Presupposition Projection in Quantified Sentences and Cross-Dimensional Anaphora*

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1 Introduction

Presupposition projection is one of the most widely discussed topics in modern formal semantics and pragmatics, and a multitude of theories in various formal frameworks have been put forward in the literature (for overviews, see Beaver 2001, Chierchia & McConnell-Ginet 2000, Geurts 1999, Kadmon 2001, Beaver & Geurts 2011). Although these theories significantly differ both in empirical predictions and in ways in which they conceive of presuppositional inferences, the problem of presupposition projection through quantificational expressions is a common tantalizing problem, and has been actively investigated in a number of different theoretical set-ups (e.g. Karttunen & Peters 1979, Cooper 1983, Heim 1983, Van der Sandt 1992, Beaver 1994, 2001, Beaver & Krahmer 2001, Fox 2008, 2012, George 2008a,b, Schlenker 2008, Charlow 2009, Chemla 2009b, Schlenker 2009, Fox 2012).

Simply put, the problem of presuppositions in quantified sentences is how to explain the presuppositions of sentences of the form $Q(R)(S)$ from the meaning of Q and the meanings of R and S, where Q is a quantificational determiner and R and S are its restrictor and nuclear scope potentially with presuppositions. For expository purposes, we will first concentrate on cases where only the nuclear scope S has a presupposition, and will come back to presupposition projection out of the restrictor R later in the paper (Section 5).

The main empirical challenge of this puzzle consists in the fact that different quantifiers have different projection properties. Consider, for instance, the sentences in (1), where the predicate *criticized herself* has a presupposition that the subject is female (Cooper 1983, Heim & Kratzer 1998).

(1) a. Every student criticized herself.
  b. No student criticized herself.
  c. A student criticized herself.

One can infer from (1a) that every student is female. We assume that the assertive meaning of *criticized herself* is simply, $[\lambda x.\ x$ criticized $x]$, while the presupposition is the gender inference $[\lambda x.\ x$ is female$]$. On this assumption, the gender inference of (1a) that every student is female should be a purely presuppositional inference, rather than an entailment in the assertive meaning (see Sudo 2012 for more sophisticated arguments for this analysis, and its theoretical implications).[1] Similarly, one can fairly reliably infer from (1b) that every student is female. Again, this cannot be an entailment in the assertive meaning, and should be attributed to the

*[acknowledgments to be added]

[1]If one is skeptical about this analysis of the predicate *criticized herself* and insists that the assertive meaning should be analyzed as $[\lambda x.\ x$ is female and criticized $x]$, consider the negation of this predicate, *didn't criticize*

projection property of the quantifier *no student*. On the other hand, (1c) has a weaker inference that at least the student who criticized herself is female, and says nothing about the gender of other students. We will hereafter call the inference that every individual in the restrictor of the quantifier satisfies the presupposition of the nuclear scope the *universal inference*.

Although the judgments of the sentences in (1) are more or less stable and uncontroversial, it is known that those of some quantified sentences are rather delicate, and in fact conflicting opinions have been expressed by various authors (cf. Cooper 1983, Heim 1983, Van der Sandt 1992, Beaver 1994, 2001, Kadmon 2001, George 2008a,b, Charlow 2009).[2] Fortunately, recent experimental work on this topic, most notably Chemla (2009a) and Sudo, Romoli, Hackl & Fox (2011), has made it clear that the following pattern holds at least as a general tendency: Quantifiers like *none of the 5 students* are likely to have a universal inference, while quantifiers like *some of the 5 students* are much less likely to have a universal inference. Furthermore, the latter type of quantifiers sometimes give rise to a universal inference.[3] Thus, the sentences in (2) are associated with a universal inference that all of the 5 students used to smoke in the decreasing order of robustness.

(2) a. None of the 5 students stopped smoking.
 b. Some of the 5 students stopped smoking.
 c. The tallest of the 5 students stopped smoking.

To summarize so far, universal quantifiers like *every student* and negative quantifiers like *no student* and *none of the 5 students* give rise to universal inferences fairly robustly, while indefinites like *a student* do not yield universal inferences (although of course compatible with them). Partitive existential quantifiers like *some of the 5 students* have an intermediate status. It is not a purpose of the present paper to do justice to these empirical generalizations, for which the interested reader is referred to the references cited above, although part of the empirical picture is admittedly not yet entirely clear. Rather, we take these generalizations to be the main data to be accounted for by a theory of presupposition projection.

The main contribution of the present paper is a novel theory of presupposition projection in quantificational sentences. The theory is motivated by a hitherto unnoticed parallelism between the projection properties and anaphoric potentials of quantificational expressions. Specifically, we observe that a universal inference arises with quantifiers that support so-called *maxset anaphora* across sentences, and a non-universal inference arises with quantifiers that support so-called *refset anaphora* across sentences. Maxset anaphora is exemplified by (3a), where the pronoun *they* refers to the plurality consisting of all the individuals in the restrictor of the quantifier *No students majoring in linguistics* occurring in the first sentence. Refset anaphora,

herself. The assertive meaning will be $[\lambda x.$ either x is male or x didn't criticize $x]$. Now take the sentence *Every student didn't criticize herself* under the surface scope reading. The sentence still suggests that every student is female, but this universal gender inference cannot be an entailment in the assertive meaning. Thus, we arrive at the same conclusion: *Every student* gives rise to an inference that every relevant student satisfies the presupposition of the predicate.

[2] Part of the controversy is due to the *local accommodation* reading (Heim 1983, Sudo, Romoli, Hackl & Fox 2011, Fox 2012) where the presupposition of the nuclear scope seems to be optionally treated as part of the assertive meaning. This reading, however, is extremely hard to obtain with presupposition triggers like *herself* (see also fn.18).

[3] Sudo, Romoli, Hackl & Fox's (2011) results indicate that some speakers insisted on a universal inference with quantifiers like *some of the 5 students* even under the experimental bias against this interpretation. In their experiment, the participants were asked to indicate whether a non-universal inference was possible at all. Furthermore, in an unpublished work, Sudo, Romoli, Hackl and Fox observe that such quantifier are more likely to be associated with a universal inference in comparison to singular definite descriptions like *the tallest of the 5 students*.

on the other hand, involves reference to the intersection of the restrictor and the nuclear scope of the quantificational determiner, as illustrated by (3b).[4]

(3) a. *No students majoring in linguistics* got an A in the mid term exam. *They* are all interested in phonology.
(*they* = all the students majoring in linguistics)

b. *Some students majoring in linguistics* got an A in the mid term exam. *They* are all interested in phonology.
(*they* = the students majoring in linguistics who got an A in the mid term exam)

The parallelism between presupposition projection and cross-sentential anaphora is schematically summarized as follows.

(4) Consider a discourse of the following form '$Q(R)(F)$. PRONOUN G.', where $Q(R)$ is a quantifier with a restrictor R, F and G are arbitrary predicates, and PRONOUN is a pronoun with appropriate phi-features, e.g. *they*.

a. If PRONOUN can be resolved to R (i.e. $Q(R)$ supports maxset anaphora), then sentences of the form $Q(R)(\lambda x.S(x)_{p(x)})$ can have a universal inference that $\forall x \in R : p(x)$, and

b. If PRONOUN can be resolved to $R \cap F$ (i.e. $Q(R)$ supports refset anaphora), then sentences of the form $Q(R)(\lambda x.S(x)_{p(x)})$ can have an inference weaker than a universal inference.

Combined with the earlier observations about presupposition projection in quantified sentences, we expect the following state of affairs.

(5) a. *Every student, no student,* and *none of the students* only support maxset anaphora.[5] Consequently they always give rise to a universal inference in presuppositional sentences.

b. *A student* only supports refset anaphora. Consequently it always gives rise an inference weaker than a universal inference in presuppositional sentences.

c. *Some of the students* supports both maxset anaphora and refset anaphora. Consequently it optionally gives rise to a universal inference.

As will be demonstrated with concrete examples later, these generalizations are empirically reasonable.[6]

In order to explain the above correlation between the presuppositional and anaphoric properties of quantifiers, we claim that the exact same mechanism is at play in both cases, namely the mechanism of anaphora. While we have nothing new to add to the theory of cross-sentential anaphora itself, this idea provides us with a new perspective on how presuppositions project in

[4]This seems to be a preferred, if not only, reading of (3b), but an ambiguity between maxset and refset readings seems to obtain more robustly with partitive quantifiers like *some of the students majoring in linguistics*. This ambiguity plays a crucial role in accounting for the projection properties of partitive existential quantifiers, as we will see in later sections.

[5]In the case of universal quantifiers, the maxset reading is equivalent to the refset reading, since every$(R)(S)$ asserts that $R = (R \cap S)$. Therefore, strictly speaking, it is not possible to know whether they support maxset anaphora or refset anaphora, or both. This is not a problem for us, since the predictions of our theory depend on the (possible) referent(s) of the pronoun, rather than the type of the anaphora, as will be explained in detail in the following sections. For the sake of brevity, we use 'refset anaphora' to mean anaphora to a set strictly smaller than the maxset.

[6]We will not discuss complement anaphora in the body of this paper. See Appendix for discussion.

quantified sentences. Specifically, we propose that the assertive meaning and presupposition of a quantified sentence are treated just like a sequence of two independent sentences involving a pronoun in the second sentence that is anaphoric to a quantifier in the first sentence. In order to see this more concretely, consider the following example, where the relevant presupposition trigger is *stop*.

(6) Two students stopped smoking.

Adopting a two-dimensional theory of presupposition, we represent the assertive meaning and presupposition of (6) in different dimensions of meaning as in (7).[7]

(7) a. ASSERTIVE MEANING: There are two students who used to smoke and stopped
 b. PRESUPPOSITION: **They** used to smoke

Against this theoretical backdrop, we propose that the presupposition of a quantificational sentence contains an anaphoric term—represented as *they* in (7b)— anaphorically related to the quantificational term in the assertive meaning—*two students* in (7a). To reiterate the main idea, the quantificational term and the anaphoric term in these two dimensions of meaning stand in essentially the same relation to two independent sentences occurring in a discourse like (3) above. We call this anaphoric relation across the two-dimensions of meaning *cross-dimensional anaphora*.

To formalize cross-dimensional anaphora, we will capitalize on an independently developed theory of cross-sentential anaphora with various types of quantificational expressions, namely (a version of) Van den Berg's (1996) Plural Predicate Logic (PPL), which is an extension of Groenendijk & Stokhof's (1991) Dynamic Predicate Logic (DPL). Our main theoretical contribution is the formulation of a Context Update Rule that enables an anaphoric link between the quantificational term in the assertive meaning and the anaphoric term in the presuppositions, and at the same time derives what is effectively an existential presupposition for sentences like (7). Since the exact same mechanism is deployed for cross-sentential and cross-dimensional anaphora, our theory straightforwardly capture the correlation stated in (3) above. Furthermore, we will demonstrate that this theory is capable of presupposition projection from the restrictor argument without further ado. Moreover, the resulting theory is not only empirically adequate but also conceptually appealing in that the projection properties of quantifiers are systematically explained by their independently justified cross-sentential anaphoric properties. We will argue in Section 6 that none of its predecessors achieves the same degree of empirical coverage while maintaining a uniform analysis of presuppositional properties of quantifiers.

The paper is organized as follows. In Section 2, we will introduce a simple version of the theory using a relatively simple dynamic language, namely Groenendijk & Stokhof's (1991) DPL, and formulate the Context Update Rule that enables cross-dimensional anaphora. In Section 3, we will examine the correlation between presupposition projection and cross-sentential anaphora with concrete examples, and in Section 4, we will enrich the simple theory introduced in Section 2 with a more complex dynamic semantics, namely Van den Berg's (1996)

[7] Karttunen & Peters (1979) formulated a two-dimensional theory as an extension of Montague Grammar, which remains quite influential in a number of respects. However, as Karttunen & Peters themselves point out, two-dimensional theories in general run into the so-called 'Binding Problem' in quantified sentences (see fn.8 and Section 6). Although the Binding Problem was sometimes thought to be fatal to two-dimensional theories, the theory proposed in this paper can be understood as a solution to it. Also there are empirical considerations that motivate departure from the standard three-valued uni-dimensional theories (Stalnaker 1973, Oshima 2006, Sudo 2012), some of which will be discussion in Section 6. See also Bergmann (1981), Dekker (2008), Herzberger (1973), Oshima (2006), Van Rooij (2005, 2010) for many-valued but uni-dimensional theories of presupposition.

PPL. Section 5 is devoted to discussion of presuppositions triggered in restrictors, which will be shown to be straightforwardly accounted for without further ado. Section 6 mentions appealing features of the present theory in comparison to major theories of presuppositions of quantified sentences found in the literature. We will discuss several open issues in Section 7 and finally conclude in Section 8.

2 Cross-Dimensional Anaphora with Singular Indefinites

This section introduces a simplified version of our formal theory that embodies cross-dimensional anaphora. Due to its simplicity, it can only deals with singular indefinites like *a student*, but it makes it easier to grasp the underlying formal mechanism. Our full theory will be presented in Section 4.

As already stated in the introduction, we adopt a two-dimensional theory of presupposition where the assertive meaning and presupposition of a sentence are treated as independent dimensions of meaning (Karttunen & Peters 1979; see also Herzberger 1973, Bergmann 1981). This is a crucial assumption for our theory, but it is beyond the scope of the present paper to raise the full set of empirical motivation for it (see especially Dekker 2008, Oshima 2006,Van Rooij 2005, 2010, Stalnaker 1973 and Sudo 2012 for relevant discussion), but we will present some crucial arguments that favor it over more standard uni-dimensional theories in Section 6.

For an illustration, consider the following simple sentence.

(8) Jesse criticized herself.

Under the two-dimensional theory of presupposition assumed here, it is (informally) analyzed as follows.

(9) a. ASSERTIVE MEANING: Jesse criticized Jesse
    b. PRESUPPOSITION: Jesse is female

We will not be concerned with the compositional derivation for the moment, and simply assume that something roughly equivalent to this is what is derived by the compositional semantics (for sub-sentential compositionality, see Section 7.1).

Adopting this framework, we propose the following general recipe for the meanings of quantifiers: a quantifier introduces an anaphoric term in the presupposition that is anaphorically related to the quantifier in the assertion. With an indefinite subject, we will therefore obtain (10).

(10) A student criticized herself.
    a. ASSERTIVE MEANING: There is a student x such that x criticized x
    b. PRESUPPOSITION: x is female

Here, the existential quantifier *a student* introduces an existentially quantified variable x in the assertive meaning, and an anaphoric term x in the presupposition. It is crucial that the same variable name is used in the two dimensions of meaning, which is meant to capture the idea that an anaphoric term is 'anaphorically related' to a quantificational term. We will show shortly how the occurrences of x in the assertion and presuppositions are ensured to co-vary, but to informally state the underlying intuition first, the assertive meaning says 'There is a student x who criticized x', and the presupposition says '*That same* x is female'. In other words, the two dimensions of meaning are likened to a discourse with cross-sentential anaphora with a quantificational antecedent, except that the anaphoric relation in this case is cross-dimensional, rather than cross-sentential.[8] Notice importantly that the resulting presupposition is 'non-universal' in the sense that only the student introduced in the assertive meaning is presupposed to be female, rather than every student is. As will be shown below, this will amount to an existential presupposition.

To repeat the core idea, the cross-dimensional relation between the quantifier in the assertive meaning and the anaphoric term in the presupposition is analogous to that of cross-sentential anaphora involving a quantificational antecedent like (11).

(11) A student is singing. He is loud.

Here, the pronoun *he* can refer back to the student that the quantifier *a student* in the first sentence introduces. As a result, the overall meaning of the sentence will be equivalent to that of *A student is singing and is loud*.[9] Our proposal is simply that the same mechanism that ensures the anaphoric link between *a student* and *he* in (11) is used for cross-dimensional anaphora in sentences like (10) above. This idea will allow us to straightforwardly capture the parallelism between these two phenomena, as we will examine more closely in Section 4.

Cross-sentential anaphora is a well studied topic in formal semantics and pragmatics, and we will borrow technical machinery from previous studies for the purposes of formalization. Approaches to cross-sentential anaphora can be broadly classified into two kinds: E-type approaches (Evans 1980, Heim 1990, Elbourne 2005) and dynamic semantics (Heim 1982, Kamp 1981, Groenendijk & Stokhof 1990, 1991, Kamp & Reyle 1993, Van den Berg 1996, Nouwen 2003a,b, 2007b, Brasoveanu 2007, 2010, Dekker 2012). Since the latter offers a rigorous treatment of plural cross-sentential anaphora and quantificational dependencies that will prove to be useful for our purposes, we will make use of a version of dynamic semantics as our meta-language.[10]

For the rest of this section, we will introduce a formal theory of presupposition projection with cross-dimensional anaphora that deals with sentences with singular indefinite quantifiers like (10). The key component of the proposal is the Context Update Rule that ensures the anaphoric relation between the quantificational term in the assertive meaning and the anaphoric term in the presupposition. We will first introduce a dynamic semantics as a theory of cross-sentential anaphora, and then use it to formulate the Context Update Rule.

[8] This constitutes our solution to the 'Binding Problem' that has plagued two-dimensional theories of presupposition (Karttunen & Peters 1979, Cooper 1983, Beaver 2001). Karttunen & Peters (1979) pointed out that if the presupposition of a sentence like (10) is analyzed as involving existential quantification as well, the resulting presupposition is going to be too weak. That is, (10) is predicted to be felicitous whenever there is a female student, and to be true when any student criticized themselves, regardless of their gender. In other words, such an analysis will fail to capture that the two dimensions of meaning are about the same individual. This was once considered to be a fatal problem for two-dimensional theories in general (Cooper 1983, Beaver 2001), but as demonstrated below, this conclusion is not warranted. See also Section 6.

[9] As with advocates of the major theories of cross-sentential anaphora, we accept this as a welcome result. However, Evans (1977, 1980), Van Rooij (2001), Slater (2000, 2006) among others point out that this might not the whole story. That is, intuitively, the following pair (can) have different meanings.

(i) a. There is a doctor in Manchester who is a woman.
b. There is a doctor in Manchester and she is a woman. (adopted from Evans 1977:508)

Standard dynamic semantics, including DPL and PPL, cannot distinguish these two sentences. While it would be an interesting extension of the present theory to try to incorporate this aspect of anaphora, we will leave it as a future task for the moment.

[10] This is of course not to deny the E-type approach. Alternative formulations are entirely left open here.

2.1 Dynamic Predicate Logic

A principled account of cross-sentential anaphora involving singular indefinites as antecedents is one of the earliest achievements of dynamic semantics. In this section, we will briefly review Groenendijk & Stokhof's (1991) Dynamic Predicate Logic (DPL), which will be used as the tentative metalanguage of our theory of presupposition projection. The reason for choosing this framework is primarily because of its simplicity. In particular, it has a very simple and intuitive syntax, which is identical to that of Predicate Logic (PL), which allows us to stay away from most of the formal details that do not concern us here.[11]

In order to see how DPL works, consider the example in (11), repeated here, under the reading where *he* refers to the student who is singing.

(11) A student is singing. He is loud.

In DPL, it is assigned the following formula as its meaning, where the same variable x is used throughout.

(12) $\exists x[\mathbf{linguist}(x) \wedge \mathbf{singing}(x)] \wedge \mathbf{loud}(x)$

In PL, the last occurrence of the variable x is not bound by the existential quantifier $\exists x$, and it fails to capture the intuition that *he* refers to the student who is singing. On the other hand in DPL, x will co-vary with the other occurrences of x, which is ensured by its semantics where each formula is interpreted as a relation between pairs consisting of an assignment function f and a possible world w.[12] More specifically, DPL sentences are interpreted in the following manner with respect to a first-order model $\mathcal{M} = \langle D, W, I \rangle$, where D is a non-empty set of individuals, W is a non-empty set of worlds such that $W \cap D = \varnothing$, and I is an interpretation function.

Definition 1 DPL Semantics

$$\begin{array}{rll} \langle f,w\rangle \| \exists x[\phi] \| \langle g,v\rangle & \text{iff} & \text{there is } h \text{ such that } f \text{ and } h \text{ differ at most in the value of } x, \\ & & \text{and } \langle h,w\rangle \| \phi \| \langle g,v\rangle \\ \langle f,w\rangle \| P(x_1,\ldots,x_n) \| \langle g,v\rangle & \text{iff} & w = v \text{ and } f = g \text{ and } \langle f(x_1),\ldots,f(x_n)\rangle \in I_w(P) \\ \langle f,w\rangle \| \phi \wedge \psi \| \langle g,v\rangle & \text{iff} & \text{there are } h \text{ and } u \text{ such that } \langle f,w\rangle \| \phi \| \langle h,u\rangle \\ & & \text{and } \langle h,u\rangle \| \psi \| \langle g,v\rangle \\ \langle f,w\rangle \| \neg\phi \| \langle g,v\rangle & \text{iff} & w = v \text{ and } f = g \text{ and there is no } \langle h,u\rangle \text{ such that } \langle f,w\rangle \| \phi \| \langle h,u\rangle \end{array}$$

What is crucial for our purposes is that this semantics validates the so-called *donkey equivalence*: $\langle f,w\rangle \| \exists x[\phi] \wedge \psi \| \langle g,v\rangle$ iff $\langle f,w\rangle \| \exists x[\phi \wedge \psi] \| \langle g,v\rangle$.[13] Applying this to (12), the following formula is semantically equivalent to (12).

(13) $\exists x[\mathbf{linguist}(x) \wedge \mathbf{singing}(x) \wedge \mathbf{loud}(x)]$

[11] For further details of DPL (and other dynamic systems), the interested reader is referred to Chierchia (1995), Nouwen (2003b), Brasoveanu (2007), Dekker (2012), as well as the original paper by Groenendijk & Stokhof (1991).

[12] For Groenendijk & Stokhof (1991), DPL formulae are interpreted as relations between assignment functions. We include possible worlds, anticipating an application to presupposition, which is essentially an intensional phenomenon (cf. Heim 1982, 1983).

[13] This is one side of the donkey equivalence. The other side says, $\langle f,w\rangle \| [\exists x[\phi]] \Rightarrow \psi \| \langle g,v\rangle$ iff $\langle f,w\rangle \| \forall x[\phi \Rightarrow \psi] \| \langle g,v\rangle$, which doesn't concern us here. Also as a corollary of the donkey equivalence, there is no need to assign a scope to $\exists x$ in DPL and we can regard it as a formula in itself: $\langle f,w\rangle \| \exists x \| \langle g,v\rangle$ iff $w = v$ and f and g differ at most in the value of x.

This formula has a meaning akin to what it would mean in PL, i.e. there is a student who is singing and loud, and hence all the occurrences of the variable x co-vary in (13). Since (12) and (13) are semantically identical, this is also the case in (12), which captures the reading we are after. We will make use of this very mechanism to enable an anaphoric link between the two dimensions of meaning of a quantified sentence.

2.2 Cross-Dimensional Anaphora and the Context Update Rule

Using DPL as our meta-language, let us analyze the meaning of the sentence in (10), repeated in (14) below. The assumption is that a sentence S is assigned as its meaning a pair of DPL formulae, $\alpha(S)$ and $\pi(S)$, representing the assertive meaning and presupposition, respectively.

(14) A student criticized herself.

a. $\alpha(14)$: $\exists x[\mathbf{criticized}(x, x)]$

b. $\pi(14)$: $\mathbf{female}(x)$

it is crucial that the same variable x is used in both dimensions, which we assume is ensured in syntax (see Section 7.1 for details). The question that immediately arises is how these representations are interpreted. To this end, we propose a Context Update Rule that takes two dimensions of meaning and alters the conversational context against which the sentence (14) is uttered. This rule enables the cross-dimensional anaphora between $\exists x$ in (14a) and x in (14b).

Following Heim (1982, 1983) among others, we model a conversational context (or common ground) by a set C of pairs $\langle f, w\rangle$ of an assignment function f and a possible world w. The update of a context C with a sentence ϕ proceeds in the following three steps.[14] We use the following notations here: $\mathrm{fnc}(C) := \{\, f \mid \exists w[\langle f, w\rangle \in C] \,\}$ and $\mathrm{wld}(C) := \{\, w \mid \exists f[\langle f, w\rangle \in C] \,\}$.

Definition 2 2D Context Update Rule

1. **Assertive Update**
   Update C with $\alpha(\phi)$ and obtain $C' = \{\, \langle f', w'\rangle \mid \exists\langle f, w\rangle \in C[\langle f, w\rangle \|\alpha(\phi)\| \langle f' w'\rangle] \,\}$.

2. **Presuppositional Felicity Check**
   Check if the worlds of the initial context $\mathrm{wld}(C)$ satisfy the presupposition 'in relation to C'', i.e. check if for each $w \in \mathrm{wld}(C)$, there is an updated assignment function $f' \in \mathrm{fnc}(C')$ such that $\langle f', w\rangle \|\pi(\phi)\| \langle g', v\rangle$ for some g' and v.
   If yes, proceed to Step 3; if no, presupposition failure (i.e. infelicity).

3. **Presuppositional Update**
   Update C' with $\pi(\phi)$ and obtain $C'' = \{\langle f'', w''\rangle | \exists\langle f', w'\rangle \in C[\langle f', w'\rangle \|\pi(\phi)\| \langle f'', w''\rangle]\}$

According to this rule, the context is updated with the assertive meaning first in Step 1, and then the presupposition is checked in Step 2. This order might look counter-intuitive at first, as the presupposition is commonly thought to precede the assertion (hence the name *pre*-supposition), but is crucial here in order for cross-dimensional anaphora to be possible. That is, the update with the assertive meaning in Step 1 yields those assignment functions f' that encode the anaphoric information of the assertive meaning. These assignment functions are used to evaluate the presupposition in Step 2.

[14] We thank an anonymous reviewer and Philippe Schlenker (p.c.) for pointing out problems of an earlier formulation, and Benjamin Spector (p.c.) and Guillaume Thomas (p.c.) for detailed discussions on the predictions of the current version.

In Step 2, the presupposition is evaluated against the worlds of the original context C, which captures the felicity condition of presuppositions that they need to be already part of the common ground when the sentence is uttered. Crucially, according to the present rule, the presupposition is not just checked against C but by making reference to C' so that the cross-dimensional anaphora is enabled. More specifically, in all the worlds of the original context C, the presupposition should already be true, *with respect to some assignment function* $f' \in \mathrm{fnc}(C')$. If Step 2 succeeds, then the intermediate context C' is updated with the presupposition $\pi(\phi)$, which yields the final context C''. If Step 2 fails, a presupposition failure ensues.

For a concrete illustration, let us take (14). Let C be the initial context such that it is not known in C whether any students criticized themselves, so that the assertion will be informative (Stalnaker 1973, 1974, 1978). Thus, in some of the worlds in C, some students criticized themselves, and in others none did. After the update with the assertive meaning (14a), the latter worlds are eliminated. Consequently, in each of the worlds of the intermediate context C', at least one student x criticized x. Each assignment function f' of C' assigns to x some particular student who criticized himself or herself. Notice that at this point there is no constraint on the gender. Now we move on to Step 2 and using these assignment functions f' in C', we check whether the original context C satisfies the presupposition 'in relation to C''. That is, for each world $w \in \mathrm{wld}(C)$, we must be able to make the presupposition true in w using some assignment function $f' \in \mathrm{fnc}(C')$. Assuming that it is known that there are some female students and at least one of them could have criticized herself, there must be such an f' for each world w. Then, in the final output context C'', we discard those assignments f' of C' that assign x a male individual. In other words, (14) is a presupposition failure just in case there is no student who could be a self-criticizing female student. Notice that the presupposition is effectively existential: for each $w \in \mathrm{wld}(C)$, there must be be some $f' \in \mathrm{fnc}(C')$ that assigns x an individual who is female and who criticized herself in some $w' \in \mathrm{wld}(C')$, which does not require every student to be female. This captures the intuitive meaning of (14) reasonably well.

Before proceeding, several remarks are in order. According to the Context Update Rule, the presupposition plays two roles: in Step 2, it imposes a felicity condition, and in Step 3, it updates the intermediate context C'. This dual role is not at all strange, given that the assertive meaning can also be regarded as having a felicity condition. That is, as is widely assumed implicitly or explicitly (Stalnaker 1973, 1974, 1978 among others), the assertive meaning is required to be informative with respect to the original context C , i.e. $C \neq C'$ (the assertive meaning is already known to be true) and $C \neq \varnothing$ (the assertive meaning is already known to be false). From this perspective, therefore, we can say that each dimension of meaning puts a felicity condition on C, and performs an update when the felicity condition is met. But the assertive meaning and presupposition put different kinds of felicity conditions, i.e. the former must be informative, while the latter must be non-informative. Also, the felicity condition of the presupposition is processed after the assertive meaning is processed and the felicity condition of the latter refers to both the original context C and the intermediate context C', which ensures cross-dimensional anaphora.

Also that the update with the presupposition in Step 3 is performed on the intermediate context C', and unlike the update with the assertive meaningin Step 1, it is allowed to be vacuous with respect to C'. That is, in a context where it is known that all the students are female, the final step is trivial in the sense that it doesn't eliminate any pair from C', i.e. $C'' = C'$, although in a different context, this can well be informative, i.e. $C'' \subset C'$, as in the case discussed above.

Finally, it should be remarked here that one might think the felicity condition is different from what is usually considered to be the presupposition of (14) by other authors such as Karttunen & Peters (1979), Heim (1983) and Beaver (2001), according to whom the presupposition is that there is a female student (in each world in C). Rather, it requires that there be a female individual who *could* be a self-criticizing student. The possibility modal 'could' in this paraphrase reflects the fact that the felicity condition says that the female individual in question be a self-criticizing student in *some world in* C'. Thus, it allows a situation where for some worlds in $w \in C$, there is an individual x who is female in w, but is a self-criticizing male individual in $w' \in C'$. Although it is beyond the scope of the present paper to empirically motivate the present analysis, we maintain that this prediction isat least empirically unproblematic. In particular, due to the update by presupposition in Step 3 (with respect to the intermediate context C'), the final context C'' necessarily ends up validating the implication that the student in question is female.[15] Thus, it is predicted that whenever felicitous, the sentence as a whole entails that the self-criticizing student is female. This means that the empirical difference between these two claims lies just in the felicity condition imposed by the presupposition. Unfortunately, it is not easy to appraise them in empirical terms, as the judgments are extremely subtle, and this point is left open for the moment.

2.3 Section Summary

To sum up, there are two main ingredients in the proposed theory. First, it is assumed that a quantifier introduces a quantificational term in the assertive meaning and an anaphoric term in the presupposition that stands in the relation of cross-dimensional anaphora with the quantificational term. Informally put, a sentence like *A student criticized herself* presupposes "*that individual* is female." Second, the Context Update Rule enables cross-dimensional anaphora by requiring the presuppositions' felicity condition to refer to both the original context C and the intermediate context C'.

In this section we employed DPL to achieve this result in formal terms, but one limitation of this is that its expressive power is limited and cannot account for other quantifiers than singular indefinites.[16] In Section 4 the theory presented here will be enriched so as to deal with a number of quantifiers. Although the enrichment is not trivial, the core idea will stay intact: a quantifier introduces a quantificational term in the assertive meaning and an anaphoric term in the presupposition which stand in the relation of cross-dimensional anaphora ensured by the Context Update Rule. Before embarking on this, we will convince ourselves of the general correlation between cross-sentential and cross-dimensional anaphora.

3 Cross-Dimensional and Cross-Sentential Anaphora

In the present section, we will demonstrate with concrete examples that presupposition projection and cross-sentential anaphora are related in the manner suggested in the generalization in (4), repeated below.

(4) Consider a discourse of the following form '$Q(R)(F)$. PRONOUN G.', where $Q(R)$ is a quantifier with a restrictor R, F and G are arbitrary predicates, and PRONOUN is a pronoun with appropriate phi-features, e.g. *they*.

[15] I thank an anonymous reviewer for drawing my attention to this particular point, which was problematic for an earlier formulation.

[16] Singular unselective quantifiers are available in DPL and other classical dynamic semantics, which involve quantification over assignments, but they are known to give rise to independent problems, most notably the 'Proportion Problem' of donkey anaphora (Kadmon 1987, Rooth 1987, Heim 1990, Chierchia 1995, Brasoveanu 2007, 2010). In PPL that we will introduce in Section 4 will allow us to incorporate singular and plural selective quantifiers. See Brasoveanu (2007) for detailed discussion on this.

a. If PRONOUN can be resolved to R (i.e. $Q(R)$ supports maxset anaphora), then sentences of the form $Q(R)(\lambda x.S(x)_{p(x)})$ can have a universal inference that $\forall x \in R : p(x)$, and
b. If PRONOUN can be resolved to $R \cap F$ (i.e. $Q(R)$ supports refset anaphora), then sentences of the form $Q(R)(\lambda x.S(x)_{p(x)})$ can have an inference weaker than a universal inference.

We will see how the theory of cross-dimensional anaphora captures this parallelism. Since the formal theory presented in Section 2 is not expressive enough to deal with most of the quantifiers discussed in the present section, the exposition here will be kept at an informal level, and a full formal theory will be offered in the next section. Also, we will only discuss unembedded quantified sentences, as projection involves some complications, to which we will come back in Section 7.1.

Recall how quantifiers are analyzed in the present theory: they introduce a quantificational term in the assertive meaning and an anaphoric term in the presupposition. Schematically, for a quantificational determiner Q, its restrictor R and a presuppositional nuclear scope S with a presupposition p,

(15) $Q(R)(S_p)$
a. ASSERTIVE MEANING: $Q(R)(S)$
b. PRESUPPOSITION: x satisfies the presupposition p

The variable x in the presupposition will be resolved to the quantifier in the assertive meaning. We saw in the previous section how this works with a singular indefinite *a student*. That is, x is resolved to the student mentioned in the assertive meaning in the manner similar to how cross-sentential anaphora with a singular indefinite is resolved. In what follows we will apply this general recipe to other kinds of quantifiers. There are generally two types of cross-sentential anaphora to consider, namely maxset anaphora (anaphora to the maxset R) and refset anaphora (anaphora to the refset $R \cap S$), and it depends on the quantifier which type of anaphora is possible cross-sententially. For example, singular indefinites generally only license refset anaphora, and as we saw in the previous section, this amounts to an existential presupposition due to the mechanism of cross-dimensional anaphora. We now demonstrate that when applied to other quantifiers, the same analysis straightforwardly explains the parallelism between cross-sentential anaphoric potentials and presuppositional properties of quantifiers stated in (4).

Let us begin with universal quantifiers. As briefly discussed at the outset, they give rise to universal inferences. For instance, (16) presupposes that every student is female.

(16) Every student criticized herself.

Given that the gender inference of *criticized herself* is purely presuppositional (cf. fn.1), the universal inference that every student is female should be due to the projection property of *every student*. We account for this with cross-dimensional anaphora as follows. Applying (15) to the present case, the two dimensions of meaning for (16) are analyzed as (17).

(17) a. ASSERTIVE MEANING: Every student x criticized x
b. PRESUPPOSITION: x is female

How is the variable x in the presupposition resolved? Our answer is that it depends on the cross-sentential anaphoric potential of *every*. That is, we liken the two dimensions of meaning in (17) to two separate sentences occurring in a discourse like (18).

(18) Every student passed the test. They were happy.

The pronoun *they* in this discourse refers to all the students, or in other words, the quantifier *every student* supports maxset anaphora.[17] According to the present analysis, the same mechanism of anaphora resolution is at work in the case of (17), except that it operates cross-dimensionally rather than cross-sententially. Specifically, assuming that the variable x in (17b) is number neutral, the predicted presupposition will be that x is resolved to all the students, and hence the resulting presupposition is that all of the students are female. Notice that this presupposition is 'universal' in that it requires every individual in the restrictor to satisfy the presupposition of the predicate *criticized herself*, i.e. every student is female. This is a desirable prediction.

Furthermore, since other universal quantifiers like *each of the students* and *all the students* also support maxset anaphora, as shown in (19), we predict that they also should give rise to universal inferences. This is a correct prediction as witnessed by (20).

(19) Each of the students/All the students passed the test. They are happy.

(20) a. Each of the students criticized herself.
b. All the students didn't stop smoking.

While (20a) is exactly parallel to (18), (20b) merits some discussion. Since English does not have a gendered plural pronoun, a different presupposition trigger, *stop*, is used in this example. It additionally contains negation, and we are interested in the surface scope reading where *all the students* takes wide scope. Focusing on this reading, observe that (20b) is associated with a universal inference that all the students used to smoke. Importantly, this universal inference is not entailed by the assertive meaning, which is that all the students are either still smoking or never smoked (Sudo 2012; see also fn.1 for relevant discussion). Thus, the projection property of the quantifier should be responsible for the universal inference.

Let us now turn to *no student*. As remarked in the introduction, a negative quantifier like *no student* tends to give rise to a universal inference as well.[18] This is illustrated by the sentence in (21), which suggests that every relevant student is female.

(21) No student criticized herself.

According to the present account, this universal inference should be due to the cross-sentential anaphoric property of *no student*. More precisely, (21) is analyzed as (22).

(22) a. ASSERTIVE MEANING: No student x criticized x
b. PRESUPPOSITION: x is female

What does x resolve to in this case? As demonstrated by (23) below, *no student* only supports maxset anaphora cross-sententially (cf. Kamp & Reyle 1993, Nouwen 2003a,b, 2007a,b).

[17] As noted in fn.5, we reserve the term 'refset anaphora' for anaphora to a set strictly smaller than the restrictor.

[18] But we do not exclude the possibility of 'local accommodation' of the presupposition at the VP level (Heim 1983, Van der Sandt 1992, Beaver 2001, Kadmon 2001, Beaver & Zeevat 2007), in which case the universal inference does not obtain. That is, the resulting reading will be: No students are female self-criticizer. This reading is hard to obtain with this particular sentence (presumably due to the presupposition trigger *herself*), but similar readings are attested with other presupposition triggers (see Klinedinst 2010 for related discussion). We assume that local accommodation is due to a separate mechanism and hence leave it undiscussed here. Relatedly, for a universal quantifier, a reading with local accommodation still entails the universal inference. This might account for Chemla's (2009a) experimental results that negative quantifiers are associated with universal inferences relatively less robustly than universal quantifiers.

(23) No student passed the test. They were unhappy.

The pronoun *they* in the second sentence of (23) is read as all the relevant students. Just as in the case of a universal quantifier, therefore, (22b) amounts to the universal presupposition that every student is female. The exact same pattern obtains with other negative quantifiers like *none of the students*.

Turning next to indefinites, recall that unlike *every student* and *no student*, *a student* does not give rise to a universal inference. The present analysis nicely captures this contrast. That is, *a student* does not support maxset anaphora, unlike *every student* and *no student*. Rather it supports refset anaphora. This is illustrated in (24). In order to avoid so-called singular *they*, we will use an inanimate antecedent.

(24) An analysis was offered.
  a. It was convincing. (it = the analysis that was offered)
  b. #They were convincing. (they = all analyses)

Correspondingly, in a sentence like the following, the presupposition is predicted to be about the relevant individual in the refset, rather than all the individuals in the maxset.[19] Thus, the presupposition will not be about all students but about just one student who self-criticized.

(25) A student criticized herself.
  a. ASSERTIVE MEANING: there is a student x who criticized x
  b. PRESUPPOSITION: x is female

As already demonstrated in the previous section, this amounts to an existential presupposition.

So far, we have only considered those quantifiers that only allow, or at least strongly prefer, either maxset anaphora or refset anaphora, but there are also quantifiers that support both types of cross-sentential anaphora. For example, unlike *a NP*, partitive existential quantifiers support maxset anaphora, as well as refset anaphora. This is illustrated by (26).

(26) One of the students passed the test.
  a. He was happy.
  b. They were all happy.

Correspondingly, we expect the following sentence (27) to be ambiguous between a reading with a universal presupposition and a reading with a weaker presupposition.

(27) One of the students criticized herself.

This prediction indeed seems to be borne out, as suggested by the experimental data reported in Chemla (2009a) and Sudo, Romoli, Hackl & Fox (2011). Furthermore, it also generalizes to other kinds of partitive quantifiers, such as the following:

(28) a. Most of the students stopped smoking.

[19]Generally, there can be more then on individual in the refset for sentences like (24), e.g. maybe multiple analyses were offered, but the speaker is using a singular phrase because there's a particular one that they have in mind. Pronominal anaphora generally keeps track of such referential intention of the speaker, and many versions of dynamic semantics, including the one we adopt here, can handle it (although we are not explicit about the meaning of number morphology in this paper, to avoid unnecessary complexity). Roughly put, the first sentence (24) introduces a singular individual that is an analysis that has been offered, to which only a singular pronoun can refer *it*. I thank Robert van Rooij (p.c.) for highly helpful comments on this issue.

b. Few of the students stopped smoking.
c. Many of the students stopped smoking.
d. Less than 30% of the students stopped smoking.
e. At least three of the students stopped smoking.
f. At most five of the students stopped smoking.

Take (28a). The quantifier *most of the students* also supports both types of cross-sentential anaphora.

(29) Most of the students criticized John. They are not so smart.
a. They = the students who criticized John (Maxset Anaphora)
b. They = all the students (Refset Anaphora)

Thus, we predict that (28a) can optionally have a universal presupposition. Again, as suggested by Chemla's (2009a) experimental data, this seems to be on the right track. That is, (28a) is associated with a universal inference less robustly than *every student* or *no student*, but much more robustly than *a student*.

Although the judgments might be very subtle with partitive quantifiers, the predictions of our theory are clear: the robustness of the universal inference in presuppositional sentences like (28) should be correlated with the robustness of maxset anaphora cross-sententially. According to the experimental data reported in the previous studies mentioned above, these predictions seem to be largely correct, although in order to evaluate the theory more thoroughly, further carefully designed experimental research is needed. In particular, those ambiguous quantifiers conceivably prefer maxset anaphora to different degrees, and if so, what is predicted is a correlation between the two gradient semantic judgments, rather than categorial distinctions, which is arguably too hard to estimate by informal introspection alone. We will leave this issue for future research.[20]

4 Incorporating Generalized Quantifiers

In the previous section, we saw with concrete data that presupposition projection in quantified sentences and cross-sentential anaphora with quantificational antecedents are closely related. The main purpose of this section is to formalize our analysis of cross-dimensional anaphora that directly captures this correlation. The core idea is the same as in Section 2, namely that a quantifier introduces an anaphoric term in the presupposition, which refers back to the relevant individual or individuals in the assertive meaning, and the Context Update Rule ensures the cross-dimensional anaphoric relation between them. The theory based on DPL in Section 2 was, however, only capable of singular indefinite quantifiers like *a student*, since DPL does not contain plural (selective) generalized quantifiers. Fortunately, generalized quantifiers have been implemented in dynamic semantics by many authors (Van Eijck & De Vries 1992, Chierchia 1995, Van den Berg 1996, Nouwen 2007a,b, Brasoveanu 2007, 2010). We will borrow necessary machinery from Van den Berg's (1996) extension of DPL called Plural Predicate Logic (PPL). We have nothing to add to PPL itself as a theory of cross-sentential anaphora. Rather, the novelty of our theory consists in the claim that the same anaphoric mechanism is responsible for presupposition projection.

[20] I would like to thank an anonymous reviewer for comments on this issue. As the same reviewer suggests, a stochastic theory is perhaps ultimately necessary to account for the data, but it is left open here whether it should be part of the underlying discourse logic, or more of a matter of (arguably ill-understood) pragmatic factors such as saliency, attention, etc.

This section proceeds as follows. The core components of PPL will be introduced in Section 4.1 and we will show how our theory augmented with PPL accounts for presupposition projection through various generalized quantifiers in Section 4.2. In Section 4.3, we will suggest that the current theory can be augmented with Van den Berg (1996) analysis of quantificational dependency to account for presupposition projection through multiple quantifiers, although a concrete implementation is left for another occasion.

4.1 Generalized Quantifiers in Dynamic Semantics

DPL as formulated by Groenendijk & Stokhof (1991) does not contain plural generalized quantifiers, but later authors offered several ways to implement them in dynamic semantics (Van Eijck & De Vries 1992, Chierchia 1995, Van den Berg 1996, Nouwen 2007a,b, Brasoveanu 2007, 2010).[21] For the sake of formalization, we will make use of Van den Berg's (1996) Plural Predicate Logic (PPL), which is an extension of DPL. The main advantage of this analysis is that both the maxset and refset of generalized quantifiers are explicitly represented.[22]

As PPL is considerably more complicated than DPL, we will introduce it step-by-step (mirroring Van den Berg's 1996:Ch.3 original exposition). The first ingredient we need is plurality, and we will introduce a way to combine the semantics of plurality and the classical theory of generalized quantifiers in a completely static fashion (Section 4.1.1). Then, we will dynamicize the resulting analysis of plural and singular generalized quantifiers, which we will use as our metalanguage in place of DPL (Section 4.1.2).

4.1.1 Generalized Quantifiers and Plurality

The classical theory of generalized quantifiers (Barwise & Cooper 1981, Keenan & Stavi 1986, Peters & Westerståhl 2006) does not take plurality into consideration, and treats all generalized quantifiers as relations between sets of individuals. However, it is evident that quantifiers can be singular or plural—e.g. *every student* vs. *all the students*—and their differences need to be accounted for. In particular, singular and plural quantifiers are known to exhibit different behavior with respect to collective predication (Van den Berg 1996, Schwarzschild 1996, Landman 2000, Winter 2001). Generally, singular quantifiers can only have distributive readings, while plural quantifiers allow collective readings. Thus, singular quantifiers are incompatible with inherently collective predicates like *gathered*, as shown below (see Schwarzschild 1996, Brisson 2003, Winter 2001 for complications).

(30) a. *Every student gathered.
     b. All the students gathered.

In order to account for plural predication, we need to introduce plural individuals in our ontology. Following Van den Berg (1996), Schwarzschild (1996), Winter (2001) and many others, we assume that plural individuals are simply sets of individuals, and singular individuals are singleton sets.[23] In this setting, predicates like $\mathtt{students}$ are interpreted as sets of sets of in-

[21] As mentioned in fn.16, singular unselective quantifiers, which are relations between sets of assignment functions, were available in the earlier works of dynamic semantics (Kamp 1981, Heim 1982, Groenendijk & Stokhof 1990, 1991). For our purposes, however, selective generalized quantifiers over individuals are crucial.

[22] See Kamp & Reyle (1993) and Nouwen (2003b, 2007b) for other dynamic systems with plural selective quantifiers.

[23] It should be stressed that nothing crucial hinges on this assumption. Link (1983) presents philosophical arguments for treating plural individuals as non-sets, but as Landman (1989) claims, the two theories are isomorphic and their ontological differences do not matter much for semantic analyses (see also Van den Berg 1996, Schwarzschild 1996, Landman 2000, Winter 2001 for relevant discussion).

dividuals, rather than simply sets of individuals. Correspondingly, quantificational determiners take two such predicates, so their meanings are relations between sets of sets of individuals, unlike in the classical generalized quantifier theory where they are analyzed as relations between sets of individuals.

How do we analyze quantifiers as relations between two sets of sets of individuals? Van den Berg (1996) provides an ingenious way of defining plural generalized quantifiers using their meanings in the classical generalized quantifier theory. His idea is roughly as follows. A plural quantifier $\mathcal{Q}$ takes two sets of sets of individuals, $\mathcal{R}$ and $\mathcal{S}$, but the meaning of $\mathcal{Q}$ can be described using the corresponding classical generalized quantifier Q applied to the 'representative sets' of individuals in $\mathcal{R}$ and $\mathcal{S}$. Now the question is, which sets are representative sets of a given plural predicate (i.e. a set of sets of individuals)? According to Van den Berg, they are the maximal sets of individuals. He defines an operator $\mathbf{max}$ that picks out the maximal sets of individuals from a predicate:

(31) $\mathbf{max}(\mathcal{X}) := \{ A \in \mathcal{X} \mid \neg\exists B \in \mathcal{X}[A \subset B] \}$

For inherently distributive predicates like $\mathbf{students}$, there is a unique maximum set (as its denotation is a semi-lattice with respect to $\subseteq$), but for predicates like $\mathbf{gather}$ and $\mathbf{lifted.the.piano}$ that are not inherently distributive, there can be more than one maximal set.[24] More concretely, when there are two gatherings by $\{ a, b, c \}$ and $\{ e, f, g \}$, $\mathbf{gather} = \{ \{ a, b, c \}, \{ e, f, g \} \}$. Then, both of these sets are maximal sets.

In the context of presupposition projection, we are for the most part interested in distributive predication (with presuppositional predicates being inherently distributive such as $\mathbf{former.smoker}$ and $\mathbf{female}$), since presupposition projection is more or less straightforward with collective presuppositional predicates like *stopped meeting* (we will come back to this below). This will allow us to simplify the discussion considerably, as we can practically assume that there always is a unique maximal set.

Now using the $\mathbf{max}$-operator, Van den Berg (1996) shows a way to derive distributive and collective readings of plural generalized quantifiers based on classical generalized quantifiers such as (32) (where A and B here are sets of individuals).

(32) *Classical Generalized Quantifiers*

a. $\mathbf{all}(A, B) :\Leftrightarrow A \subseteq B$

b. $\mathbf{some}(A, B) :\Leftrightarrow A \cap B \neq \emptyset$

c. $\mathbf{exactly.three}(A, B) :\Leftrightarrow |A \cap B| = 3$

Schematically, the interpretation of $\mathcal{Q}(\mathcal{R})(\mathcal{S})$ for a plural quantificational determiner $\mathcal{Q}$ with two plural predicative arguments $\mathcal{R}$ and $\mathcal{S}$ will look as (33).

(33) $\exists A \in \mathbf{max}(\mathcal{R}) \exists B \in \mathbf{max}(\wp(A) \cap \mathcal{S})[Q'(A, B)]$

Here Q' is the corresponding classical generalized quantifier like (32). The first existential quantifier introduces a maximal set A of $\mathcal{R}$, and the second existential quantifier introduces a maximal set B of $\mathcal{S}$ that consists only of members of A (this embodies the conservativity universal; see Van den Berg 1996 for details). Or in other words, A is the maxset and B is the refset. Then the classical generalized quantifier Q' is applied to A and B.

The reading represented in (33) is a collective reading. That is, the refset B is a member of $\mathcal{S}$ (and a subset of A), but it is not required that each member of B should also satisfy the

[24] Van den Berg (1996) notes that while in principle allowed, cases that have no maximal sets are rare in real natural language examples. We will also ignore such cases in the present paper.

predicate. More concretely, consider (34a), whose interpretation will look like (34b).

(34) a. Exactly three students gathered.
b. $\exists A \in \mathbf{max}(\mathbf{students})\exists B \in \mathbf{max}(\wp(A) \cap \mathbf{gather})[\mathbf{exactly}.\mathbf{three}(A, B)]$

Recall that $\mathbf{students}$ is assumed to be sets of sets that are consisting only of students. Since this is an inherently distributive predicate, A will be the biggest set in $\mathbf{students}$, i.e. the set containing all the students. On the other hand, B will be the biggest set of students that gathered. Notice that it's not required that for each $b \in B$, $\{b\} \in \mathbf{gather}$ (and in fact singleton sets are never members of inherently collective predicates like $\mathbf{gather}$), so this is a collective reading. In the last conjunct, $\mathbf{exactly}.\mathbf{three}(A, B)$ says that the cardinality of $A \cap B$—which is incidentally equivalent to B due to conservativity—is exactly three.

Based on the reading represented in (33), Van den Berg (1996) proposes to derive distributive readings of plural generalized quantifiers using the distributivity operator δ defined as follows.[25]

(35) $\delta(\mathcal{X}) := \wp(\{\, a \mid \{\, a \,\} \in \mathcal{X} \,\})$

The function of this operator is to create a semi-lattice out of $\mathcal{X}$ based on all singleton individuals in $\mathcal{X}$. Therefore, if $\mathcal{X}$ already has such a structure, i.e. it is inherently distributive, its effect is redundant, e.g. in the case of $\mathbf{students}$, while for other predicates like $\mathbf{lifted}.\mathbf{the}.\mathbf{piano}$ that are neither inherently distributive or inherently collective, the presence of the operator has significant effects. For an illustration, consider the distributive reading of (36a), which is analyzed as (36b).

(36) a. Exactly three students lifted the piano.
b. $\exists A \in \mathbf{max}(\delta(\mathbf{students}))\exists B \in \mathbf{max}(\delta(\wp(A)\cap\mathbf{lifted}.\mathbf{the}.\mathbf{piano}))[\mathbf{exactly}.\mathbf{three}(A, B)]$

In words, A is the biggest set of individuals whose members are all students, and B is the biggest subset of A that only contains students that lifted the piano individually, and finally $\mathbf{exactly}.\mathbf{three}(A, B)$ says that the cardinality of $A \cap B$—which is equivalent to B due to conservativity—is exactly three. Notice that the first occurrence of δ is semantically null, as $\mathbf{students}$ is inherently distributive, whereas the second occurrence of δ has a non-trivial effect. That is, it requires each member of B to have lifted the piano.

Finally, consider an example containing a singular quantifier *each student*. On the assumption that it always comes with δ, the distributive reading is the only available reading.

(37) a. Each student lifted the piano.
b. $\exists A \in \mathbf{max}(\mathbf{students})\exists B \in \mathbf{max}(\delta(\wp(A) \cap \mathbf{lifted}.\mathbf{the}.\mathbf{piano}))[\mathbf{each}(A, B)]$

A is the set containing all the students, and due to the δ-operator, B is the set of students each of whom lifted the piano. As a consequence, singular quantifiers are not compatible with collective predicates whose extensions do not include singleton sets, by assumption.

To summarize, wehave added plural individuals, which are sets of individuals, in the ontology, and analyzed plural quantifiers in terms of classical generalized quantifiers. Specifically, using the $\mathbf{max}$-operator, plural quantifiers introduce the maxset and refset, and apply the classical generalized quantifier to them. Distributive readings are derived using the δ-operator. In the next subsection, we will dynamicize this analysis of plural generalized quantifiers.

[25] Van den Berg (1996) also discusses 'pseudo-distributive' readings where the distribution is not down to the atomic individuals. We will not deal with this reading of plural quantifiers in this paper. See also Schwarzschild (1996), Landman (2000) and Winter (2001) for relevant discussion.

4.1.2 Plural Predicate Logic

Recall that what we want is a dynamic theory of plural generalized quantifiers that we can extend to cross-dimensional anaphora for presupposition projection, but the theory of plural generalized quantifiers introduced in the previous section is a static semantics, so we need to dynamicize it. To this end, we will adopt Van den Berg's (1996) Plural Predicate Logic (PPL). The syntax of PPL is essentially the same as DPL, and its semantics is also very similar, as shown below, except for the crucial difference that assignment functions now assign sets of individuals, rather than mere individuals, to variables.

Definition 3 PPL Semantics

$$\begin{array}{rll} \langle f, w\rangle \| \exists x[\phi] \| \langle g, v\rangle & \text{iff} & \text{there is } h \text{ such that } f \text{ and } h \text{ differ at most in the value of } x, \\ & & \text{and } \langle h, w\rangle \| \phi \| \langle g, v\rangle = 1 \\ \langle f, w\rangle \| P(x_1, \ldots, x_n) \| \langle g, v\rangle & \text{iff} & w = v \text{ and } f = g \text{ and } \langle f(x_1), \ldots, f(x_n)\rangle \in I_w(P) \\ \langle f, w\rangle \| \phi \wedge \psi \| \langle g, v\rangle & \text{iff} & \text{there are } h \text{ and } u \text{ such that } \langle f, w\rangle \| \phi \| \langle h, u\rangle = 1 \\ & & \text{and } \langle h, u\rangle \| \psi \| \langle g, v\rangle = 1 \\ \langle f, w\rangle \| \neg\phi \| \langle g, v\rangle & \text{iff} & \text{it's not the case that } \langle f, w\rangle \| \phi \| \langle g, v\rangle \end{array}$$

Within this framework, Van den Berg (1996) offers the following definitions for the dynamic versions of the key operators introduced above. Notice that these operators now take a formula, rather than a predicate, as their argument, but with the help of substitution, we can achieve the same meaning. We write $\phi[x/y]$ to mean the formula obtained from ϕ by simultaneously replacing all occurrences of x in ϕ with y such that y is free in ϕ.

(38) a. $\mathbf{max}_x(\phi) := \neg\exists y(x \subset y \wedge \phi[x/y]) \wedge \phi$
b. $\delta_x(\phi) := \forall y \subset x(\mathbf{singleton}(y) \rightarrow \phi[x/y])$

In words, $\mathbf{max}_x(\phi)$ says that x denotes a maximal set satisfying ϕ (again recall that variables now denote sets of individuals), and $\delta_x(\phi)$ says that all the singleton subsets y of x satisfy ϕ.

In this setting, the distributive readings of plural generalized quantifiers $\mathcal{Q}$ look schematically as follows.[26]

(39) $\|\mathcal{Q}x(\phi, \psi)\| := \exists x \exists y[\mathbf{max}_y(\delta_y(\phi[x/y])) \wedge \mathbf{max}_x(\delta_x(x \subseteq y \wedge \psi)) \wedge Q'(y, x)]$

Here, y represents a maximal set of individuals that satisfies the restrictor ϕ, and x represents a maximal subset of y whose members satisfy the nuclear scope ψ. Q' is the dynamic version of the corresponding classical generalized quantifier, e.g. (40).

(40) a. $\langle f, w\rangle \|\mathbf{all}(y, x)\| \langle g, v\rangle$ iff $f = g$ and $w = v$ and $f(y) \subseteq f(x)$
b. $\langle f, w\rangle \|\mathbf{some}(y, x)\| \langle g, v\rangle$ iff $f = g$ and $w = v$ and $f(y) \cap f(x) \neq \varnothing$
b. $\langle f, w\rangle \|\mathbf{exactly.three}(y, x)\| \langle g, v\rangle$ iff $f = g$ and $w = v$ and $|f(y) \cap f(x)| = 3$

For an illustration, let us consider (41).

(41) Exactly three of the students are former smokers.

[26] PPL handles singular generalized quantifiers, such as *each* and *every*, which always forces distributive readings, with an extra conjunct $\mathbf{singleton}(x)$.

(i) $\|\mathcal{Q}x(\phi, \psi)\| := \exists x \exists y[\mathbf{max}_y(\phi[x/y]) \wedge \mathbf{max}_x(x \subseteq y \wedge \mathbf{singleton}(x) \wedge \psi) \wedge Q'(y, x)]$

The conjunct $\mathbf{singleton}(x)$ forces each member of x to satisfy the nuclear scope ψ. However, as explained above, we are only interested in distributive readings here and its effect is vacuous when δ is present. For this reason we will ignore this in what follows.

In PPL, this sentence is given the following representation (as before, the δ-operators here are redundant given that the predicates in this sentence are inherently distributive).

(42) $$\exists x \exists y \left[\begin{array}{l} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{exactly.three}(x, y) \end{array} \right]$$

In words, y is the set of all the relevant students, and x is the maximal subset of it such that each of its members is a former smoker. In the final clause, it is expressed that the cardinality of $x \cap y$ (which is equivalent to y) is exactly three. This captures the meaning of (41).[27]

Now, we wanted this dynamic semantics to be able to account for cross-sentential anaphora, and it in fact does for the same reason as in DPL. That is, x and y in (42) are bound by existential quantifiers, and as in DPL, PPL validates the donkey equivalence. In PPL, x and y are sets of individuals, rather than individuals, but they can still be referred back to in a following discourse. For instance, suppose that (43) is uttered after (41).

(43) They don't drink at all.

The pronoun *they* here can be resolved to y, in which case it will denote all the students (i.e. maxset anaphora), or to x, in which case it will denote the three students who are former smokers (i.e. refset anaphora).[28] Our theory deploys this very mechanism of cross-sentential anaphora for cross-dimensional anaphora to account for presupposition projection through generalized quantifiers.

Other quantificational determiners can be dealt with in essentially the same manner, by changing the last conjunct of (42) to the appropriate (dynamicized) classical generalized quantifier meaning.[29] With a universal quantifier, the refset x is asserted to be identical to the maxset y, and hence maxset anaphora and refset anaphora result in the same reading.

[27] Notice that we have not yet incorporated presupposition here. We will claim in the next subsection that the meaning that y is the set of students here is not asserted but presupposed for (41), due to the partitive structure.

[28] The refset anaphora reading here is a so-called maximality reading: the pronoun refers to the maximal set of former smokers among the students. It is known that determiners including unmodified numerals give rise to a non-maximal reading (Van den Berg 1996, Brasoveanu 2007, 2010). Consider (i).

(i) Three students are former smokers. They were not very happy.

Under the refset anaphora reading, *they* here refers to three students who are former smokers, but this does not necessarily imply that they are the only former smokers among the students, unlike in the case of (41). This can be captured by analyzing *three students* as a plural indefinite, rather than a plural generalized quantifier. For instance, the meaning of the first sentence of (i) under the relevant reading will look as (ii).

(ii) $\exists x\, [(\delta_x(\mathbf{students}(x))) \wedge (\delta_x(\mathbf{former.smokers}(x))) \wedge \mathbf{three}(x)]$

It seems to us that this reading is an optionally available one, and the maximal reading is also a possibility, in which case the non-maximal reading should be treated as stemming from a systematic ambiguity of bare numeral phrases. This issue is, however, essentially orthogonal to the theory of presupposition projection put forward here, and we will not discuss it further in this paper. We thank Philippe Schlenker (p.c.), Benjamin Spector (p.c.) and Robert van Rooij (p.c.) for highly helpful discussion on this topic.

[29] Van den Berg (1996) claims that downward monotonic quantifiers should be analyzed as negations of upward monotonic quantifiers. For instance, according to him, the meaning of (ia) should be represented as (ib).

(i) a. Fewer than three students are former smokers.

b. $$\neg \exists x \exists y \left[\begin{array}{l} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{three}(y, x) \end{array} \right]$$

(44) a. All the students are former smokers.

b. $$\exists x \exists y \begin{bmatrix} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{all}(y, x) \end{bmatrix}$$

In the case of *no student*, the refset x is asserted to be $\varnothing$ (see the discussion in fn.29 for an alternative treatment).

(45) a. No students are former smokers.

b. $$\exists x \exists y \begin{bmatrix} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{no}(y, x) \end{bmatrix}$$

Assuming that a pronoun may not refer to $\varnothing$, the only possible reading will be the reading where it is resolved to the maxset y, i.e. maxset anaphora.

4.2 Cross-Dimensional Anaphora with Generalized Quantifiers

Adopting the dynamic treatment of generalized quantifiers introduced in the previous subsection, we will now propose a modification of our fragment given in Section 2. Consider the following sentence containing a presupposition trigger *stopped*.

(46) Exactly three of the students stopped smoking.

Let us assume that the assertive meaning of this sentence is (47), which is analogous to (42) above.[30]

(47) $$\exists x \exists y \begin{bmatrix} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{exactly.three}(y, x) \end{bmatrix}$$

In addition to this assertive meaning, the sentence has a presupposition. We assume that for (46), the following two presuppositions are available (the δ-operator is idle due to the inherent distributivity of $\mathbf{used.to.smoke}$, but is added here to emphasize that we are interested in a purely distributive reading).

(48) a. $\delta_x(\mathbf{used.to.smoke}(x))$ (REFSET PRESUPPOSITION)
b. $\delta_y(\mathbf{used.to.smoke}(y))$ (MAXSET PRESUPPOSITION)

The intuition here is that a quantifier introduces two individuals, y and x, which correspond to the maxset and refset respectively, and the presupposition in principle can be about either of them. Thus (48a) corresponds to cross-dimensional refset anaphora and gives rise to a non-universal inference, while (48b) corresponds to cross-dimensional maxset anaphora and gives rise to a universal inference.

In order to interpret (46) with these assertive meaning and presupposition, we do not need to modify the Context Update Rule. More concretely, the initial context C is first updated with the assertive meaning (47), yielding an intermediate context C' where each $\langle f', w' \rangle \in C'$ is such

The main motivation for this treatment of downward monotonic quantifiers has to do with non-distributive readings with multiple maxima. However, as discussed by Spector (2014), the reading predicted by (i) might be too strong. Since our focus here is distributive readings, we will not delve into this complication in the present paper.

[30]See Sudo (2012) for empirical arguments to assign to *stopped smoking* an assertive meaning that entails the presupposition that the subject used to smoke.

that $f'(y)$ is the set of all the students in w' and $f'(x)$ is a set consisting of three students each of whom is a former smoker in w'. Assuming that the presupposition is (48b), we check whether each world w of the original context C, there is $f' \in \mathrm{fnc}(C')$ that satisfies the requirement that each member of $f'(y)$ used to smoke in w, i.e. all the students are known to have smoked before. This is a universal inference. On the other hand if the presupposition is (48a), there needs to be $f' \in \mathrm{fnc}(C')$ for each world $w \in \mathrm{wld}(C)$ such that each member of $f'(x)$, which are some particular three students, smoked in w. This amounts to an existential inference, just as in the case of *a student* we saw in Section 2. (Step 3 will be vacuous for this example, because the presupposition is entailed by the assertive meaning)

It should be mentioned at this moment that it is more realistic to analyze the partitive quantifier *exactly three of the students* as involving some sort of presupposition. Following many authors working in dynamic semantics, we analyze the restrictor to be anaphoric, rather than existentially quantified. This idea can be implemented as in (49) for the sentence (46).

(49) a. ASSERTIVE MEANING: $\exists x \begin{bmatrix} \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former}.\mathbf{smokers}(x))) \wedge \\ \mathbf{exactly}.\mathbf{three}(x, y) \end{bmatrix}$

b. REFSET PRESUPPOSITION: $\mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \delta_x(\mathbf{used}.\mathbf{to}.\mathbf{smoke}(x))$

c. MAXSET PRESUPPOSITION: $\mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \delta_y(\mathbf{used}.\mathbf{to}.\mathbf{smoke}(y))$

The first conjunct of the presupposition requires y to denote the total set of students. Via the Context Update Rule, this presupposition gives rise to a felicity condition that for each $\langle f, w \rangle$ in C, there must be a non-empty set S of students in w and $f(y) = S$.

It is also of some interest to look at a downward entailing quantifier like *fewer than three of the students*. For instance, consider the following sentence.

(50) Fewer than three of the students stopped smoking.

The assertive meaning of this sentence can be analyzed as (51). To simplify the discussion, we treat the presupposition of the partitive construction as part of the assertion here.

(51) $$\exists x \exists y \begin{bmatrix} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{former}.\mathbf{smokers}(x))) \wedge \\ \mathbf{fewer}.\mathbf{than}.\mathbf{three}(y, x) \end{bmatrix}$$

The dynamic meaning of the classical generalized quantifier $\mathbf{fewer}.\mathbf{than}.\mathbf{three}$ is as follows.

(52) $\langle f, w \rangle \| \mathbf{fewer}.\mathbf{than}.\mathbf{three}(y, x) \| \langle g, v \rangle$ iff $f = g$ and $w = v$ and $|f(x) \cap f(y)| < 3$

The presupposition is the same as (48), and maxset anaphora (y) and refset anaphora (x) are both possible. Notice, however, that in the case of the refset reading, i.e. the reading with a non-universal presupposition (48b), there needs to be at least one student who used to smoke, which is not guaranteed by the assertive meaning. Thus, after the update with the assertive meaning, there might be some $\langle f', w' \rangle \in C'$ such that $f'(y) = \varnothing$. But these pairs will not survive in the final output C'' after the presupposition (48b) is processed. As a consequence, the resulting context implies that there are some students who used to smoke. In other words, the presupposition is informative here, and is used to eliminate certain possibilities from the intermediate context C'.

This seems to us to be a good prediction. In fact one tends to draw an existential inference more robustly when the predicate is presuppositional than when it is not, as the following minimal pair illustrates.

(53) a. Fewer than three of the students are former smokers.
b. Fewer than three of the students stopped smoking.

The inference that there are some students who are former smokers follows more reliably from (53b) than from (53a).[31]

Let us now look at a universal quantifier *every student*. Notice that we have an additional clause $\mathbf{singleton}(x)$, as *every student* is a singular quantifier (fn.26), but its effects are trivial as the predicates here only allow distributive readings. We treat the maxset y to be anaphoric, as in the case of partitive quantifiers.

(54) Every student stopped smoking.

a. ASSERTIVE MEANING:

$$\exists x \left[\begin{array}{l} \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{singleton}(x) \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{every}(x, y) \end{array} \right]$$

b. REFSET PRESUPPOSITION: $\mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \delta_x(\mathbf{used.to.smoke}(x))$

c. MAXSET PRESUPPOSITION: $\mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \delta_y(\mathbf{used.to.smoke}(y))$

Since a universal quantifier asserts that the maxset y is identical to the refset x, with either choice of the presupposition, a universal inference ensues. This is a desired result.

Lastly, let us look at the analysis of *no student*. Here again, we assume that the maxset is anaphoric, but nothing crucial hinges on this.

(55) No students stopped smoking.

a. ASSERTIVE MEANING:

$$\exists x \left[\begin{array}{l} \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{singleton}(x) \wedge \mathbf{former.smokers}(x))) \wedge \\ \mathbf{no}(x, y) \end{array} \right]$$

b. REFSET PRESUPPOSITION: $\mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \delta_x(\mathbf{used.to.smoke}(x))$

c. MAXSET PRESUPPOSITION: $\mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \delta_y(\mathbf{used.to.smoke}(y))$

A negative quantifier like *no student* asserts that the refset x is empty. Assuming that the empty set does not support any anaphora (which is true in the case of cross-sentential anaphora), only the maxset presupposition (56c) makes sense. Consequently, a universal inference is derived, as desired.

To sum up, we have demonstrated that our formal theory accounts for the parallelism between presupposition projection and cross-sentential anaphora as summarized in (4).

(4) Consider a discourse of the following form '$Q(R)(F)$. PRONOUN G.', where $Q(R)$ is a quantifier with a restrictor R, F and G are arbitrary predicates, and PRONOUN is a pronoun with appropriate phi-features, e.g. *they*.

a. If PRONOUN can be resolved to R (i.e. $Q(R)$ supports maxset anaphora), then sentences of the form $Q(R)(\lambda x.S(x)_{p(x)})$ can have a universal inference that $\forall x \in R : p(x)$, and

b. If PRONOUN can be resolved to $R \cap F$ (i.e. $Q(R)$ supports refset anaphora), then sentences of the form $Q(R)(\lambda x.S(x)_{p(x)})$ can have an inference weaker than a universal inference.

[31] Arguably, both sentences in (53) are associated with an existential scalar implicature to begin with, but the point is that it is harder to cancel this implicature for (53b). Admittedly the judgments of these sentences are subtle and should be ultimately assessed in a controlled experiment, which is left for future research.

Although our formal theory might look complex, the underlying idea is simple: a quantifier introduces a quantificational term in the assertive meaning and an anaphoric term in the presupposition, and they stand in a cross-dimensional anaphora via the Context Update Rule.

Before proceeding, it should be mentioned at this moment that (4) is meant to only cover distributive readings, but our theory has no problem accounting for collective presuppositions. To see this, consider the following example.

(56) Exactly three students stopped meeting.

a. ASSERTIVE MEANING:

$$\exists x \exists y \left[\begin{array}{l} \mathbf{max}_y(\delta_y(\mathbf{students}(y))) \wedge \\ \mathbf{max}_x(x \subseteq y \wedge \mathbf{used.to.meet.but.not.anymore}(x)) \wedge \\ \mathbf{exactly.three}(y, x) \end{array} \right]$$

b. REFSET PRESUPPOSITION: $\mathbf{used.to.meet}(x)$

c. MAXSET PRESUPPOSITION: $\mathbf{used.to.meet}(y)$

As before, y is the maxset, i.e. the set containing all the students. Under the collective reading, the refset x is a maximal group of students that used to meet but does not meet anymore. To simplify the discussion, suppose there is only one group of three students that used to meet (see fn.29 for relevant discussion). Then the refset presupposition is going to be satisfied, and the reading is captured. On the other hand, the maxset presupposition will be satisfied if the group contains all the boys, in which case the refset and maxset presupposition coincide, but not otherwise. This seems to be the correct prediction.

4.3 Multiple Quantifiers and Quantificational Dependency

Before closing this section, we suggest that the present analysis can also be extended to more complex cases that involve more than one quantifier interacting with the same presupposition. Although a technical implementation is omitted here, we will briefly sketch the analysis, drawing an analogy to cross-sentential anaphora. In order to see the problem, consider the following example.

(57) None of the boys introduced exactly one girl to both of the people that he wanted her to meet.

The intended reading is the one where *he* is bound by *none of the boys* and *her* is bound by *exactly one girl*. Under this reading, this sentence has a presupposition triggered by *both* that is informally paraphrased as (58).

(58) For all of the boys x, there is at least one girl y such that there are exactly two people that x wanted y to meet.

Cases like this that involve multiple quantifiers are, to the best of our knowledge, not discussed in the previous literature.

At first blush, the analysis put forward here seems to miss the dependency between x and y in (58). Specifically, if each of the quantifiers introduces an anaphoric term, the predicted presupposition of (57) will simply be (59).

(59) There are exactly two people that x wanted y to meet.

However, if x and y are simply resolved to two particular plural individuals, this analysis does not capture how x and y are related, and fails account for the presupposition paraphrased in

(58).

Pushing further the analogy between presupposition projection and cross-sentential anaphora, we suggest that such cases can be likened to the so-called *quantificational subordination* observed in cross-sentential anaphora (Brasoveanu 2007, 2010, Van den Berg 1996, Nouwen 2003b, 2007b). For example, consider (60).

(60) Three students each wrote exactly two papers. They each sent them to L&P.
(Nouwen 2003b:117)

Under one possible reading of (60), *they* denotes the three students, while *them* denotes the two papers that *each of the three students wrote*, rather than the six papers at the same time. Thus, the denotation of *them* can be dependent on *they* in such a way that for each x of the three students, *them* denotes the papers that x wrote.

Under the present analysis of presuppositions of quantified sentences, we would expect the same kind of dependency should be possible for cross-dimensional anaphora. If so, the presupposition of the sentence (57) does look like (59), but the interpretations of the relevant variables x and y are analogous to the interpretations of the pronouns in (60), i.e. (59) amounts to the presupposition that for each of the boys x, there are exactly two people z that x wanted the girl who x introduced to z to meet, which is the desired reading.

There are several ways to formally capture this dependency that are compatible with the formal analysis of quantifiers adopted here (see the works cited above). However, since they require non-trivial complications in our metalanguage, we refrain from presenting a formal extension of our theory here.

5 Presupposition Projection from the Restrictor

In this section, we will demonstrate that the analysis put forward in this paper straightforwardly covers the presuppositions triggered in the restrictors of quantificational determiners, in addition to those triggered in the nuclear scope, which we have focused on so far. Generally, a presupposition triggered in the restrictor of a quantificational determiner results in a weak inference, regardless of what the quantificational determiner is, compared to presuppositions triggered in the nuclear scope (Beaver 1994, 2001, George 2008a, Chemla 2009a), and the present theory nicely accounts for this.

Let us first look at non-downward monotonic symmetric determiners:

(61) a. Some students who stopped smoking drank.
b. Many students who stopped smoking drank.
c. Exactly three students who stopped smoking drank.
d. Between five and ten students who stopped smoking drank.

None of these sentences have a universal inference that all of the students used to smoke (or an absurdly strong universal inference that all individuals in the universe, students or not, used to smoke), but they merely presuppose that there are some students who used to smoke. This state of affairs is expected under the proposed analysis. That is, since these determiners are symmetric, the predictions are the same as in the case of projection out of the nuclear scope, except that the maxset is now the set of students who used to smoke, rather than the whole set of students, due to the additional relative clause. To see this more concretely, take (61c), whose meaning is represented as follows.

(62) a. ASSERTIVE MEANING: $\exists x \exists y \begin{bmatrix} \mathbf{max}_y(\delta_y(\mathbf{students}(y) \wedge \mathbf{former.smoker}(y))) \wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{drank}(x))) \wedge \\ \mathbf{exactly.three}(y, x) \end{bmatrix}$

b. REFSET PRESUPPOSITION: $\delta_x(\mathbf{used.to.smoke}(x))$

c. MAXSET PRESUPPOSITION: $\delta_y(\mathbf{used.to.smoke}(y))$

The maxset presupposition merely requires that these students who are asserted to be former smokers used to smoke, which is weaker than a universal inference that all of the students used to smoke. Due to the Context Update Rule, this will be an existential presupposition, i.e. in each $w \in \mathrm{wld}(C)$, there's $f' \in \mathrm{fnc}(C')$ such that each member of $f'(y)$ used to smoke in w. The refset presupposition is even weaker, requiring the subset x of y that drank (and has exactly three members) to satisfy the presupposition.

The exact same analysis also applies to downward monotonic existential determiners like the following.

(63) a. Few students who stopped smoking drank.

b. At most five students who stopped smoking drank.

c. No more than five students who stopped smoking drank.

The predicted presuppositions of these sentences are again much weaker than a universal inference that all of the students used to smoke. Concretely, the maxset presupposition will be the same as before and amounts to an existential presupposition, i.e. in each $w \in \mathrm{wld}(C)$, there's $f' \in \mathrm{fnc}(C')$ such that each member of $f'(y)$ used to smoke in w. The refset presupposition will be weaker than this, i.e. in each $w \in \mathrm{wld}(C)$, there's $f' \in \mathrm{fnc}(C')$ such that each member of $f'(x)$ used to smoke in w, where $f'(x)$ is a subset of $f'(y)$ that drank (in w'). Notice that in the latter case, the assertive meaning itself does not exclude the possibility that there are no students who are former smokers and drank, but, in order for the anaphora to succeed, there must be some such students. In other words, the sentences in (63) are predicted to be associated with an existential inference, which seems to be correct (cf. the discussion in §4.2).

Unlike these determiners, determiners like *every, no*, *most* and partitive determiners involve an additional presupposition regarding their restrictor. As before, we assume that the maxsets of these determiners are anaphoric terms (cf. Van den Berg 1996, Geurts & Van der Sandt 1999, Brasoveanu 2007, 2010). By way of illustration, consider (64).

(64) Every student who stopped smoking drank.

a. ASSERTIVE MEANING:
$\exists x[\mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{singleton}(x) \wedge \mathbf{drank}(x))) \wedge \mathbf{every}(y, x)]$

b. REFSET PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{students}(y) \wedge \mathbf{former.smokers}(y))) \wedge \delta_y(\mathbf{used.to.smoke}(y))$

c. MAXSET PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{students}(y) \wedge \mathbf{former.smokers}(y))) \wedge \delta_x(\mathbf{used.to.smoke}(x))$

Recall with a universal quantifier, the two presuppositions result in an identical reading, since x and y are asserted to be identical. The set of individuals denoted by y serves as the domain of quantification here, which is presupposed to be the students who are former smokers. In other words, the presupposition of (64) can only be satisfied if the initial context C is such that for all $\langle f, w \rangle \in C$, $f(y)$ is the set of all the students who are former smokers in w (notice that $f(y) = f'(y)$ because the assertive meaning only adds x). Then, since y is the set of students who are former smokers, the inference added by *stopped smoking* that each member of y used to smoke is already entailed. All in all, (64) only requires that y be the set of students who are former smokers. This is a desired result.

Determiners like *no* and *most* can be analyzed in the same way, by changing the quantificational force in the assertive meaning as follows.

(65) No student who stopped smoking drank.

a. ASSERTIVE MEANING:
$\exists x[\mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{drank}(x))) \wedge \mathbf{no}(y,x)]$

b. PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{students}(y) \wedge \mathbf{former.smokers}(y))) \wedge \delta_x(\mathbf{used.to.smoke}(x))$

(66) Most of the students who stopped smoking drank.

a. ASSERTIVE MEANING:
$\exists x[\mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{drank}(x))) \wedge \mathbf{most}(y,x)]$

b. REFSET PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{students}(y) \wedge \mathbf{former.smokers}(y))) \wedge \delta_y(\mathbf{used.to.smoke}(x))$

c. MAXSET PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{students}(y) \wedge \mathbf{former.smokers}(y))) \wedge \delta_x(\mathbf{used.to.smoke}(y))$

Recall that *no* does not support refset anaphora, because the refset x is asserted to be null. Thus, the refset presupposition is not represented in (67). But for (66) we predict two readings based on maxset and refset anaphora, and the latter reading will be weaker than the former. However, this is harmless, as the resulting presupposition is already entailed. That is, since y is presupposed to be the set of all students who used to smoke, its subset x also consists of students who used to smoke. All in all, this means that we predict that the above three sentences are practically synonymous with the following sentences that have the same assertive meaning but no presupposition in the restrictor, which seems to be correct.

(67) a. Every student who is a former smoker drank.
b. No student who is a former smoker drank.
c. Most of the students who are former smokers drank.

Interestingly, with predicates like *criticized herself* that have a presupposition that is not entailed by the assertive meaning, the resulting inferences become stronger. The present theory has no problem account for this state of affairs. Consider the following examples.

(68) Every student who criticized herself was drunk.

a. ASSERTIVE MEANING:
$\exists x[\mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{singleton}(y) \wedge \mathbf{drank}(x))) \wedge \mathbf{every}(y,x)]$

b. PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{student}(y) \wedge \mathbf{criticized}(y,y))) \wedge \delta_y(\mathbf{female}(y))$

(69) No student who criticized herself was drunk.

a. ASSERTIVE MEANING:
$\exists x[\mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{singleton}(y) \wedge \mathbf{drank}(x))) \wedge \mathbf{no}(y,x)]$

b. PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{student}(y) \wedge \mathbf{criticized}(y,y))) \wedge \delta_y(\mathbf{female}(y))$

These sentences presuppose that the students who criticized themselves are all female. As one can see from the representations here, this is exactly what is predicted. That is, for each $\langle f, w\rangle \in C$, $f(y)$ must be the set of all students who criticized themselves and each of them must be female. In a similar vein, consider (70).

(70) Most of the students who didn't stop smoking drank.

a. ASSERTIVE MEANING:
$\exists x[\mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{drank}(x))) \wedge \mathbf{most}(y,x)]$

b. REFSET PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{student}(y) \wedge \neg\mathbf{former.smoker}(y))) \wedge \delta_y(\mathbf{used.to.smoke}(x))$

c. MAXSET PRESUPPOSITION:
$\mathbf{max}_y(\delta_y(\mathbf{student}(y) \wedge \neg\mathbf{former.smoker}(y))) \wedge \delta_x(\mathbf{used.to.smoke}(y))$

The maxset presupposition (70c) is that y is the set of students who are not former smokers but used to smoke, i.e. all of them are still smoking. This seems to be the most prominent reading of (70). We also predict the refset presupposition (70b), which has a weaker presupposition. That is, each member of y either has been smoking or never smoked, but its subset x is a set of smokers. Although this latter reading might be hard to obtain, we believe this prediction is not outrageous.

6 Comparison with Previous Theories

In the preceding three sections, we saw in detail how the presuppositions of quantified sentences are accounted for. In the present section, we will briefly mention some appealing features of the present theory in comparison to major previous theories found in the literature. For reasons of space, we need to skip details of the technical aspect of these alternative theories.

Firstly, under the present theory, the presuppositional properties of determiners follow from their cross-sentential anaphoric properties and need not be independently stipulated as in Beaver, Beaver's (1994, 2001). Beaver claims that there is no evidence for universal presuppositions for any quantifiers, and assigns existential presuppositions to all quantifiers uniformly. However, experimental results reported in Chemla (2009a) and Sudo et al. (2011) strongly suggest otherwise. Although Beaver's theory in principle allows a universal presupposition (as in Heim 1983, discussed below), it does not have a principled way of explaining which quantifier has a universal presupposition. It is also worth mentioning that the correlation between the two phenomena has not been pointed out before, and to the best of our knowledge, no other theory captures it.

Secondly, being a two-dimensional theory, it is more expressive than more standard accounts of presuppositions of quantified sentences like Heim (1983), Beaver & Krahmer (2001), George (2008a,b) and Fox (2012) (see Cooper 1983 for a related view), which have a serious undergeneration problem. Let us discuss this point in some detail.

Putting the technical details aside, Heim (1983) generally assigns universal presuppositions to all quantified sentences of the form $Q(R)(S_p)$, including those involving singular indefinites, and in order to derive non-universal inferences, she makes use of local accommodation. To illustrate, consider the following sentence.

(71) A student criticized herself.

Without local accommodation, (71) is assigned a universal presupposition that every student is female. With local accommodation, (71) is predicted to mean that there is a student who is female and criticized herself.

There are several objections to this analysis. Firstly, it is doubtful that the universal presupposition is a possible reading for (71). Secondly, this theory becomes problematic for quantifiers that are not up-ward monotonic, because for such quantifiers, local accommodation will not yield the correct reading. To see this concretely, consider the following sentence.

(72) Exactly one student criticized herself.

This sentence has a reading with a non-universal presupposition, but it cannot be generated with local accommodation, which would mean that exactly one student is female and criticized herself. Although this reading might be available, the reading of (72) we are after here only requires one student to be female.

An analogous problem arises under trivalent theories like Beaver & Krahmer (2001), George (2008a,b) and Fox (2012).[32] These theories make use of a third truth-value, #, which specifies when the presupposition of the sentence fails. More concretely, (71) is predicted to be true when there is one student who is female and criticized herself, false when every student is female and none of them criticized herself, and # in all other cases. Thus, the presupposition amounts to the disjunction of the former two situations, i.e. either there is a female student who criticized herself or every student is female and none of them criticized herself. This is weaker than a universal presupposition, as desired. However, (72) is problematic. It is predicted to be true when all the students are female and only three of them criticized themselves, false when all the students are female and less than three or more than three of them criticized themselves, and # in all other cases. Since the presupposition is the disjunction of the former two situations, (71) is predicted to have a universal presupposition, just as Heim (1983) predicts. Furthermore, local accommodation does not yield the correct reading.

This problem is a fundamental one for these previous theories (see Stalnaker 1973, Oshima 2006, Van Rooij 2005, 2010, Sudo 2012 for related discussion). They are 'uni-dimensional' in the sense that presuppositions are treated as part of the assertive meaning—which circumvents the Binding Problem (see fn.7 and fn.8)—but consequently, they only distinguish three kinds of sentential meanings, i.e. true, false and presupposition failure. However, the meaning of (72) requires reference to a fourth kind of meaning. More specifically, what (72) means is that for one student, both assertive meaning and presupposition are true, and for all the other students, the assertive meaning is false but the presupposition might or might not be true. In uni-dimensional theories, there is no way to access the assertive meaning without also referring to the presupposition, but the latter part of the meaning of (72) requires exactly that. Thus, this is an issue of expressive power, and in order to deal with sentences like (72), a theory needs to be able to distinguish four situations, rather than three: (i) both assertive meaning and presupposition are true, (ii) the assertive meaning is true but the presupposition is false, (iii) the assertive meaning is false but the presupposition is true, and (iv) both of them are false. Being two-dimensional, our theory is expressive enoughand can distinguish these four cases. In fact it accounts for the correct meaning of (72), as we saw in the preceding sections.[33]

Related to this issue of expressive power, the 'Binding Theory' of presupposition (couched in Discourse Representation Theory, championed by Van der Sandt (1992) and Geurts (1999) among others, has a related problem, although it is not a uni-dimensional theory. More specifically, it predicts two readings for (72). One reading corresponds to the local accommodation reading that other theories also predict. In addition, it also generates a so-called 'intermediate accommodation' reading where the presupposition becomes part of the assertive meaning

[32] Cooper (1983) uses four truth-values, but its predictions for quantified sentences are essentially the same as trivalent theories'.

[33] In a recent paper (Sauerland 2013), Uli Sauerland gave a more conservative solution to (72) by giving a focus-sensitive semantics for *exactly* (see Spathas 2010 and Jacobson 2012 for related ideas). However, his account crucially hinges on focus sensitivity of relevant operators, which might make sense for *exactly* but less so for other non-upward monotonic quantifiers like *fewer than 5*.

of the restrictor, which can be paraphrased by *Exactly one female student criticized herself*. This reading seems to be unavailable. Thus, the 'Binding Theory' not only fails to capture the intended reading but also generates a reading whose availability is highly suspicious.[34]

It should also be mentioned that our theory is not the first two-dimensional theory of presupposition. Karttunen & Peters (1979) is probably one of the earliest to give an explicit formulation of a two-dimensional version of Montague Grammar (Montague 1973). However, as they themselves remark in the end note, their theory run into the Binding Problem. Specifically, their theory assigns the following meaning to (71).

(73) a. ASSERTIVE MEANING: A student x criticized x
b. PRESUPPOSITION: A student is female

This analysis fails to identify the students mentioned in the two dimensions of meaning. A similar problem arises for Chemla's (2009b) two-dimensional theory. Our theory avoids this problem by the mechanism of cross-dimensional anaphora.

7 Further Issues

Before closing the paper, we would like mention to three further issues for the present theory. The first one has to do with compositionality and presupposition projection in more complex sentences. We will sketch in Section 7.1 how meanings of quantifiers and predicates give rise to the meanings of atomic quantificational sentences, and how the presuppositions so derived will project when atomic quantificational sentences are embedded, although it is beyond the scope of the present paper to offer a comprehensive picture. Section 7.2 discusses the issue regarding different types of presupposition triggers. Section 7.3 briefly mentions quantification in other domains than individuals.

7.1 Compositionality Issues

The problem of presupposition projection is essentially the problem of compositionality, i.e. how to predict the presuppositions of complex phrases from the syntax and semantics of their parts. In this subsection, we will sketch a way to achieve sub-sentential compositionality (how to combine the meanings of quantificational determiners and predicates to arrive at the meanings of atomic quantificational sentences) and presupposition projection in complex sentences (how the presuppositions of atomic quantificational sentences project up when they are embedded).

In order to achieve sub-sentential compositionality in our theory, we need a way to map sentences in the object language, i.e. English, to pairs of PPL formulae, which get interpreted model-theoretically (Montague 1973, Groenendijk & Stokhof 1990, Muskens 1996, Brasoveanu 2007). Our theory is two-dimensional and in this respect, it is similar to Karttunen & Peters (1979), for whom a sentence meaning is a pair of formulae in a static language, namely Montague's IL. In dealing with sub-sentential compositionality, Karttunen & Peters (1979) propose to decompose the meaning of each expression in the object language into three components: (i) the assertive meaning, (ii) the presupposition it triggers, and (iii) the 'heritage function' which specifies how it treats the presupposition of its arguments. The latter two make up the presuppositional meaning of the expression. This technique is directly applicable to our dynamic theory, as we will demonstrate now.

[34] Also, as discussed in detail by Beaver (2001) and Chemla (2009a), it fails to derive universal presuppositions in general, which is empirically unsatisfactory (but see Geurts & Van der Sandt 1999 for complications regarding the presuppositionality of certain determiners).

Let us take the presuppositional predicate *stopped smoking* as an example. Since our theory is two-dimensional, its meaning $[\![\text{stopped smoking}]\!]$ is a pair consisting of its assertive meaning, $(\text{stopped smoking})_a$ and presuppositional meaning $|\text{stopped smoking}|_p$.

(74) $[\![\text{stopped smoking}]\!] = \langle (\text{stopped smoking})_a,\ |\text{stopped smoking}|_p \rangle$

a. $(\text{stopped smoking})_a = \lambda\langle x_a, x_p\rangle.\ \mathbf{former.smoker}(x_a)$

b. $|\text{stopped smoking}|_p = \lambda\langle x_a, x_p\rangle.\ x_p \wedge \mathbf{used.to.smoke}(x_a)$

These functions take a pair $\langle x_a, x_p\rangle$, consisting of the subject's assertive meaning x_a and its presupposition x_p. The presuppositional meaning (74b) says that the resulting sentence will presuppose the conjunction of whatever the subject presupposes (i.e. x_p) and the proposition that the subject used to smoke. For instance, if the subject is *the man who is dating Mary*, the entire sentence will presuppose that there is a unique man who is dating Mary, which comes from the presupposition of the subject, and that he used to smoke, which is newly added by *stop smoking*.

In this setting, the indefinite determiner *a* can be analyzed in the following manner.

(75) $[\![\text{a}^x]\!] = \langle (\text{a}^x)_a,\ |\text{a}^x|_p \rangle$

a. $(\text{a}^x)_a = \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\ \exists x[\mathbf{singleton}(x) \wedge P_a(\langle x, \top\rangle) \wedge Q_a(\langle x, \top\rangle)]$

b. $|\text{a}^x|_p = \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\ P_p(\langle x, \top\rangle) \wedge Q_p(\langle x, \top\rangle)$

As we just saw, one-place predicates denote pairs $\langle P_a, P_p\rangle$ where P_a and P_p are functions of type $\langle e \times e,\ t \times t\rangle$, where $\tau \times \tau$ is the type of pairs of objects of type τ. The assertive meaning (75a) introduces $\exists x$, which refers to the variable x appearing as the superscript in the object language, and the assertive parts of the predicates, P_a and Q_a, are applied to x. Generally, P_a and Q_a ignore the second coordinates of their inputs (see (74a) above), so the second coordinate of their argument can be anything, and we just use $\top$ here, which represents a tautologous formula (i.e. $\langle f, w\rangle \|\top\| \langle f, w\rangle$ for any $\langle f, w\rangle$). Turning now to the presupposition (75a), since an indefinite article does not trigger a presupposition, (75b) just specifies how the presupposition of the predicates are manipulated. Crucially, the presupposition of the predicates P_p and Q_p apply to the same variable x that is introduced by the $\exists x$ in the assertive meaning (and is mentioned in the object language as the supercript). Together with the Context Update Rule given in Section 2, this meaning will derive the correct presupposition for sentences like *A student stopped smoking*. More specifically, the meanings of *a student* and *A student stopped smoking* will look like the following (assuming that *student* does not have a (non-trivial) presupposition).

(76) $[\![\text{a}^x\ \text{student}]\!] = \langle (\text{a}^x)_a([\![\text{student}]\!]),\ |\text{a}^x|_p([\![\text{student}]\!]) \rangle$

a. $(\text{a}^x\ \text{student})_a = \lambda\langle Q_a, Q_p\rangle.\ \exists x[\mathbf{singleton}(x) \wedge \mathbf{student}(x) \wedge Q_a(\langle x, \top\rangle)]$

b. $|\text{a}^x\ \text{student}|_p = \lambda\langle Q_a, Q_p\rangle.\ Q_p(\langle x, \top\rangle)$

(77) $[\![\text{a}^x\ \text{student stopped smoking}]\!]$
$= \langle (\text{a}^x\ \text{student})_a([\![\text{stopped smoking}]\!]),\ |\text{a}^x\ \text{student}|_p([\![\text{stopped smoking}]\!]) \rangle$

a. $(\text{a}^x\ \text{student stopped smoking})_a = \exists x[\mathbf{singleton}(x) \wedge \mathbf{student}(x) \wedge \mathbf{former.smoker}(x)]$

b. $|\text{a}^x\ \text{student stopped smoking}|_p = \mathbf{used.to.smoke}(x)$

As demonstrated in Section 2, the Context Update Rule ensures that the variable x in the presupposition co-varies with the other occurrences of x in the assertive meaning, which results in an existential presupposition.

Similarly, *exactly three* is given the following meaning (under the distributive reading). Since *exactly three* introduces two variables x and y, we assume that it bears two superscripts.

Also, we assume that one of them is marked by * as occurring in the presupposition.[35]

(78) a. $\llparenthesis\text{exactly three}^{y^*,x}\rrparenthesis_a = \llparenthesis\text{exactly three}^{y,x^*}\rrparenthesis_a$

$$= \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\exists x\exists y \left[\begin{array}{l} \mathbf{max}_y(\delta_y(P_a(\langle y, \top\rangle)))\wedge \\ \mathbf{max}_x(\delta_x(x \subseteq y \wedge Q_a(\langle x, \top\rangle)))\wedge \\ \mathbf{exactly}.\mathbf{three}(y, x) \end{array}\right]$$

b. $|\text{exactly three}^{y,x^*}|_p = \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\ \delta_x(P_p(x, \top)) \wedge \delta_x(Q_p(x, \top))$

c. $|\text{exactly three}^{y^*,x}|_p = \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\ \delta_y(P_p(y, \top)) \wedge \delta_y(Q_p(y, \top))$

As explained earlier, other quantifiers can be dealt with by changing the last clause of (78a) with the dynamic versions of the corresponding classical generalized quantifiers. It should be emphasized here that the attractiveness of the present theory is that the presuppositional meanings of all quantification determiners are essentially the same: they introduce an anaphoric term that refers to one of the individuals (or the only individual in the case of indefinites) that they are about. To be more precise, presuppositional determiners like *every* can be analyzed as follows. Unlike the determiners we saw above, it introduces its own presupposition, which is the first clause of (79b) (since the maxset and refset presuppositions are essentially the same in this case, only the former is given here).

(79) a. $\llparenthesis\text{every}^{y,x}\rrparenthesis_a = \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\ \exists x \left[\begin{array}{l} \mathbf{max}_x(\delta_x(x \subseteq y \wedge \mathbf{singleton}(x)\wedge \\ Q_a(\langle x, \top\rangle))) \wedge \mathbf{every}(y, x) \end{array}\right]$

b. $|\text{every}^{y,x}|_p = \lambda\langle P_a, P_p\rangle.\lambda\langle Q_a, Q_p\rangle.\ \mathbf{max}_y(\delta_y(P_a(\langle y, \top\rangle)))\wedge\delta_y(P_p(y, \top))\wedge\delta_y(Q_p(y, \top))$

Let us now discuss complex sentences containing atomic quantificational sentences as their parts.[36] Given that Karttunen & Peters (1979) already propose analyses of various sentential operators in their static two-dimensional theory, one might think that all we have to do is to dynamicize these meanings. However, this turns out to be more complicated. We will sketch the problem with negation. Consider the following sentence.

(80) It's not the case that a student criticized herself.

In order to analyze this sentence, we need the meaning of negation. If we take Karttunen & Peters's (1979) meaning and simply fit it to the present framework, it will look as follows.

(81) $[\![\text{it is not the case}]\!] = \langle\llparenthesis\text{it is not the case}\rrparenthesis_a, |\text{it is not the case}|_p\rangle$

a. $\llparenthesis\text{it is not the case}\rrparenthesis_a = \lambda\langle S_a, S_p\rangle.\ \neg S_a$

b. $|\text{it is not the case}|_p = \lambda\langle S_a, S_p\rangle.\ S_p$

This is meant to capture the widely accepted assumption that negation is a 'presupposition

[35]The analysis presented here might not be fine-grained enough to distinguish between partitive and non-partitive quantifiers in the relevant respect. That is, since both partitive and non-partitive quantifiers introduce new variables x and y, it is predicted that both restrictor and refset anaphora should be supported, and correspondingly that both universal and non-universal presuppositions should be available. As far as we know, no convincing evidence is available for or against this prediction, and we would like to leave this issue open for the moment. Notice however that this is coming from the underlying dynamic semantics of plural generalized quantifiers, which does not draw this distinction, and crucially, the proposed analysis of presupposition projection can in principle be coupled with a more fine-grained dynamic theory of cross-sentential anaphora. Thus, if it turns out that non-partitive quantifiers do not support maxset anaphora as robustly as the corresponding partitive quantifier, a dynamic theory that captures this fact can be used to predict that the universal inference does not arise as robustly.

[36]A thank an anonymous reviewer for pointing out a problem of an earlier solution.

hole', i.e. it only negates the assertive meaning and simply passes the presupposition of the input to the next level.

However, this meaning makes the wrong prediction for (80), which will be analyzed as follows.

(82) a. $\llbracket(80)\rrbracket_a = \neg\exists x[\mathbf{student} \wedge \mathbf{criticized}(x, x)]$
b. $|(80)|_p = \mathbf{female}(x)$

The problem here is that due to the wide scope negation in the assertive meaning, x in the presupposition can no longer be bound by $\exists x$, just as anaphora fails in the following case of cross-sentential anaphora.

(83) It is not the case that a man entered the room. #He sat down.

Generally, Karttunen & Peters's (1979) way of capturing presupposition projection is not compatible with our theory (and independently it is criticized on conceptual grounds; see Gazdar 1979, Soames 1979, 1982, 1989).

Instead, we suggest here that in complex sentences the Context Update Rule is applied multiple times. The idea is very similar in spirit to Heim's (1983) and related theories (Karttunen 1974, Schlenker 2009, Stalnaker 1978) that make use of so-called *local contexts*. In a negative sentence like (80), for example, the input context C is first updated with the embedded sentence using the Context Update Rule, and then the resulting context is subtracted from C. Then the presupposition that the sentence has is the same for all intents and purposes as the positive counterpart, i.e. for each $w \in C$, there must be $f'(x)$ who is female in w, because its presupposition is evaluated against C, which is its local context. This seems to be a reasonable analysis of the sentence.

Furthermore, Heim (1983) and others have developed sophisticated analyses of various sentential connectives using this idea, and we can capitalize on them.[37] Adopting this approach, we can analyze various sentential operator as creating local contexts where the Context Update Rule apply. For instance, sentential conjunction ϕ *and* ψ updates the input context C with ϕ first and creates a local context that gets updated with ψ, where both updates are done by the Context Update Rule.

Since pursing this approach to other connectives requires another paper, we will refrain from presenting the whole system here. However, one crucial difference from the previous approaches of this sort should be mentioned. That is, in local context theories, it is usually assume that quantificational determiners also create local contexts, but for us, they do not. In fact, in our system the Context Update Rule never applies within an atomic sentence.

7.2 Strong Triggers

Charlow (2009) observes that the so-called strong presupposition triggers, such as additive particles, give rise to universal inferences through all quantifiers. For instance, according to him, both negative and existential quantifiers are infelicitous in the following discourse.

(84) Just five of those 100 students smoke. They all smoke Newports.
a. #Fortunately, none of those 100 students also smokes Marlboros.
b. #Unfortunately, some/at least two of those 100 students also smoke Marlboros. (Charlow 2009)

[37] Needless to say, these analyses are not uncontroversial, but so are alternatives. See Geurts (1999), Schlenker (2008, 2009), Rothschild (2011), Chemla & Schlenker (2012) for discussion.

This state of affairs is not predicted under our theory, as there is no principled reason to think that refset anaphora is not available in (84b). In other words, there is nothing in the present theory that predicts a difference among presupposition triggers.

However, we would like to point out that the data Charlow offers is confounded by the fact that the *of*-phrase is independently infelicitous. Specifically, the use of *of those 100 students* sounds much less natural compared to *of those five students* in the particular context presented above.

Instead of Charlow's example above, let us consider the following examples, which eliminate this confound.

(85) Less than half of the students in each year have been to Paris.
  a. 1/3 of the first years have also been to Toulouse.
  b. Two of the first years have also been to Toulouse.
  c. Few of the first years have also been to Toulouse.

These sentences sound coherent, i.e. they do not have universal inferences. If this is correct, it contradicts Charlow's verdict that additive particles like *also* always give rise to universal inferences.

However, there is still a possibility that a weaker version of Charlow's claim is true. That is, different presupposition triggers give rise to universal inferences to different degrees, and additive particles generally prefer universal inferences. As the empirical facts are not clear at this moment, and carefully designed experiments are likely to be necessary to assess reliable data, we will leave this topic for future research.

7.3 Quantification in Other Domains

Another problem that is left open here is presupposition projection through other kinds of quantifiers. Firstly, the theory put forward in this paper seems to be applicable to the projection properties of temporal quantifiers. Generally, universal temporal quantifiers give rise to universal inferences and existential temporal quantifiers give rise to non-universal inferences, just as in the case of quantifiers over individuals. For instance, (86a) suggests that whenever John woke up early, it was raining, while (86b) has a weaker inference.

(86) a. Whenever John woke up early, he knew that it was raining.
  b. On some of the days, John knew that it was raining.

Thus, we can analyze these sentences in a very similar way. That is, we can enrich the metalanguage with variables ranging over temporal intervals and straightforwardly extend our analysis of quantifiers in the individual domain.

However, modals seem to require a different treatment. Modals are standardly modeled as quantifiers over possible worlds, and as quantifiers, they can be given analyses similar to quantifiers in the individual and temporal domains. However, there seems to be a discrepancy between their actual projection properties and what is predicted by the extension of the present theory. Consider the following examples, where the relevant presupposition trigger is *both*.

(87) a. John is allowed to invite both of his sisters.
  b. John might have invited both of his sisters.

Both of these sentences imply that John has exactly two sisters. However, the present analysis applied to modals predicts a weaker presupposition to be available. More specifically, if we maintain the claim that a quantifier, including a modal, introduces an anaphoric term in the presupposition that needs to be resolved with respect to the assertive meaning, the following presuppositions will be predicted for (87), where w' is to be resolved to the worlds that are existentially quantified in the assertive meaning.

(88) John has exactly two sisters in w'

The resulting presuppositions of (88), then, will be roughly paraphrased by the following, which are much weaker than what we actually observe.

(89) a. John is allowed to have exactly two sisters.
b. John might have exactly two sisters.

Crucially, possibility modals do support 'refset' anaphora (which is better known as *modal subordination*; Roberts 1989, 1997, Brasoveanu 2007, 2010). For instance, the second sentences of (90) are about the worlds in which John comes, which are worlds that the first sentence talks about.

(90) a. John is allowed to come. He will bring a bottle of red wine.
b. John might come. He would bring a bottle of wine.

The whole topic of presupposition projection through modals has been relatively understudied (but see Heim 1992, Geurts 1999, Sudo 2013). We will leave this issue for future research.

8 Conclusion

In the present paper, we pursued the following idea: the presuppositional properties of quantificational determiners correlate with their discourse anaphoric properties. Specifically, if a given quantifier supports maxset anaphora cross-sententially, it is able to give rise to a universal presupposition, and if it supports refset anaphora, it is able to give rise to a non-universal presupposition. We offered a formal theory of cross-dimensional anaphora that captures this correlation in a straightforwardly manner by treating the two dimensions of meaning as essentially analogous to the relation of two independent sentences with a quantificational antecedent and an anaphoric term that is anchored to it. The key mechanism is the Context Update Rule that enables the cross-dimensional anaphoric relation. Although there are a number of further issues to deal with, as mentioned in the previous section, we hope that this contributes to our understanding of the recalcitrant problem of how presuppositions and quantification interact.

A Complement Anaphora

In Section 3 , we saw several basic cases where the predicted correlation between cross-sentential and cross-dimensional anaphora nicely holds. In this appendix, we discuss one place where the correlation seems to break down, namely so-called *complement anaphora*.

Sentences with certain determiners such as *few* are known to allow complement anaphora cross-sententially, where a pronoun in the second sentence is resolved to the individuals in the restrictor that do *not* satisfy the nuclear scope, or the complement set. As a baseline, consider the sentence in (91).

(91) Most of the students showed up today. They were all sick.

The pronoun *they* in the second sentence of this example can be read as the students who showed up today (refset anaphora), or all the relevant students (maxset anaphora), but it is extremely hard, if possible at all, to construe it as denoting the students who did not come today. Now, consider the following sentences.

(92) a. Few of the students showed up today. They were all sick.
b. Only one student showed up today. They were all sick.

In these sentences, *they* can be read as the students who did not show up, unlike in (91). Thus, certain quantifiers seem to support anaphora to their complement sets.

On the other hand, presuppositions can never be about the complement set, even if the quantifier is one that supports complement anaphora. More concretely, (93) cannot be read as presupposing that those students that did not criticized themselves are female.

(93) Only one of the students criticized herself.

Therefore, the parallelism between presupposition projection and cross-sentential anaphora seems to fail here.

However, there is ample reason to believe that complement anaphora is licensed via pragmatic inferences and is not solely due to the anaphoric potentials of the quantifiers (Moxey 2006, Moxey & Sanford 1993, Moxey et al. 2001, Nouwen 2003a,b, 2010, Paterson et al. 1997, Sanford et al. 1996, 2001, 2007). For instance, Moxey (2006) observes that although positive quantifiers like *a small number of* is said to disallow complement anaphora like *most*, when put in contexts where the refset is expected to be large, its complement anaphora reading is accepted more often than in contexts where such expectations are not explicit. This observation strongly suggests that complement anaphora requires a mediation of some contextual factor. Furthermore, as many of the above studies point out, there is gradient variation among negative quantifiers: some quantifiers (e.g. *not many*) strongly prefer complement set anaphora, some (e.g. *at most n*) do not readily allow for it, and still others (e.g. *few*) fall somewhere between in this spectrum. These observations all point to a view, as propounded by the studies cited here, that complement anaphora is enabled by some pragmatic inference regarding the discrepancy between the previously expected and the asserted cardinalities of the refset.

Adopting this view, we suggest that an inference that gives rise to a possible antecedent for complement anaphora is inherently unavailable in the case of cross-dimensional anaphora. That is, the relation between the two dimensions of meaning does not completely mirror that of two independent sentences occurring in a real discourse in that no extra pragmatic inference that would license complement anaphora is allowed between the assertive meaning and presupposition. As a consequence, the predicted presupposition for a given quantificational sentence is solely determined by the anaphoric potential of the quantifier.[38]

[38]Something similar can be said about complex quantifiers like *not every student* and *almost no student*. It seems that *not every student* is associated with a universal inference very robustly. For instance, (i) suggests that every student is female.

(i) Not every student criticized herself.

However, as far as the assertive meaning is concerned, (i) is equivalent to the existential statement that some students didn't criticized themselves, and indeed these individuals can be referred to by a pronoun in a later discourse.

(ii) Not every student came. They stayed home.

If this is a possibility, (i) should allow an inference that the students who did not criticized themselves are female, contrary to fact. It seems to us that refset anaphora like (ii) involves an inference similar to that of complement

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anaphora. If this analysis is on the right track, this discrepancy between cross-dimensional and cross-sentential anaphora can be attributed to the pragmatic inference that is only available in the latter case. I thank Philippe Schlenker (p.c.) and Benjamin Spector (p.c.) for raising this issue.

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