo is partly red.
b. These six letters are partly red.

(10) a. This deck of playing cards is partly transparent.
b. These play cards are partly transparent.

(11) a. The furniture in this room is partly wooden.
b. The table, chairs and shelf in this room are partly wooden.

The singular/mass versions of these sentences show that *partly* actually can directly quantify over parts of complex entities, as long as they are not denoted by plural DPs.

But how does *partly* 'know' how the (complex) entity is referred to??

1.4 An intensional theory of Relative Atomicity

- I will develop an **'intensional' theory of** Relative Atomicity.
  – Extension (reference) = the entity
  – Intension (sense) = the way the entity is identified
- One and the same object can be atomic or not depending on how we describe it.

- The Google logo can be seen as *one logo* (atomic), or *six letters* (complex).
- Similarly, *this deck of cards* (atomic) vs. *these cards* (complex).
- How about: *the table, chairs, and shelf* (complex) vs. *the furniture* (complex?).

• **Core idea**: The model contains no atoms. Count nouns are intensional operators that temporarily restrict our attention to a subset of the model, which may contain atoms.

- Plural count nouns introduce **relative atoms** and (complex) entities they make up.
- Singular count nouns are about entities (usually) with parts, so these entities are not atoms.

Plural count (*flags*) Singular count (*flag*)

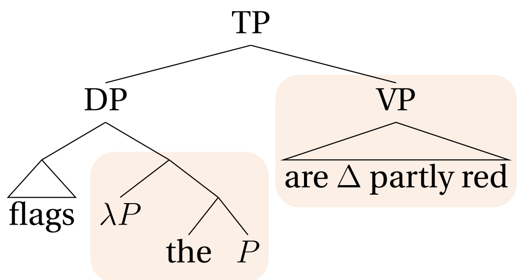
- Mass nouns do not restrict the intensional parameter.

> The mass-count distinction is (partly) intensional.

• **Compositional puzzle**: The noun in the subject DP and the operator *partly* are structurally far from each other.

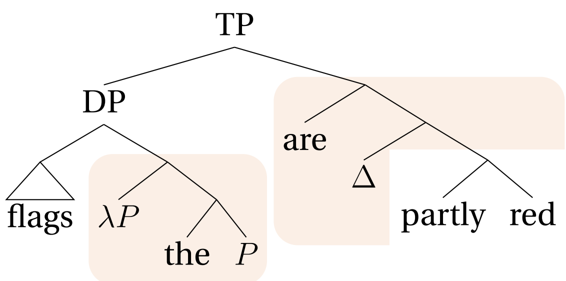
(12) a. The Google logo is partly red.
b. These six letters are partly red.

• **Proposal**: A count noun's intension may affect DP-external operators, as it takes scope at the edge of its local DP and extends its intensional effect to the VP (cf. Charlow 2014, 2020).



The extensional aspect of the noun's meaning reconstructs.

• Atomic quantifiers like Δ (must) quantify over **relative atoms** but after that, they reset the intensional parameter to the entire model, so subsequent quantifiers can quantify over parts of these relative atoms.



- We need **relative atoms** to account for the puzzle. Then there will be no need for **absolute atoms**.

- I won't have time for detailed comparisons with previous theories of relative atomicity (Rothstein 2010, Landman 2011, 2016, Sutton & Filip 2016, Rothstein 2017, Sutton & Filip 2017), but here are some brief remarks.

  - Some specifically target the context sensitivity of atomicity with certain count nouns (e.g., *wall*) (Rothstein 2010, 2017, Sutton & Filip 2016).
  - None discusses the issue of DP-external sub-atomic quantifiers (save brief discussion on distributivity in Landman 2016).

2 Absolute Atomicity

2.1 Atomic entities vs. complex entities

- Each model has a non-null domain D of entities.

- D is ordered by a part-whole relation $\leqslant_i$.

- $\leqslant_i$ induces the join operation $\sqcup_i$ via the absorption law: $x \leqslant_i y$ iff $x \sqcup_i y = y$.

- $(D, \leqslant_i)$ is a join semilattice, i.e., for each $x, y \in D$, the least upper bound of x and y with respect to $\leqslant_i$, namely $x \sqcup_i y$, is a member of D (or equivalently, $\sqcup_i$ is associative, commutative, and idempotent).

- The minimal elements of $(D, \leqslant_i)$ are said to be atomic.

  (13) $A \coloneqq \min_{\leqslant_i} D = \{\, x \in D \mid \neg\exists y[y \leqslant_i x \wedge y \neq_i x] \,\}$

  (14) a. $x \in D$ is atomic iff $x \in A$.
  b. $x \in D$ is complex iff x is not atomic.

  I will call the members of A '**absolute atoms**'.

- Based on these assumptions, analyses have been developed for:

  - Number marking and mass/count
  - Counting modifiers and quantifiers
  - Distributivity

  The standard analyses of these phenomena in **Absolute Atomicity** crucially refer to **absolute atoms**.

> Giving up on **Absolute Atomicity** will force us to reconsider these phenomena.

2.2 Number marking and mass/count

- The extension of a singular count noun is a set of **absolute atoms**.

(15) $[\![\textbf{cat}]\!] = \{\, x \in A \mid x \text{ is a cat} \,\}$

- The extension of a plural count noun is the closure of the singular counterpart under $\sqcup_i$ (= the $*$-operator of Link 1983).

(16) $[\![\textbf{cats}]\!] = \{\, \bigsqcup_i S \mid S \subseteq [\![\textbf{cat}]\!] \wedge S \neq \varnothing \,\}$

Note: (16) includes atomic entities, as well as non-atomic entities, i.e., morphosyntactically plural nouns are not semantically plural, but number-neutral (see Sauerland 2003, Sauerland, Anderssen & Yatsushiro 2005, Spector 2007, Zweig 2009, Sudo 2023 for arguments).

- For mass nouns, there are many different views (Chierchia 1998a,b, 2010, Landman 2011, 2016, Link 1983, Rothstein 2010, 2017, Sutton & Filip 2016, 2017, among many, many others).
  - Canonical mass nouns like *mud* and *space* intuitively describe things that do not have atoms.
  - But Chierchia 1998a,b, 2010 assumes that every entity in the model is made up of atoms, so even such mass nouns denote sets of model-theoretic entities that are made up of atoms.
  - Others postulate entities that are not made of atoms (which requires a non-standard theory of sets). We'll come back to this point.

> Most (though not all) accounts of number marking and mass/count are entirely extensional. I will claim that intensionality plays an important role.

2.3 Counting modifiers and quantifiers

- Counting modifiers count the number of **absolute atoms** (and never of complex entities).

(17) I saw **three** students.

This sentence is *not* true when I saw two atomic entities s_1 and s_2 and one complex entity $s_1 \sqcup_i s_2$. The sentence entails there are three **absolute atoms**, each of which is a student I saw.

- Some counting quantifiers quantify over **absolute atoms**.

(18) **Each** essay was given written feedback.

(19) Most of the doors are wooden.

a. $\approx$ The number of atomic doors that is wooden is (far) greater than the number of atomic doors that is not wooden.

b. $\not\approx$ The total area/volume of the wooden parts of the doors is (far) greater than the total area/volume of the non-wooden parts of the doors.

- Some do not quantify over absolute atoms.

(20) a. I drank a lot of coffee.
b. I drank most of the coffee.

2.4 Distributivity

- Distributivity is characterised in terms of atomicity.

(21) The children made a snowman.
a. Distributive reading: Each child made a snowman.
b. Collective reading: The children collaborated and made a snowman together.

This ambiguity is standardly analysed with an implicit distributivity operator Δ, which universally quantifies over **absolute atoms** that constitute the complex entity that the subject denotes.

(22) $[\![$**The children** Δ **made a snowman**$]\!] = 1$ iff
for each $x \in A$ such that $x \leqslant_i [\![$**the children**$]\!]$, $[\![$**made a snowman**$]\!](x) = 1$.

- More complex cases

(23) The three professors hate one another.

(24) The five dogs chased the three cats.

- In some cases Δ quantifies over complex entities..

(25) a. The linguists and philosophers in the department hate each other.
b. The first-year linguists and philosophers took different courses.

We will reanalyse counting modifiers/quantifiers and distributivity in terms of **relative atoms**.

3 Sub-atomicity in Absolute Atomicity

Certain expressions and constructions access sub-atomic parts of **absolute atoms** (Link 1983, Krifka 1990, Wągiel 2018, 2019).

⋆ **Sub-atomic quantifiers** quantify over parts of their associate.

(26) DP-internal

a. **Part of** the flag is red.
b. **Half** the door is transparent.
c. **Most of** the book is in French.

(27) DP-external

a. The table is **entirely** wooden.
b. The building is **partly** visible.
c. The flag is **half** red.
d. The door is **mostly** white.

- Cumulativity/co-distributivity

(28) a. The flag is red and white.
b. The kids ate my hamburger.

- (Pluralia tantum?)

Absolute Atomicity needs to be augmented with extra machinery to account for sub-atomic phenomena.

In addition to some conceptual issues (to be skipped), we will discuss the puzzle of **DP-external sub-atomic quantifiers**.

3.1 Introducing sub-atomic entities

$\leqslant_i$ is not meant to account for the intuitive part-whole relation.

- By definition, *the dog* refers to , say d.
- Similarly, *the dog's tail* refers to an atomic entity, say t.
- But we cannot have $t \leqslant_i d$, because d is atomic by assumption.

To account for the part-whole relation between the dog d and its tail t, we need a different partial order, $\leqslant_p$.

- The corresponding join operation is $\sqcup_p$.
- $(D, \leqslant_p)$ is assumed to be a join semilattice, i.e., for any two entities, x, y in D, their least upper bound with respect to $\leqslant_p$ (namely $x \sqcup_p y$) is also a member of D.
- Whenever $x \leqslant_i y$, we should have $x \leqslant_p y$.
- But $x \leqslant_p y$ doesn't imply $x \leqslant_i y$. E.g., a complex entity with respect to $\leqslant_i$ may be sub-atomic part of an atomic entity.
  - Suppose $[\![$**my right hand**$]\!] = r$.
  - Suppose also $[\![$**the five fingers of my right hand**$]\!] = f_1 \sqcup f_2 \sqcup f_3 \sqcup f_4 \sqcup f_5$.
  - $f_1 \sqcup f_2 \sqcup f_3 \sqcup f_4 \sqcup f_5 <_p r$ but $f_1 \sqcup f_2 \sqcup f_3 \sqcup f_4 \sqcup f_5 \nleqslant_i r$

(29) My right hand is partly frostbitten.

With $\leqslant_p$ we can state our analysis of sub-atomic quantifiers. Roughly (ignoring presupposition and scalar implicature):

(30) a. By assumption, $[\![$**the flag**$]\!] \in A$.
b. $[\![$**The flag is partly red**$]\!] = 1$ iff for some $x \leqslant_p [\![$**the flag**$]\!]$, x is red.

3.2 Individuated domain?

Given sub-atomic phenomena, any theory of **Absolute Atomicity** needs two partial orders, $\leqslant_i$ and $\leqslant_p$, on D.

- $\leqslant_i$ is used for for number marking, atomic (counting) quantifiers, distributivity
- $\leqslant_p$ is used for sub-atomic quantifiers.

There's a technical point to be mentioned here.

- $\leqslant_p$ needs to be defined for things that might not be made up of minimal elements, e.g. *space, time, line segment, reason, explanation, advice,* etc. They intuitively have parts, and natural language can quantify over them.

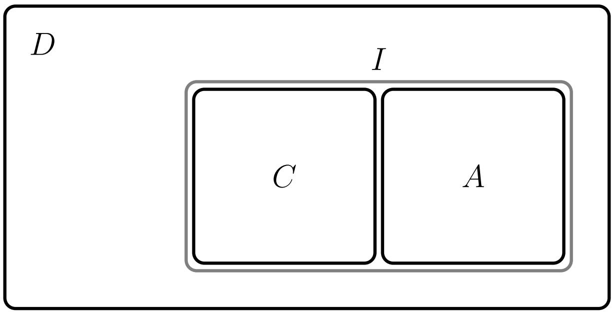
  (31) a. **Part of** the reason is financial.
  b. The remaining time will **mostly** spent on philosophical matters.

- As mentioned, Chierchia 1998a,b, 2010 nonetheless maintains that all such cases are also built on atomic entities, but if the purpose of formal semantics is to account for the way we reason and draw inferences in natural language, I doubt that we reason about atoms of time and reason.
- If the model contains atom-less entities (e.g., members of $[\![$**time**$]\!]$, $[\![$**space**$]\!]$, $[\![$**hole**$]\!]$), $\leqslant_i$ will have to be defined only for a sub-part of D.
  - $(D, \leqslant_p)$ is a join semilattice, i.e., for each $x, y \in D$, $x \sqcup_p y \in D$.
  - But it cannot be the case that $(D, \leqslant_i)$ is a join semilattice, because it's not true that for any $x, y \in [\![$**time**$]\!]$, we can find a complex entity z such that $x \leqslant_i z$ and $y \leqslant_i z$ (as $\leqslant_i$ is not defined for x and y).
- But we still want to say that any two entities x and y that are absolute atoms themselves or made up of absolute atomes do form a complex entity $x \sqcup_i y$.

So we are forced to say that some subpart I (for 'individuated') of D forms a join semilattice with respect to $\leqslant_i$, but not the entire domain D.

- $A \subseteq I$ is the set of atomic entities with respect to $\leqslant_i$, which are members of the denotations of singular count nouns.

- The non-atomic part of I ($I \backslash A$) is denoted by C (for 'complex entities').
- If you are Chierchia, $D = I$.
- $(I, \leqslant_i)$ and $(D, \leqslant_p)$ are semilattices. $(D, \leqslant_i)$ is not, unless you are Chierchia.



- $[\![$**cat**$]\!] \subseteq A$
- $[\![$**the cat**$]\!] \in A$ (if the model contains exactly one cat)
- $[\![$**cats**$]\!] \subseteq I$
- $[\![$**the cats**$]\!] \in C$ (if the model contains more than one cat)
- The extension of a mass noun might include things inside or outside I (depending on your theory).
- $[\![$**the dog's tail**$]\!] \leqslant_p [\![$**the dog**$]\!]$ but $[\![$**the dog's tail**$]\!] \nleqslant_i [\![$**the dog**$]\!]$
- $[\![$**my hands**$]\!] \leqslant_p [\![$**my body**$]\!]$ but $[\![$**my hands**$]\!] \nleqslant_i [\![$**my body**$]\!]$.
- $[\![$**my thumbs**$]\!] \leqslant_p [\![$**my hands**$]\!]$ but $[\![$**my thumbs**$]\!] \nleqslant_i [\![$**my hands**$]\!]$
- Whenever $x \leqslant_i y$, $x \leqslant_p y$.

.... The rest of this section will be skipped ....

3.3 A conceptual concern

Consider hybrid nouns:

(32) a. There is a lot of reason to be happy.
b. There are a lot of reasons to be happy.

According to the above (non-Chierchian) view, you have to be referring to different things by *reason* and *reasons.*

- By assumption, $[\![$**reasons**$]\!] \subseteq I$, since $[\![\textbf{reason}_{\textbf{sg.count}}]\!] \subseteq A$.
- Intuitively, $[\![\textbf{reason}_{\textbf{mass}}]\!] \subseteq D \backslash I$.
- So $[\![\textbf{reason}_{\textbf{mass}}]\!] \neq [\![\textbf{reason}_{\textbf{sg.count}}]\!]$.

But we want to capture the fact that the sentences in (32) are (near-)synonymous.

This is only a conceptual issue. One could maintain:

1. *Reason*$_{mass}$ is an 'object mass noun' and refers to a subset of I, the same set as *reason*$_{sg.count}$; or

2. *Reasons* is a 'fake plural noun' and refers to a subset of $D\backslash I$, the same set as *reason*$_{mass}$.[1]

Chierchia's atomic theory is an extreme version of 1., where every noun denotes a subset of I.

3.4 DP-internal sub-atomic quantifiers

With $\leqslant_p$, we can analyse DP-internal sub-atomic quantifiers as follows.

(33) $[\![$**Part of the door is wooden**$]\!] = 1$ iff for some $x \leqslant_p [\![$**the door**$]\!]$, $[\![$**wooden**$]\!](x) = 1$

One might think that we could give the following denotation for $[\![$**part**$]\!]$:

(34) $[\![$**part (of)**$]\!] = \lambda x_e.\lambda P_{(e,t)}.$ for some $y \leqslant_p x$, $P(y) = 1$

But this fails to account for the fact that *Part of the NP* is incompatible with plural count nouns.

(35) a. Part of the door is wooden.
b. Part of the money will be used to buy books.
c. *Part of the doors are wooden.

Relatedly, *some of the NP* is only compatible with mass nouns and plural count nouns.

(36) a. *Some of the door is wooden.
b. Some of the money will be used to buy books.
c. Some of the doors are wooden.

Neither of the following accounts for this restriction.

(37) a. $[\![$**some (of)**$]\!] = \lambda x_e.\lambda P_{(e,t)}.$ for some $y \leqslant_i x$, $P(y) = 1$
b. $[\![$**some (of)**$]\!] = \lambda x_e.\lambda P_{(e,t)}.$ for some $y \leqslant_p x$, $P(y) = 1$

3.4.1 First attempt

One way to account for these restrictions in semantic terms:

[1] In a recent talk with Kurt Erbach, we argued that *potatoes* is a fake plural noun, as it can refer to any forms of potatoes, included uncountable instances, unlike *apples*.

(38) a. $[\![\textbf{part (of)}]\!] = \lambda x_e\colon\ x \notin C.\ \lambda f_{(e,t)}.\ \exists y \in D[y \leqslant_p x \wedge f(x)]$
b. $[\![\textbf{some (of)}]\!] = \lambda x_e\colon\ x \notin A.\ \lambda f_{(e,t)}.\ \exists y \in D[y \leqslant_p x \wedge f(x)]$

But (38b) derives the wrong truth-conditions for (36c). It should be false when small portions of some of the doors are wooden, and none are wholly wooden. It's only true if each of the doors is (almost wholly) wooden (see the discussion on non-maximality below).

Let us postulate two versions of *some*:

(39) a. $[\![\textbf{some}_{\textbf{mass}}\ \textbf{(of)}]\!] = \lambda x_e\colon\ x \notin I.\ \lambda f_{(e,t)}.\ \exists y \in D[y \leqslant_p x \wedge f(x)]$
b. $[\![\textbf{some}_{\textbf{plural}}\ \textbf{(of)}]\!] = \lambda x_e\colon\ x \in C.\ \lambda f_{(e,t)}.\ \exists y \in A[y \leqslant_i x \wedge f(x)]$

Crucially, (39b) quantifies over atomic entities.

But this analysis has potential issues.

- Object mass nouns denote members of I. Then, the mass version of *some* needs to be compatible with elements of I.

  (40) a. Some of my suitcases are missing.
  b. Some of my luggage is missing.

  (41) $[\![\textbf{some (of)}]\!] = \lambda x_e.\ \lambda f_{(e,t)}.\ \exists y \in D[y \leqslant_p x \wedge f(x)]$

  With this version of *some*, we derive the wrong truth-conditions for (36c).

- When $[\![\textbf{my suitcases}]\!] = [\![\textbf{my luggage}]\!]$, how do we account for the contrast in (42).

  (42) a. *Part of my suitcases is/are missing.
  b. Part of my luggage is missing.

3.4.2 Second attempt

Let us assume that the selectional restrictions of *part of* and *some of* are morphosyntactic in nature (similarly to *many* vs. *much*), and the denotations are as follows.

(43) a. $[\![\textbf{part (of)}]\!] = \lambda x_e.\lambda f_{(e,t)}.\ \exists y \in D[y \leqslant_p x \wedge f(x)]$
b. $[\![\textbf{some}_{\textbf{mass}}\ \textbf{(of)}]\!] = \lambda x_e.\lambda f_{(e,t)}.\ \exists y \in D[y \leqslant_p x \wedge f(x)]$
c. $[\![\textbf{some}_{\textbf{plural}}\ \textbf{(of)}]\!] = \lambda x_e.\lambda f_{(e,t)}.\ \exists y \in A[y \leqslant_i x \wedge f(x)]$

Assume:

(44) a. $[\![\textbf{my baggage}]\!] = [\![\textbf{my suitcases}]\!] \in C$
b. $[\![\textbf{the homework}]\!] = [\![\textbf{the homework assignments}]\!] \in C$

We account for the entailment from (a) to (b) in (45) and (46).

(45) a. Some of my suitcases have gone missing.

b. Some of my luggage has gone missing.

(46) a. You will need lambda calculus for some of the homework assignments for next week.
b. You will need lambda calculus for part of the homework for next week.

3.4.3 Some more thoughts

- *Part* in *part of the NP* is potentially simply a mass noun.
- *Some of the NP* might be derived from *some NP of the NP*. This could account for the incompatibility with singular count nouns (together with an account of the partitivity entailment).

4 The puzzle of DP-external sub-atomic quantifiers

I will characterise the puzzle in terms of the above theory of **Absolute Atomicity** with two partial orders, $\leqslant_i$ and $\leqslant_p$.

4.1 Denotations

Using $\leqslant_p$, we can analyse DP-external sub-atomic quantifiers as follows.

(47) $[\![\textbf{The flag is partly red}]\!] = 1$ iff for some $x \leqslant_p [\![\textbf{the flag}]\!]$, $[\![\textbf{red}]\!](x) = 1$.

(48) $[\![\textbf{partly}]\!] = \lambda P_{(e,t)}.\lambda x_e.$ for some $y \leqslant_p x$, $P(y) = 1$

Mostly will refers to a non-counting measurement.

(49) $[\![\textbf{The flag is mostly red}]\!] = 1$ iff
$\mu(\sup_{\leqslant_p}\{\, x \leqslant_p [\![\textbf{the flag}]\!] \mid [\![\textbf{red}]\!](x) = 1 \,\}) \gg \mu(\sup_{\leqslant_p}\{\, x \leqslant_p [\![\textbf{the flag}]\!] \mid [\![\textbf{red}]\!](x) = 0 \,\})$

where μ maps any entity to its area/volume.

These will work for mass associates:

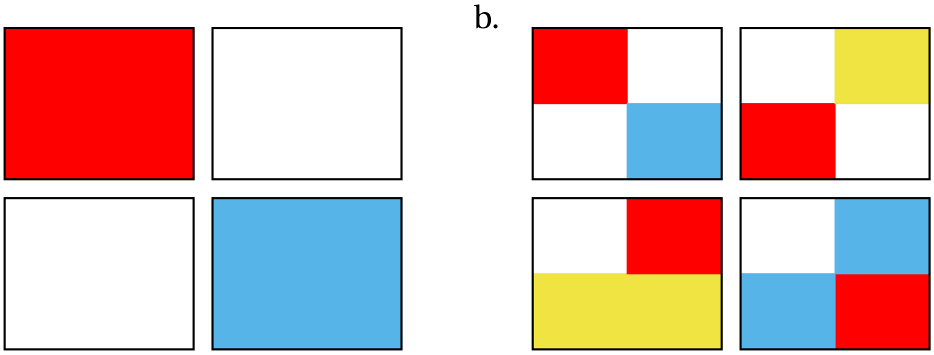
(50) a. The money will be partly spent on books.
b. The bread is mostly gone.

4.2 The puzzle of plural associates

The above analysis overgenerates for plural count associates.

(51) These flags are partly red.

a. b.



Why is it true with respect to (51b) but not with respect to (51a)???

(Near-)minimal pairs:

(52) a. The Google logo is partly red.
b. These six letters are partly red.

(53) a. This deck of playing cards is partly wet.
b. These play cards are partly wet.

(54) a. The furniture in this room is partly wooden.
b. The table, chairs and shelve in this room are partly wooden.

> *Partly* needs to know how its associate is being referred to!

This is the main puzzle, but to understand why these sentences with plural associates can mean what they can mean, we need to touch on homogeneity.

4.3 Homogeneity and Δ

Predicates like *red* and *wooden* are so-called 'summative predicates', which are about properties of parts of entities.

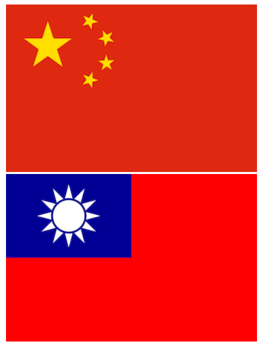
(55) a. The flag is (partly) red.
b. The door is (partly) wooden.

Non-summative ('integrative') predicates.

(56) a. The professor is (#partly) unhappy. (vs. The professors are unhappy)
b. James is (#partly) taller than Katie.
c. The soldiers surrounded the fortress.

Summative predicates without quantifiers give rise to **homogeneous readings** (see Križ 2019 for an overview).

- Homogeneous readings give rise to truth-value gaps: they are like universal readings in the positive and like existential readings in the negative.



(57) a. The flag is (not) red.
b. The table is (not) wooden.
c. The building is (not) visible.

(58) a. The professors are (not) unhappy.
b. The applications were (not) successful.

- Homogeneous readings easily allow for exceptions (**non-maximality**), e.g., *The flag is red* is often judged to be true for the Chinese flag, and even for the Taiwanese flag in some contexts. Compare:

(59) The flag is completely/entirely red.

Following the literature on homogeneity, we postulate a phonologically null distributivity operator Δ.

- If we followed Križ 2015, we'd give it a trivalent denotation along the following lines (see Bar-Lev 2021, Križ & Spector 2021, Paillé 2022 for other theories of homogeneity).

(60) $$[\![\textbf{Subj}\ \Delta\ \textbf{Pred}]\!] = \begin{cases} 1 & \text{if for every 'part' } x \text{ of } [\![\textbf{Subj}]\!],\ [\![\textbf{Pred}]\!](x) = 1 \\ 0 & \text{if for every 'part' } x \text{ of } [\![\textbf{Subj}]\!],\ [\![\textbf{Pred}]\!](x) = 0 \\ \# & \text{otherwise} \end{cases}$$

Non-maximality arises because we sometimes treat some cases of $\#$ as practically true or practically false.

- One might want to postulate different versions of Δ for different notions of 'part' (atomic vs. sub-atomic/non-atomic). See below.
- Δ can be seen as another DP-external quantifier. The only difference from overt universal quantifiers is that it triggers homogeneity. In the Križ-style analysis:

(61) $$[\![\textbf{Subj all Pred}]\!] = \begin{cases} 1 & \text{if for every 'part' } x \text{ of } [\![\textbf{Subj}]\!],\ [\![\textbf{Pred}]\!](x) = 1 \\ 0 & \text{otherwise} \end{cases}$$

- In **Absolute Atomicity** plural morphology is often analyzed as having the same meaning as Δ, but I'll propose that there's more than that.

4.4 The syntax of atomic and subatomic quantifiers

- We postulate Δ for (62).

(62) The flags are Δ partly red.
$\approx$ Each atomic part of the subject denotation is partly red (modulo homogeneity/non-maximality).

This captures the intuitively available meaning of the sentence.

It also accounts for why its meaning is similar to (63) (modulo homogeneity/non-maximality).

(63) The flags are all partly red.

NB: ∆ and *all* in (62) and (63) are **atomic quantifiers**, i.e., they quantify over absolute atoms.

- We postulate ∆ for (64), as well.

(64) The flag is ∆ red.

Compare this to:

(65) The flag is entirely red.

NB: ∆ and *entirely* in (64) and (65) are **sub-atomic quantifiers**, i.e., they quantify over parts of absolute atoms.

- In the following case, we need an atomic ∆ and a sub-atomic ∆.

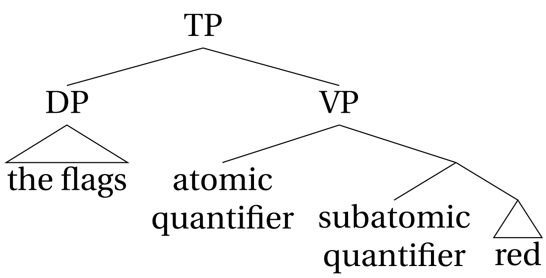
(66) The flags are ∆ ∆ red.

Compare:

(67) The flags are all entirely red.

- With a plural associate, there are two positions for DP-external quantifiers (Aldridge & Neeleman 2015).
  - The higher position hosts an atomic quantifier.
  - The lower position hosts a sub-atomic quantifier.

(68) *The flags are partly all red.



(69) The flags are ∆/all (for each flag) partly (for some sub-atomic part of the flag) red.

- With a singular count associate, only the lower position is available.

(70) *The door is all partly wooden.

- A mass associate (sometimes) allows for both.

(71) The furniture in this room is all partly wooden.

4.5 Variation among DP-external quantifiers

- We've been using *partly* in our examples, because it can only function as a sub-atomic quantifier.

(72) a. The flags are all partly red.
b. *The flags are partly entirely red.

- Other DP-external quantifiers can appear in either position.

(73) a. The flags are all mostly red.
b. The flags are mostly entirely red.

(74) a. The flags are all half red.
b. The flags are half entirely red.

Consequently, the following is ambiguous.

(75) The flags are mostly red.
a. The flags are Δ mostly red.
≈ Each flag is mostly red.
b. The flags are mostly Δ red.
≈ Most of the flags are entirely red.

	Atomic	Sub-atomic
partly	*	OK
all	OK	OK
mostly	OK	OK
half	OK	OK
20%	OK	OK
each	OK	*
Δ	OK	OK

- The atomic and sub-atomic versions of the same quantifier need to be given different denotations.

(76) a. $[\![\textbf{all}_{\textbf{atomic}}]\!] = \lambda P_{\langle e,t\rangle}.\lambda x_e.$ for each $y \leqslant_i x$ such that $y \in A$, $P(y) = 1$.
b. $[\![\textbf{all}_{\textbf{sub-atomic}}]\!] = \lambda P_{\langle e,t\rangle}.\lambda x_e.$ for each $y \leqslant_p x$, $P(y) = 1$

(77) a. $$[\![\Delta_{\textbf{atomic}}]\!] = \lambda P_{\langle e,t\rangle}.\lambda x_e. \begin{cases} 1 & \text{for each } y \leqslant_i x \text{ such that } y \in A, P(y) = 1 \\ 0 & \text{for each } y \leqslant_i x \text{ such that } y \in A, P(y) = 0 \\ \# & \text{otherwise} \end{cases}$$
b. $$[\![\Delta_{\textbf{sub-atomic}}]\!] = \lambda P_{\langle e,t\rangle}.\lambda x_e. \begin{cases} 1 & \text{for each } y \leqslant_p x, P(y) = 1 \\ 0 & \text{for each } y \leqslant_p x, P(y) = 0 \\ \# & \text{otherwise} \end{cases}$$

4.6 Restating the puzzle

- With singular and mass associates, direct sub-atomic quantification.

(78) a. The door is partly wooden.
b. The furniture is partly wooden.

- With plural associates, sub-atomic quantification can only happen after atomic quantification.

(79) a. The flags are Δ partly red.
b. The tables are all partly wooden.

Puzzle: Why isn't a plural associate compatible with direct sub-atomic quantification?

(80) *The flags are *(Δ) partly red.

- Recall that other ways of referring to complex entities allow for direct sub-atomic quantification.

(81) a. The furniture in this room is partly wooden.
b. The table, chairs and shelve in this room are partly wooden.

(82) a. The Google logo is partly red.
b. These six letters are partly red.

(83) a. This deck of playing cards is partly wet.
b. These play cards are partly wet.

- It's also not the case that plural subjects syntactically require the position for an atomic quantifier to be filled, because they are compatible with integrative predicates.

(84) a. The problems are diverse.
b. The members are John, Paul, George, and Ringo.

.... The rest of this section will be skipped ....

4.7 Other readings of DP-external quantifiers

4.7.1 Quality readings

Some of these quantifiers give rise to 'quality readings' (Aldridge & Neeleman 2015).

(85) The door is half transparent.
a. Half the door is transparent. — Sub-atomic reading
b. The transparency of the (entire) door is 50%. — Quality reading

The quality reading is accounted for with a third position, which is lower than the other two.

(86) The doors are $\underbrace{\text{all}}_{\text{each door}}$ $\overbrace{\text{partly}}^{\text{for some part of the door}}$ $\underbrace{\text{entirely}}_{\text{quality}}$ transparent.

The ambiguity of (85) is analysed as structural ambiguity:

(87) a. The door is half Δ transparent
b. The door is Δ half transparent.

4.7.2 Occasion readings

(88) I mostly danced.

Ultimately, we want to give a uniform(-ish) analysis for all readings.

5 An Intensional Theory of Relative Atomicity

WARNING: This and the next section are formally sloppy in certain compositional respects. See the Appendix for the full details of semantic composition.

5.1 The model

- One domain and one partial order $\leqslant$, reflecting the intuitive notion of part-whole ($\approx\leqslant_p$ in the above theory of **Absolute Atomicity**).
  - Link 1983 postulated two domains and two partial orders.
  - Subsequent theories of **Absolute Atomicity** postulated one domain and two partial orders, $\leqslant_i$ and $\leqslant_p$.
  - I will argue that $\leqslant_i$ is theoretically superfluous (and conceptually unnatural). So one domain and one partial order.
- $(D, \leqslant)$ is a join semilattice. For any two entities x, y we can always talk about their join $x \sqcup y$.
- Most, perhaps all, entities in D are non-atomic, as they all have parts.
  - Physical objects are perceived as having parts (possibly except for elementary particles, but it's unclear how we mentally represent them).
  - Non-physical objects may also have parts (e.g., time, ideas, theories, reasons, advice, or even holes?).

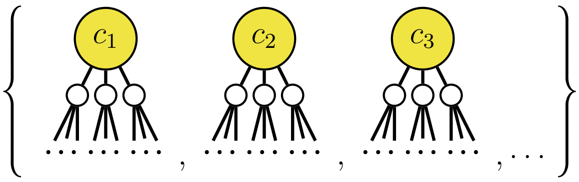
  In the following discussion, I will assume that the model has no absolute atoms.

5.2 Number morphology and noun extensions

- Things describable by singular count nouns are not absolute atoms.

  (89) $[\![\textbf{cat}]\!] = \{ x \in D \mid x \text{ is a cat} \}$

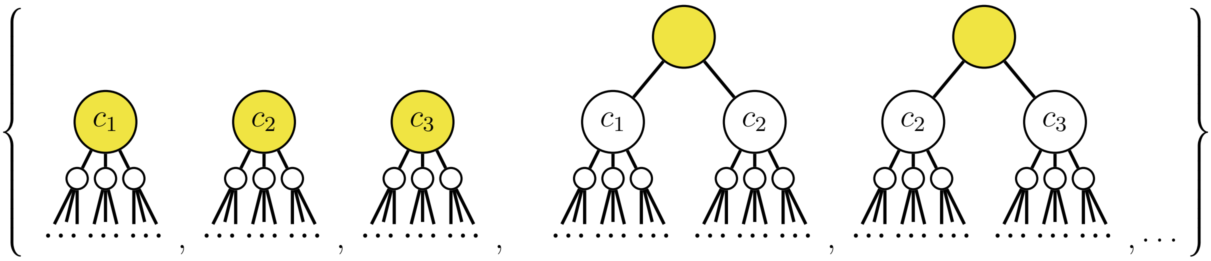
Each $x \in [\![\textbf{cat}]\!]$ is a single cat but has parts, its tail, face, paws, etc., so not atomic.



Subatomic quantifiers will quantify over these parts.

- The extension of a plural count noun contains any combination of the members of $[\![\textbf{cat}]\!]$.

(90) $[\![\textbf{cats}]\!] = \{ \sqcup S \mid S \subseteq [\![\textbf{cat}]\!] \text{ and } S \neq \varnothing \}$



- The members of the extension of *cats* still have parts, so we are still not account for the puzzle.
- Because of these parts, there are no absolute atoms in the extension.
- We'll introduce **relative atoms** in the intension and solve the puzzle.

- Note that the sets that these nouns denote have the same structure as usual. But as is well known this is not enough to distinguish plural nouns and object mass nouns. So we may have (depending on the model):

(91) a. $[\![$**the table, chairs, and shelf in this room**$]\!]$ = $[\![$**the furniture in this room**$]\!]$
b. $[\![$**the table in this room**$]\!]$ = $[\![$**the furniture in this room**$]\!]$

Some mass nouns denote infinite sets, e.g., *time* and *space*.

5.3 Intensionality

- An expression is assigned an extension relative to a model and a number of intensional parameters (assignment function, possible world, time, situation, context, etc.).

$$[\![\alpha]\!]_{\mathcal{M}}^{g,w,t,s,c}$$

- The model parameter $\mathcal{M}$ is usually omitted. But keep in mind that you can always access $(D, \leqslant)$, as it is part of the model.
- I'm not interested in these usual intensional parameters, so I will not use them. (Assignments are an exception but I will only be explicit about them in the Appendix).

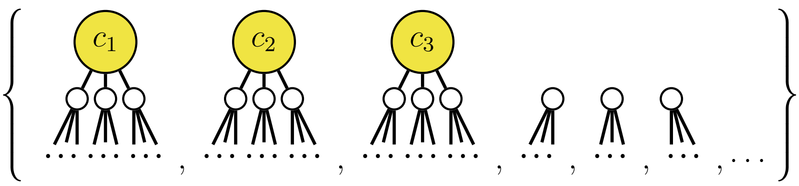
Proposal: A new intensional parameter that indicates entities under discussion.

- The new parameter is a restriction $(S, \leqslant\restriction_S)$ of $(D, \leqslant)$.
  - $S \subseteq D$; and
  - $\leqslant\restriction_S \coloneqq \{\, (x, y) \mid x \leqslant y \text{ and } x, y \in S \,\}$.
- Instead of $(S, \leqslant\restriction_S)$, let's write $S^{\leqslant}$.
- $S^{\leqslant}$ encodes **relative atomicity** when it has minimal elements = **relative atoms**.
- A singular count noun introduces a restriction using its extension. The interpretations of nouns themselves are insensitive to the intensional parameter, so for any $S^{\leqslant}$:

(92) $[\![\textbf{cat}]\!]^{S^{\leqslant}} = \{\, x \in D \mid x \text{ is a cat} \,\}$

  - Remember that the members of this set have parts (with respect to $\leqslant$).
  - The new restriction it introduces includes these cats and their parts.

(93) a. ${}^{\downarrow}\{\, x \in D \mid x \text{ is a cat} \,\}^{\leqslant} = {}^{\downarrow}\texttt{CAT}^{\leqslant}$.
b. ${}^{\downarrow}S = \{\, x \in D \mid x \leqslant y \text{ for some } y \in S \,\}$



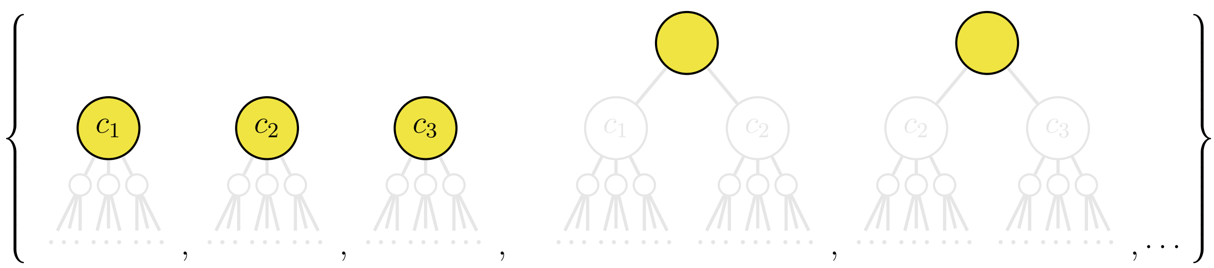
- A plural count noun introduces **relative atoms**.

(94) For any $S^{\leqslant}$,
$[\![\textbf{cats}]\!]^{S^{\leqslant}} = \{\, \bigsqcup C \mid C \subseteq [\![\textbf{cat}]\!]^{S^{\leqslant}} \text{ and } C \neq \varnothing \,\}$

  - The members of this set also have parts with respect to $\leqslant$.
  - But with respect to the restriction it introduces, we don't see their parts.

(95) a. ${}^{\uparrow}\texttt{CAT}^{\leqslant}$
b. ${}^{\uparrow}S = \{\, x \in D \mid y \leqslant x \text{ for some } y \in S \,\}$

  - When $S^{\leqslant}$ has minimal elements, I write $\min(S^{\leqslant}) \neq \varnothing$.



- Mass nouns generally have extensions similar to count nouns, but they do *not* introduce new intensional parameters and use the full model $(D, \leqslant)$.

> The mass/count distinction is partly intensional.

- It is sometimes remarked that the weak extension for plural nouns is conceptually problematic, given their morphological markedness relative to their singular counterparts (Farkas & de Swart 2010, Bale, Gagnon & Khanjian 2011). In the current theory with intensional effects, plural nouns are the only device that introduce **relative atoms**.

5.4 Atomic DP-external quantifiers

- Recall that our model has no **absolute atoms**.
- Atomic DP-external quantifiers quantify over **relative atoms** encoded in the intensional parameter, if any. We treat this as a presupposition.

(96) The cats each meowed twice.

(97) a. $\ulcorner$**each**$\urcorner$ $\in \mathrm{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$ iff $\min(S^{\leqslant}) \neq \varnothing$.
b. Whenever $\ulcorner$**each**$\urcorner$ $\in \mathrm{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$,
$\llbracket\textbf{each}\rrbracket^{S^{\leqslant}} = \lambda P_{\langle e,t\rangle}.\lambda x_e\colon\ x \in S.$ each $y \leqslant x$ such that $y \in \min(S^{\leqslant})$, $P(y) = 1$.

- Whenever *each* is used, there needs to be a plural noun that introduces a restriction with **relative atoms**.
  - A DP inherits the intensional parameter that its head noun introduces (We'll discuss how this happens shortly).

    (98) $\llbracket\textbf{the cats}\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}$

  - The DP transfers the intensional parameter to the VP.

    (99) $\llbracket\textbf{the cats VP}\rrbracket^{S^{\leqslant}} = 1$ iff $\llbracket\textbf{VP}\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}(\llbracket\textbf{the cats}\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}) = 1$

    (This is done via Intensional Functional Application, and to do it, we Montague-lift entities; see the Appendix for details)
  - If the VP contains *each*, it will quantify over the relative atoms, each of which is a cat in this case.

    (100) The cats each meowed twice.

  - If there is no plural DP in the right position, then presupposition failure ensues.

    (101) a. The desks, chairs, and cabinets in this office were each disinfected twice.
    b. *The furniture in this office was each disinfected twice.

- Other atomic quantifiers have the same presupposition.[2]

(102) a. $\ulcorner$**all**$_{\textbf{atomic}}$$\urcorner$ $\in \mathrm{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$ iff $\min(S^{\leqslant}) \neq \varnothing$.

[2] I will not try to account for possible differences between *all*$_{atomic}$ and *each*, so I will tentatively give it the same meaning as *each*. In reality, *each* wants to have something to distribute over in its semantic scope.

b. Whenever $\ulcorner \textbf{all}_{\textbf{atomic}} \urcorner \in \text{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$,
$\llbracket \textbf{all}_{\textbf{atomic}} \rrbracket^{S^{\leqslant}} = \lambda P_{(e,t)}.\lambda x_e.$ each $y \leqslant x$ such that $y \in \min(S^{\leqslant})$, $P(y) = 1$.

(103) a. $\ulcorner \Delta_{\text{atomic}} \urcorner \in \text{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$ iff $\min(S^{\leqslant}) \neq \varnothing$.
b. Whenever $\ulcorner \Delta_{\text{atomic}} \urcorner \in \text{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$,

$$\llbracket \Delta_{\textbf{atomic}} \rrbracket^{S^{\leqslant}} = \lambda P_{(e,t)}.\lambda x_e. \begin{cases} 1 & \text{each } y \leqslant x \text{ such that } y \in \min(S^{\leqslant}),\ P(y) = 1 \\ 0 & \text{each } y \leqslant x \text{ such that } y \in \min(S^{\leqslant}),\ P(y) = 0 \\ \# & \text{otherwise} \end{cases}$$

- Not only this, these atomic quantifiers reset the intensional parameter to the entire model $(D, \leqslant)$. Schematically:

(104) $\llbracket$**the cats each meowed twice**$\rrbracket^{S^{\leqslant}} = 1$
iff $\llbracket$**each meowed twice**$\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}(\llbracket$**the cats**$\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}) = 1$
iff $\llbracket$**each**$\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}(\llbracket$**meowed twice**$\rrbracket^{(D,\leqslant)})(\llbracket$**the cats**$\rrbracket^{\uparrow\texttt{CAT}^{\leqslant}}) = 1$

To ensure this, we need to turn the quantifiers into intensional operators. See the Appendix for details.

5.5 Sub-atomic DP-external quantifiers

- Sub-atomic DP-external quantifiers are incompatible with parameters with relative atoms. We again encode this as a presupposition.

(105) a. $\ulcorner \textbf{partly} \urcorner \in \text{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$ iff $\min(S^{\leqslant}) = \varnothing$.
b. Whenever $\ulcorner \textbf{partly} \urcorner \in \text{dom}(\llbracket\cdot\rrbracket^{S^{\leqslant}})$,
$\llbracket \textbf{partly} \rrbracket^{S^{\leqslant}} = \lambda P_{(e,t)}.\lambda x_e\colon x \in S.$ for some $y \leqslant x$, $P(y) = 1$.

- This presupposition accounts for why direct sub-atomic quantification is impossible with plural associates.

(106) The flags are partly red.

- *The flags* introduces a restriction with **relative atoms**, $\uparrow\texttt{FLAG}^{\leqslant}$.
- Consequently *partly* cannot be the next operator down.
- If Δ_{atomic} intervenes, it resets the parameter to $(D, \leqslant)$, which has no relative atoms, so the presupposition of *partly* will be satisfied.[3]

(107) $\llbracket$**the flags were** $\Delta_{\textbf{atomic}}$ **partly red**$\rrbracket^{S^{\leqslant}} = 1$
iff $\llbracket \Delta_{\textbf{atomic}}$ **partly red**$\rrbracket^{\uparrow\texttt{FLAG}^{\leqslant}}(\llbracket$**the flags**$\rrbracket^{\uparrow\texttt{FLAG}^{\leqslant}}) = 1$
iff $\llbracket \Delta_{\textbf{atomic}} \rrbracket^{\uparrow\texttt{FLAG}^{\leqslant}}(\llbracket$**partly red**$\rrbracket^{(D,\leqslant)})(\llbracket$**the flags**$\rrbracket^{\uparrow\texttt{FLAG}^{\leqslant}}) = 1$

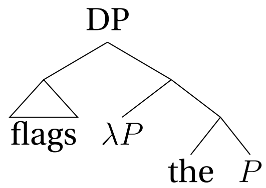
[3]Here we are crucially assuming that the model has no absolute atoms. What if it does have some? Then we could reanalyse the presupposition to be one where $S^{\leqslant}$ is not an atomic semilattice, i.e., at least one thing in S is not based on relative atoms (and also require a model to always have something that is not made up of atoms, e.g., space and time).

The ban on direct sub-atomic quantification comes from the requirement of sub-atomic quantifiers that entities with parts (non-atoms) are being talked about.

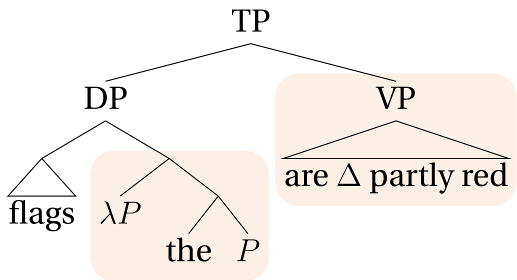
6 DP-internal matters

6.1 Scope

- Above we said that a DP inherits the intensional parameter that its head noun introduces.
- Technically, we achieve this by having the noun scope at the edge of the DP. We could use any theory of scope, but let us use movement theory, as it is visually easy to understand.



  - The extension of the noun (a set of entities) will reconstruct to the original position.
  - *The* P will be affected by the intension of the noun, i.e., it will be interpreted with respect to the intensional parameter that the noun introduces.
- Furthermore, the noun's intensional parameter will be further inherited by the next level (ensured by Intensional Functional Application; see the Appendix). So there will be two domains affected by the noun's intensional parameter.



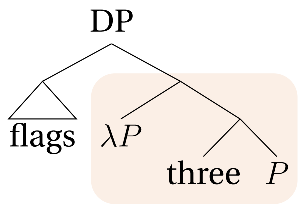
  See Charlow 2014, 2020 for more general discussion of scope extension from the edge.
- The intensional scope of the noun is its syntactic agreement domain. This is potentially syntactic relevant(?).

6.2 Counting modifiers and quantifiers

- Recall that **Absolute Atomicity** makes use of **absolute atoms** in the analyses of:
  - Number morphology and mass/count
  - Distributivity
  - Counting modifiers and quantifiers

  We've taken care of 1. and 2. in our theory of **Relative Atomicity**.

- Counting modifiers and quantifiers can be accounted for with **relative atoms** in a way similar to DP-external atomic quantifiers. Importantly, they will be in the immediate scope of the noun.



(108) a. $\ulcorner$**three**$\urcorner \in \mathrm{dom}([\![\cdot]\!]^{S^{\leqslant}})$ iff $\min(S^{\leqslant}) \neq \varnothing$
b. Whenever $\ulcorner$**three**$\urcorner \in \mathrm{dom}([\![\cdot]\!]^{S^{\leqslant}})$,
$[\![$**three**$]\!]^{S^{\leqslant}} = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$ there are three distinct $x, y, z \in \min(S^{\leqslant})$ such that $P(x \sqcup y \sqcup z) = Q(x \sqcup y \sqcup z) = 1$.

- Those that select for singular nouns require $S^{\leqslant}$ to be distinct from $(D, \leqslant)$ (so that the attention is sufficiently restricted) and to have no atoms. This excludes mass nouns and plural nouns. Note that quantification is over the maxima of $S^{\leqslant}$.

(109) a. $\ulcorner$**each**$\urcorner \in \mathrm{dom}([\![\cdot]\!]^{S^{\leqslant}})$ iff $S^{\leqslant} \neq (D, \leqslant)$ and $\min(S^{\leqslant}) \neq \varnothing$
b. Whenever $\ulcorner$**each**$\urcorner \in \mathrm{dom}([\![\cdot]\!]^{S^{\leqslant}})$,
$[\![$**each**$]\!]^{S^{\leqslant}} = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$ for each $x \in \max(S^{\leqslant})$, $P(x) = Q(x) = 1$.

(110) a. $\ulcorner$**one**$\urcorner \in \mathrm{dom}([\![\cdot]\!]^{S^{\leqslant}})$ iff $S^{\leqslant} \neq (D, \leqslant)$ and $\min(S^{\leqslant}) \neq \varnothing$
b. Whenever $\ulcorner$**each**$\urcorner \in \mathrm{dom}([\![\cdot]\!]^{S^{\leqslant}})$,
$[\![$**one**$]\!]^{S^{\leqslant}} = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.$ for one $x \in \max(S^{\leqslant})$, $P(x) = Q(x) = 1$.

> There is no need for **absolute atoms**.

7 Summary

- The puzzle of DP-external sub-atomic quantification. Direct sub-atomic quantification is not possible with a plural subject.

(111) a. *The flags are partly red.
b. The flags are Δ_{atomic} partly red.

(112) The furniture is partly wooden.

- I argued that we need **relative atoms** to account for this, and put forward an 'intensional' theory of **Relative Atomicity**.

> Different intensional effects for different types of nouns:
>
> – Plural count nouns introduce **relative atoms**.

- Singular count nouns restrict the domain.
- Mass nouns don't do anything.

⇒ Part of mass/count is intensional.

- **Relative Atomicity** accounts for:
  - Number morphology and mass/count
  - Distributivity
  - Counting modifiers and quantifiers

No need for **absolute atoms**. We should give up on **Absolute Atomicity**.

- Further issues to think about:
  - Connectives

(113) a. the students and professors
b. the students or professors

  - Non-atomic distributivity and context sensitivity (Schwarzschild 1996).

(114) The cows and pigs are separated.

  - Sub-atomic modifiers, e.g. *half* and *double* (Wągiel 2018, 2019).

(115) a. a half baguette
b. a double expresso

  - Group nouns

(116) a. the playing cards
b. the deck of cards

(117) The committee are old.

Potentially a plain plural DP has a group interpretation (cf. Landman 1989a,b, 2000), which might feed direct subatomic quantification DP-externally.

  - Intensional sensitivity of predicates (summative/distributive vs. integrative/collective)
  - Languages without number marking
  - Classifiers and floating numeral quantifiers

Appendix: Compositional details

- We will use movement theory of scope so we will be explicit about assignment.
- We write $[\![\alpha]\!]^{g,S^{\leqslant}}$ for the extension of α, relative to g and $S^{\leqslant}$, i.e., $[\![\alpha]\!](g)(S^{\leqslant})$.
- We will be explicit about presupposition projection, which is ensured by the compositional rules and lexical entries.

Model and restrictions

- A model comes with a domain D of entities and a partial order $\leqslant$ on D such that $(D, \leqslant)$ is a join semilattice.
- The corresponding join operation is $\sqcup$.
- A restriction $(S, \leqslant\upharpoonright_S)$ of $(D, \leqslant)$:
  - $S \subseteq D$; and
  - $\leqslant\upharpoonright_S \coloneqq \{\, (x, y) \mid x \leqslant y$ and $x, y \in S \,\}$.
- Instead of $(S, \leqslant\upharpoonright_S)$, we'll write $S^{\leqslant}$

Compositional rules

(118) **Intensional Functional Application**
For any g and $S^{\leqslant}$,
if α dominates two constituents β and γ such that
a. $\ulcorner\beta\urcorner \in \mathrm{dom}(\llbracket\cdot\rrbracket^{g,S^{\leqslant}})$; and
b. $[\lambda o\colon \ulcorner\gamma\urcorner \in \mathrm{dom}(\llbracket\cdot\rrbracket^{g,o}).\ \llbracket\gamma\rrbracket^{o}] \in \mathrm{dom}(\llbracket\beta\rrbracket^{g,S^{\leqslant}})$,
then $\ulcorner\alpha\urcorner \in \mathrm{dom}\llbracket\cdot\rrbracket^{g,S^{\leqslant}}$ and $\llbracket\alpha\rrbracket^{g,S^{\leqslant}} = \llbracket\beta\rrbracket^{g,S^{\leqslant}}([\lambda o\colon \ulcorner\gamma\urcorner \in \mathrm{dom}(\llbracket\cdot\rrbracket^{g,o}).\ \llbracket\gamma\rrbracket^{g,o}])$

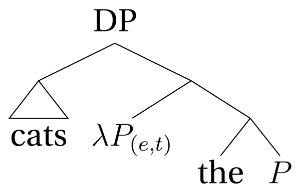
(119) **Predicate Abstraction**
For any g and $S^{\leqslant}$,
if α dominates two constituents $\lambda\xi_\tau$ for some type τ and β, then $\ulcorner\alpha\urcorner \in \mathrm{dom}\llbracket\cdot\rrbracket^{g,S^{\leqslant}}$ and $\llbracket\alpha\rrbracket^{g,S^{\leqslant}} = \lambda x_\tau\colon \ulcorner\beta\urcorner \in \mathrm{dom}(\llbracket\cdot\rrbracket^{g[\xi\mapsto x],o}).\ \llbracket\beta\rrbracket^{g[\xi\mapsto x],o}$

(120) **Variable Rule**
For any g and $S^{\leqslant}$, if α is a variable, then $\ulcorner\alpha\urcorner \in \mathrm{dom}(\llbracket\cdot\rrbracket^{g,S^{\leqslant}})$ and $\llbracket\alpha\rrbracket^{g,S^{\leqslant}} = g(\alpha)$.

We won't need Extensional Functional Application, because we'll take care of that in the lexical entries of extensional operators (they will simply pass on their intensional parameters to their arguments).

Noun intensions

Recall that nouns take scope at the edge of their local DP.



Nouns are intensional operators, taking the intension of their sister constituent, and changing its intensional parameter. For any set A, we write χ^A to denote the characteristic function of A.

For any g and $S^{\leqslant}$,

(121) $[\![\textbf{cat}]\!]^{g,S^{\leqslant}} = \lambda f_{(s,((e,t),((e,t),t)))}\colon\ {}^{\downarrow}\texttt{CAT}^{\leqslant} \in \mathrm{dom}(f) \wedge \chi^{\texttt{CAT}} \in \mathrm{dom}(f({}^{\downarrow}\texttt{CAT})).\ f(\texttt{CAT}^{\leqslant})(\chi^{\texttt{CAT}})$

(122) $[\![\textbf{cats}]\!]^{g,S^{\leqslant}} = \lambda f_{(s,((e,t),((e,t),t)))}\colon\ {}^{\uparrow}\texttt{CAT}^{\leqslant} \in \mathrm{dom}(f) \wedge \chi^{\texttt{CATS}} \in \mathrm{dom}(f({}^{\uparrow}\texttt{CAT}^{\leqslant})).\ f({}^{\uparrow}\texttt{CAT}^{\leqslant})(\chi^{\texttt{CATS}})$

(123) $[\![\textbf{mud}]\!]^{g,S^{\leqslant}} = \lambda f_{(s,((e,t),((e,t),t)))}\colon\ (D,\leqslant) \in \mathrm{dom}(f) \wedge \chi^{\texttt{MUD}} \in \mathrm{dom}(f((D,\leqslant))).\ f((D,\leqslant))(\chi^{\texttt{MUD}})$

(124)
a. $\texttt{CAT} = \{\, x \in D \mid x \text{ is a cat} \,\}$
b. $\texttt{CATS} = \{\, \bigsqcup C \mid C \subseteq \texttt{CAT} \text{ and } C \neq \varnothing \,\}$
c. $\texttt{MUD} = \{\, x \in D \mid x \text{ is mud} \,\}$

(125)
a. ${}^{\downarrow}S = \{\, x \in D \mid \exists y \in S[x \leqslant y] \,\}$
b. ${}^{\uparrow}S = \{\, x \in D \mid \exists y \in S[y \leqslant x] \,\}$

Quantificational determiners and scope extension

We analyse *the* as a quantifier. It's an extensional operator, as it passes its intensional parameter to its arguments.

(126) For any g and $S^{\leqslant}$,

$$[\![\textbf{the}]\!]^{g,S^{\leqslant}} = \lambda P_{(s,(e,t))}\colon\ S^{\leqslant} \in P \wedge |\max_{\leqslant}(\mathrm{set}(P(S^{\leqslant})))| = 1.\ \lambda Q_{(s,(e,t))}\colon\ S^{\leqslant} \in \mathrm{dom}(Q) \wedge \bigsqcup \mathrm{set}(P(S^{\leqslant})) \in \mathrm{dom}(Q(S^{\leqslant})).\ Q(S^{\leqslant})(\bigsqcup \mathrm{set}(P(S^{\leqslant}))) = 1$$

(127) $\mathrm{set}(f) \coloneqq \{\, x \in D_\tau \mid x \in \mathrm{dom}(f) \text{ and } f(x) = 1 \,\}$ for any $f \in D_{(\tau,t)}$.

DP-external atomic quantifiers

(128) For any g and $S^{\leqslant}$,

a. $\ulcorner\textbf{each}\urcorner \in \mathrm{dom}([\![\cdot]\!]^{g,S^{\leqslant}})$ iff $\min S^{\leqslant} \neq \varnothing$.

b. Whenever $\ulcorner\textbf{each}\urcorner \in \mathrm{dom}([\![\cdot]\!]^{g,S^{\leqslant}})$,

$$[\![\textbf{each}]\!]^{g,S^{\leqslant}} = \lambda P_{(s,(e,t))}\colon\ (D,\leqslant) \in \mathrm{dom}(P).\ \lambda x_e\colon\ \forall y \leqslant x[x \in \min S^{\leqslant} \rightarrow y \in \mathrm{dom}(P((D,\leqslant)))].\ \forall y \leqslant x[x \in \min S^{\leqslant} \rightarrow P((D,\leqslant))(y) = 1]$$

DP-external sub-atomic quantifiers

(129) For any g and $S^{\leqslant}$,

a. $\ulcorner\textbf{partly}\urcorner \in \mathrm{dom}([\![\cdot]\!]^{g,S^{\leqslant}})$ iff $\min S^{\leqslant} = \varnothing$.

b. Whenever $\ulcorner\textbf{partly}\urcorner \in \mathrm{dom}([\![\cdot]\!]^{g,S^{\leqslant}})$,

$$[\![\textbf{partly}]\!]^{g,S^{\leqslant}} = \lambda P_{(s,(e,t))}\colon\ (D,\leqslant) \in \mathrm{dom}(P).\ \lambda x_e\colon\ x \in S \wedge \exists y \leqslant x[y \in \mathrm{dom}(P((D,\leqslant)))].\ \exists y \leqslant x[P((D,\leqslant))(y) = 1]$$

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