Pressure-dependent water absorption cross sections for exoplanets and other atmospheres

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Many atmospheres (cool stars, brown dwarfs, giant planets, extrasolar planets) are predominately composed of molecular hydrogen and helium. H$_2^{16}$O is one of the best measured molecules in extrasolar planetary atmospheres to date and a major compound in the atmospheres of brown-dwarfs and oxygen-rich cool stars, yet the scope of experimental and theoretical studies on the pressure broadening of water vapour lines by collision with hydrogen and helium remains limited. Theoretical H$_2$- and He-broadening parameters of water vapour lines (rotational quantum number $J$ up to 50) are obtained for temperatures in the range 300–2000 K. Two approaches for calculation of line widths were used: (i) the averaged energy difference method and (ii) the empirical expression for $Jff$-dependence. Voigt profiles based on these widths and the BT2 line list are used to generate high resolution ($\Delta \nu = 0.01$ cm$^{-1}$) pressure broadened cross sections for a fixed range of temperatures and pressures between 300 and 2000 K and 0.001–10 bar. An interpolation procedure which can be used to determine cross sections at intermediate temperature and pressure is described. Pressure broadening parameters and cross sections are presented in new ExoMol format.

1. Introduction

The ExoMol project aims to provide comprehensive line lists of molecular transitions appropriate for molecules in hot atmospheres such as those found in exoplanets, brown dwarfs and cool stars [1]. Until recently the ExoMol database did not include any information on the pressure broadening of molecular lines despite the known importance of pressure effects in these environments [2]. The scope of the ExoMol database is being extended to systematically provide this additional data and other supplementary information on the pressure broadening of molecular lines despite the limited information for broadening of water vapour lines by H$_2$ and He [3].

H$_2^{16}$O is one of the few molecules detected in the atmosphere of an exoplanet to date [4,5]. In particular it has been identified in the atmospheres of several hot extrasolar giant planets including HD189733b [6–11], GJ 436b [12,13], HD 209458b [14,15], XO-1b [16,15], HAT-P-1b [17], HD 179949b [18], WASP-19b [19], WASP-12b [20] and planets orbitting HR 8799 [21,22]. Test model calculations have shown that including pressure broadening of the water spectrum significantly alters the observed spectrum of transiting hot Jupiter exoplanet [2].

Prior to the discovery of exoplanets, H$_2^{16}$O was already known to be a component of the atmospheres of cool stars and brown dwarfs [23–26], where it can dominate the observed spectrum [27]. Recent discoveries have focused very cold brown dwarfs: Faherty et al. [28] made a tentative detection of water clouds in WISE J085510.83-071442.5 and Skemer et al. [29] detected water and clouds in the cold brown dwarf WISE 0855.

H$_2^{16}$O is one of the few molecules contained in the current release of HITEMP [30] which contains a comprehensive set of air- and self-broadening parameters. These were obtained using a diet based on physical principles and statistics [31] which had a strong bias toward near-room temperatures. As the bulk of the atmosphere of giant exoplanets and cool stars is composed of a H$_2$ rich H$_2$–He mix, molecular opacities in atmospheric models for these objects should incorporate pressure broadening due to H$_2$ and He.

A number of works, reviewed in the next section, have provided pressure broadening parameters for broadening of H$_2^{16}$O by H$_2$ and/or He for this purpose, although the spectral, temperature and pressure coverage is far from comprehensive. Altogether, detailed information for broadening of water vapour lines by H$_2$ and He is available for around 1100 and 5000 lines respectively, while...
the most complete line list for water to date (BT2, [32]) contains around half a billion lines which is sufficiently complete for temperatures up to 3000 K. The temperature coverage, although appreciable for broadening by H$_2$ (40–1500 K), is more limited for broadened by He (83–600 K). Conversely the pressure coverage for broadening by He (up to 3 atm) is more extensive than for broadening by H$_2$ (up to 1330 mbar $\approx$ 1.3 atm).

The first aim of this work is to provide H$_2$O–H$_2$ and H$_2$O–He pressure broadening parameters suitable for temperatures up to 2000 K and pressures up to 10 bar. These parameters can be used to generate a pressure and temperature dependent Lorentzian half-width for every line in the BT2 line list, or indeed any water line list with at least rotational angular momentum ($J$) quantum assignments.

The implementation of full line lists in atmospheric modelling codes, although ideal, is often not practical due to their sheer size [33]. Hence the second aim of this work is to provide pressure-dependant absorption cross sections based on Voigt profiles for H$_2$O in a mixed H$_2$/He (85/15%) environment for a range of temperatures ($T$ = 300–2000 K) and pressures ($P$ = 0.001–10 bar) relevant to exoplanet and cool star atmospheres.

2. Previous work

For the H$_2^{16}$O–H$_2$ system the most extensive experimental study was performed by Brown and Plymate [34] who derived pressure broadened widths for 630 lines in the range 55–4045 cm$^{-1}$ at room temperature. Other exclusively room temperature measurements have been carried out by Steyert et al. [35] (39 lines in the range 380–600 cm$^{-1}$), Brown et al. [36] (4 lines around 1540 cm$^{-1}$), Golubiatnikov [37] (1 line at 183 GHz), Lucchesinia et al. [38] (15 lines in the range 820–830 nm), and Zeninari et al. [39] (6 lines around 1.39 μm). The temperature and/or pressure dependence of the collision induced widths for selected (2–12) H$_2$O lines has been investigated by a handful studies [39–42]. The works considering the widest range of temperatures and pressures are those by Langlois et al. [41] and Zeninari et al. [39] respectively. Langlois and combined two experimental set-ups, one incorporating a temperature controlled static cell (limited to 450 K), the other a pressure driven shock tube, to determine the temperature dependence of the pressure broadened widths of twelve H$_2$O lines over the range 300–1200 K with an expected accuracy of ± 25%. The dependence could be roughly represented by a power law of decreasing collision induced line width with increasing temperature. Zeninari et al. made measurements of six H$_2^{16}$O lines at several pressures between 6 and 1330 mbar (ambient pressure $\approx$ 1000 mbar) and concluded that the dependence could be described as a linear increase in collision induced line width with pressure.

Theoretically determined pressure broadened widths and their temperature dependence are available from Gamache et al. [43], Faure et al. [44] and Drouin and Wisenfeld [45]. Drouin and Wisenfeld focused on three lines at cold temperatures (below 200 K) while Gamache, Lynch et al. aimed to provide information for hundreds of lines suitable for high temperature applications. The maximum temperature considered by Faure et al. was hotter (1500 K vs 750 K) though Gamache et al. produced parameters for more lines (386 vs 228). All three works focused on rotational transitions.

Several of the studies already mentioned [38,35–37,43,40,42,39] also reported parameters for the H$_2$–H$_2$O–He system. Although the most extensive studies of He-broadening coefficients of H$_2^{16}$O have been performed by Petrova and co-workers [46–48], and Solodov and Starikov [49,50]. Each of these works presented measurements of pressure broadened widths for tens of lines belonging to strong vibrational bands including $\nu_2 + \nu_3$ [49,50,47,48], $\nu_1 + \nu_2$ [50,47,48], $\nu_1 + \nu_2 + \nu_3$ [46,47] and $2\nu_2$ and $\nu_1 + 2\nu_1$ [48] at room temperature and varying pressures. All found the pressure dependence to be linear (up to 3 atm [47,48]). Petrova et al. [47] combined their own measurements with literature data to determine the vibrationally and rotationally dependent intermolecular potential for the H$_2$O–He system. This potential was used by Petrova et al. [47,48] to compute, and fit an analytical expression to, helium pressure broadened width for transitions belonging to 11 and 13 vibrational bands respectively with rotational quantum numbers $J$ and $K_a$ up to 14 in the temperature interval $83 \leq T \leq 600$ K. The temperature dependence in the model was refined by comparison to previous temperature dependent studies by Goyette and De Lucia [51] (1 line at 183 GHz, 80–600 K) and Godon and Bauer [52] (2 lines at 183 and 380 GHz, 300–390 K). In addition room temperature pressure broadened widths for H$_2$O–He have been measured by Lazarev et al. [53] (1 line at 14,397.4 cm$^{-1}$), Poddar et al. [54] (14 lines in the range 11,988–12,218 cm$^{-1}$), Caveau et al. [55] (14 lines in the range 1850–2140 cm$^{-1}$) and Caveau and Valentin [56] (10 lines in the range 1170–1440 cm$^{-1}$). Caveau et al. and Caveau and Valentin also investigated narrowing due to dynamic confinement (Dicke narrowing [57,58]).

3. Calculation of H$_2$ and He pressure induced line widths of H$_2$O spectral lines

3.1. Theoretical techniques used

Two calculation techniques are used to determine the H$_2$O–H$_2$ and H$_2$O–He Lorentzian half-widths. These techniques distinguish between rigorous quantum numbers, namely the total angular momentum $J$ and total symmetry ($K_a$), which corresponds to purity and ortho/para designation, and approximate projections of the rotational motion ($K_c, K_s$). The techniques are:

1. The averaged energy difference (AED) method [59] is used in cases where the complete set of quantum numbers (both rigorous and approximate) of a transition are known.

2. The $J^T$ – dependence technique [60] has been applied in cases where only rigorous (total angular momentum $J$ and total symmetry) were known.

These approaches are described in detail in the respective papers, thus only the main features are outlined below.

The averaged energy difference method allows the calculation of line widths of asymmetric top molecules with approximately the same precision as in modern theoretical and experimental methods without the need for a complicated calculation scheme. The approach is based on matching the so called coupled energy state difference with a line broadening value. Based on the modified Robert-Bonamy formalism, the expression for the half-widths $\gamma_{ij}$ is given by [61,62]:

$$\gamma_{ij} = \frac{n_0}{2\pi c} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\nu f(\nu)dv}{\pi} \left[ 1 - \cos(S_1 + \text{Im}(S_2))e^{-\Re(S_2)} \right]$$

(1)

where indices $i$ and $f$ specify initial and final energy states, $n_0$ is the number density of perturbers, $b$ is the impact parameter, $\nu$ is the relative initial velocity and $f(\nu)$ is the Maxwell Boltzmann distribution function [59,61,62]. Real $S_1$ and complex $S_2 = \Re(S_2) + i\text{Im}(S_2)$ are the first and second order terms in the expression for the scattering matrix. These terms depend on collision dynamics, the intermolecular potential and on the ro-vibrational states of the molecule (and collision induced transitions between these states). Expressions for $S_1$ and $S_2$ can be found in Refs. [59,63–65].
The aim is to estimate how values of $\Re(S_2)$ vary with different lines of interest. Coupled energy state differences $\phi_f$ between an $^{16}$O-$^2$H line of interest and other coupled lines ($\phi_f$ is defined explicitly in [59]) are derived for thousands of lines. The averaged energy differences for lines with experimentally-determined collision-induced widths are then used to match averaged state energy differences to line broadening values using a fitting formula.

The dependence of broadening parameters on the averaged energy difference of the coupled states is smooth. Fig. 1 illustrates this for the $^{16}$O-$^2$H line widths. This figure demonstrates that one can estimate corresponding widths with a simple fitting formula. For other systems such $^{3}$H broadening of water the observed data has been found to lie almost exactly on a smooth curve [66]; here the scatter reflects, at least in part, uncertainty in the experimental data. The reconstructed widths obtained from the fitting formula agree well with the experimental values as shown in Fig. 2 for $^{16}$O-$^2$H. This method requires assignment to normal modes (via quantum numbers $v_1, v_2, v_3, K_a$ and $K_c$). However, these quantum numbers are not always known or, indeed, always valid [67].

For cases where only good quantum numbers ($J$ and $F_{\text{rot}}$) are defined, the $f_J - f_J$ dependence technique can be applied. Available widths are compiled and averaged for each value of $J$ per branch (P, Q and R) and total symmetry (ortho/para). The resulting values are used to derive functions which describe the dependence of the averaged width on $J$ per branch and per symmetry (if there is appreciable difference for the latter two). This method applied to $^{16}$O-$^2$H and $^{16}$O-$^3$He is illustrated in Fig. 3.

### 3.2. The $^{16}$O-$^2$H and $^{16}$O-$^3$He line widths

Both approaches described in the previous section were used to obtain separate $^{16}$O-$^2$H and $^{16}$O-$^3$He Lorentzian half-widths for BT2 lines in the range 500–10,000 cm$^{-1}$ with an intensity that exceeds $1 \times 10^{-30}$ cm$^2$/molecule in the temperature range 300–2000 K. Vibrational dependence is neglected as it is found to change the value of the half-widths by only a few percent. Dependence on $K_a$ and total symmetry was found to be much smaller than dependence on $K_a$ and $J - J$ respectively.

Our calculated values range between 0.0208 and 0.0927 cm$^{-1}$ atm$^{-1}$ for broadening by $^2$H$^2$ and 0.0043–0.0229 cm$^{-1}$ atm$^{-1}$ for broadening by He at 296 K. The average ratio between the $^2$H to He pressure induced line widths is 4.2. Broadening of water by He is much stronger than by He because the $^2$H molecule has a quadrupole moment. The electrostatic potential is given by an expansion of the charge distribution in terms of the electric moments of the molecules. So, the main contribution into $^{16}$O-$^2$H broadening value is from the dipole-quadrupole interaction, besides this term there is also a quadrupole–quadrupole interaction contribution. Helium does not have any electric moments, so only the polarization potential (interactions between induced moments) gives the contribution, which is very much weaker.

Tables 1 and 2 present a summary of the comparison of our calculated values with available experimental and previous theoretical studies. The root mean square deviations (RMSD) between this work and other works is given as a percentage of the current values. For the $^{16}$O-$^2$H system the RMSD is within 20% or the uncertainty quoted in the given reference for all studies. It should be noted that for the comparison to Zeninari et al., the RMSD of 19.1% is mainly down to the $6_{66} \rightarrow 6_{61}$ transition which is in fact an unresolved doublet [39]. The RMSD for the remaining five transitions is 8.4%.

For the $^{16}$O-$^3$He system the RMSD is within 26% for all studies. Again for the comparison to Zeninari et al., the RMSD of 15.0% is mainly down to the $6_{66} \rightarrow 6_{61}$ transition. The RMSD for the
and a″ is the reference temperature, 2s Code identifying quantum number set for the temperature range 300–2000 K to sufficient accuracy (within 3%). For high accuracy treatments other issues arise with use of the Voigt profile [69].

The pressure dependence of the Lorentzian half-widths in the range 0.001 - 10 bar is assumed to be linear:

$$\gamma(P) = \gamma_{ref} \times \left( \frac{P}{P_{ref}} \right)$$

(3)

where $P_{ref} = 1$ bar is the reference pressure. This is indicated by available pressure dependent experimental investigations up to around ambient pressure for broadening by H₂ [39], and up to around 3 atmospheres ($\approx 3$ bar) for broadening by He [47,48]. Although measurements at the high end of the pressure range (~10 bar) would be helpful to verify this; in particular high pressure leads to three-body effects which are implicitly neglected in our formulation.

### 3.4. The broad files

In principle parameters were calculated for around 4 million BT2 lines with an intensity that exceeds $1 \times 10^{-30}$ cm$^2$/molecule in the temperature range 300–2000 K. In practice, since vibrational dependence was neglected and dependence on total symmetry and rotational quantum number $K_z$ was found to be comparatively small, the results comprise of around 23,000 widths that depend on $J$, $J'$, $K_a$ and $K_z$ and 100 widths that depend on $J'$ only. This can be represented in new ExMoMol format [3] as two .broad files:

1H₂-16O_H₂_NLAD.broad, extract shown in Table 3.
1H₂-16O_He_NLAD.broad, extract shown in Table 4.

These files and Eq. (4) below can be used to generate a pressure and temperature dependent Lorentzian half-width for any water vapour line, with at least $J' - J$ quantum assignments, for a pure H₂, pure He or mixed H₂/He environment.

$$\gamma(T, P) = \left( \frac{T_{ref}}{T} \right)^{n} \times \left( \frac{P}{P_{ref}} \right) \times \sum_b \gamma_{b} P_b^2$$

(4)

where $\gamma_{b}$ is the Lorentzian half-width due to a specific broadener.

### Table 1

Root mean square deviations (RMSD) of calculated values from available data on H$_2^{16}$O–H$_2$ broadening; $\gamma$ is the half-width and $n$ the temperature exponent. In many cases there are no experimental data available for comparison.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Number of transitions</th>
<th>RMSD $\gamma$ (%)</th>
<th>RMSD $n$</th>
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<tbody>
<tr>
<td>Experiment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steyert et al. [35]</td>
<td>39</td>
<td>10.4</td>
<td>–</td>
</tr>
<tr>
<td>Brown and Pymate et al. [34]</td>
<td>630</td>
<td>7.4</td>
<td>–</td>
</tr>
<tr>
<td>Brown et al. [36]</td>
<td>4</td>
<td>8.8</td>
<td>–</td>
</tr>
<tr>
<td>Golubiatnikov [37]</td>
<td>1</td>
<td>13.9</td>
<td>–</td>
</tr>
<tr>
<td>Zeninari et al. [39]</td>
<td>6</td>
<td>19.1</td>
<td>–</td>
</tr>
<tr>
<td>Langlois et al. [41]</td>
<td>11</td>
<td>17</td>
<td>36.9 %</td>
</tr>
<tr>
<td>Dutta et al. [42]</td>
<td>2</td>
<td>7.3</td>
<td>–</td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gamache et al. [43]</td>
<td>386</td>
<td>6.2</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>4.9</td>
<td>%</td>
</tr>
<tr>
<td>Faure et al. [44]</td>
<td>228</td>
<td>24.1</td>
<td>18.2 %</td>
</tr>
</tbody>
</table>

### Table 2

Root mean square deviations (RMSD) of calculated values from available data on H$_2^{16}$O–He broadening; $\gamma$ is the half-width and $n$ the temperature exponent.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Number of transitions</th>
<th>RMSD $\gamma$ (%)</th>
<th>RMSD $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petrova et al. [48]</td>
<td>103</td>
<td>17.1</td>
<td>–</td>
</tr>
<tr>
<td>Petrova et al. [47]</td>
<td>150</td>
<td>17.7</td>
<td>–</td>
</tr>
<tr>
<td>Petrova et al. [46]</td>
<td>105</td>
<td>15.5</td>
<td>–</td>
</tr>
<tr>
<td>Solodov and Starikov [49]</td>
<td>32</td>
<td>14.0</td>
<td>–</td>
</tr>
<tr>
<td>Solodov and Starikov [50]</td>
<td>33</td>
<td>13.9</td>
<td>–</td>
</tr>
<tr>
<td>Coyette and De Luca [51]</td>
<td>1</td>
<td>1.7</td>
<td>42.2 %</td>
</tr>
<tr>
<td>Godon and Bauer [52]</td>
<td>2</td>
<td>10.3</td>
<td>13.6 %</td>
</tr>
<tr>
<td>Poddar et al. [54]</td>
<td>14</td>
<td>14.0</td>
<td>–</td>
</tr>
<tr>
<td>Claveau and Valentin [56]</td>
<td>10</td>
<td>25.7</td>
<td>–</td>
</tr>
<tr>
<td>Claveau et al. [55]</td>
<td>14</td>
<td>24.7</td>
<td>–</td>
</tr>
<tr>
<td>Steyert et al. [35]</td>
<td>39</td>
<td>23.3</td>
<td>–</td>
</tr>
<tr>
<td>Brown et al. [36]</td>
<td>4</td>
<td>11.1</td>
<td>–</td>
</tr>
<tr>
<td>Dutta et al. [42]</td>
<td>2</td>
<td>9.2</td>
<td>–</td>
</tr>
<tr>
<td>Zeninari et al. [39]</td>
<td>6</td>
<td>15.0</td>
<td>–</td>
</tr>
<tr>
<td>Golubiatnikov [37]</td>
<td>4</td>
<td>4.0</td>
<td>–</td>
</tr>
<tr>
<td>Lazarev et al. [51]</td>
<td>1</td>
<td>7.1</td>
<td>–</td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gamache et al. [43]</td>
<td>386</td>
<td>6.2</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>4.9</td>
<td>%</td>
</tr>
</tbody>
</table>

remaining five transitions is 8.6 %.

### 3.4. Temperature and pressure dependence of the H$_2^{16}$O–H$_2$/He line widths

In order to determine the temperature dependence, the half-width calculations were made at the following temperatures: $T = 300, 400, 500, 600, 700, 800, 900, 1000, 1200, 1400, 1600, 1800, 2000$ K. The temperature dependence, represented by exponent $n$, was obtained by fitting to the standard relation:

$$\gamma(T) = \gamma_{ref} \times \left( \frac{T_{ref}}{T} \right)^{n}.$$

(2)

where $T_{ref} = 296$ K is the reference temperature, $\gamma_{ref}$ is the Lorentzian half-width at reference temperature and $n$ is the temperature exponent. The temperature exponents vary from 0.866 to 0.027 for broadening by H₂ and from 0.5 to 0.02 for broadening by He. Comparisons with experimentally derived and previously calculated values are summarised in Tables 1 and 2.

Recently Wilzewski et al. [68] commented that a single power law of the form Eq. (2) only works well within relatively narrow temperature intervals. However they also note that there is not enough experimental data to characterise an alternative model at present, and we find Eq. (2) reproduces our calculated values for the temperature range 300–2000 K to sufficient accuracy (within 3%). For high accuracy treatments other issues arise with use of the Voigt profile [69].

### Table 3

1H₂-16O_H₂_NLAD.broad: Extract of H$_2^{16}$O–H₂ broad file: portion of the file (upper part); field specification (lower part).

<table>
<thead>
<tr>
<th>Code</th>
<th>A2</th>
<th>% s</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a3</td>
<td>0.0690</td>
<td>0.502</td>
<td>Code identifying quantum number set following $J'$</td>
</tr>
<tr>
<td>a3</td>
<td>0.0580</td>
<td>0.424</td>
<td>$J'$ quantum number</td>
</tr>
<tr>
<td>a3</td>
<td>0.0754</td>
<td>0.541</td>
<td>$K_a$ quantum number</td>
</tr>
<tr>
<td>a3</td>
<td>0.0342</td>
<td>0.253</td>
<td>$K_a$ quantum number</td>
</tr>
<tr>
<td>a3</td>
<td>0.0328</td>
<td>0.241</td>
<td>$K_a$ quantum number</td>
</tr>
<tr>
<td>a3</td>
<td>0.0317</td>
<td>0.240</td>
<td>$K_a$ quantum number</td>
</tr>
</tbody>
</table>

* Code definitions: a3—parameters presented as a function of $J'$ (compulsory) and $J$, $K_a'$ and $K_a''$, a1—parameters presented as a function of $J'$ (compulsory) and $J$. 

1H₂-16O_He_NLAD.broad: Extract of H$_2^{16}$O–He broad file: portion of the file (upper part); field specification (lower part).
Table 4
182-160_H2O_NLAD.broad: extract of H2\textsuperscript{18}O-He broad file: portion of the file (upper part); field specification (lower part).

<table>
<thead>
<tr>
<th>Field</th>
<th>Format</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a3</td>
<td>%2s</td>
<td>Code identifying quantum number set following $f^\alpha$</td>
</tr>
<tr>
<td>h retal</td>
<td>%6.4f</td>
<td>Lorentzian half-width at reference temperature and pressure in cm\textsuperscript{-1}/bar</td>
</tr>
<tr>
<td>n</td>
<td>%5.3f</td>
<td>Temperature exponent</td>
</tr>
<tr>
<td>$J'$</td>
<td>%7d</td>
<td>Lower $J$-quantum number</td>
</tr>
<tr>
<td>$J''$</td>
<td>%7d</td>
<td>Upper $J$-quantum number</td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>%2d</td>
<td>Lower rotational quantum number</td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>%2d</td>
<td>Upper rotational quantum number</td>
</tr>
</tbody>
</table>

* Code definitions: a3—parameters presented as a function of $J'$ (compulsory) and $J$, $K_\alpha$ and $K_\alpha$. a1—parameters presented as a function of $J''$ (compulsory) and $J$.

in units cm\textsuperscript{-1}/bar (ExoMol convention) and $p_\text{a}$ is the partial pressure of the broadener. Note that to convert from cm\textsuperscript{-1}/bar used by ExoMol to cm\textsuperscript{-1}/atm used by Hitran requires $\gamma$ to be multiplied by 1.01325.

The .broad files presented as part of this work differ slightly from those included in the ExoMol database. The .broad files provided with the ExoMol line lists, described in [3], contain data from multiple sources while the .broad files available as supplementary data to this work contain data from only one source, the current work. Hence, the file names are appended with a dataset name NLAD. We recommend the use of the full .broad file presented on the ExoMol website as this includes both line-by-line parameters where available, which are essential for detailed and high resolution studies, while the current model which allows a width to be generated for every molecular line.

4. Computation of H$_2^{16}$O absorption cross-sections

4.1. Method

The high resolution cross section is calculated on an evenly spaced wavenumber grid, $\tilde{\nu}$, defining bins of width $\Delta \nu$. A Voigt profile is used to model the joint contributions from thermal and collision induced broadening:

$$f_v(\tilde{\nu}, \tilde{\nu}_{0j}, \alpha_j, \gamma_j) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\tilde{\nu} - \tilde{\nu}_{0j})^2}{\alpha_j^2} + \frac{i \gamma_j}{\alpha_j} \ln 2\right)$$

where wofz is the scaled complex complementary error function, also known as the Faddeeva function. This is calculated using the Faddeeva package [70]. $\tilde{\nu}_{0j}$ is the line centre, $\gamma_j$ is the Lorentzian half-width at half-maximum and $\alpha_j$ is the Doppler half-width at half-maximum given by:

$$\alpha_j = \sqrt{\frac{2kT \ln 2 \tilde{\nu}_{0j}}{m c}}$$

at temperature $T$ in K for a molecule of mass $m$ in kg. Note that in the limit $\alpha \approx \gamma$, the profile reduces to a Gaussian:

$$f_v(\tilde{\nu}, \tilde{\nu}_{0j}, \alpha_j) = \frac{\ln 2}{\pi \alpha_j} \exp\left(-\frac{(\tilde{\nu} - \tilde{\nu}_{0j})^2}{\alpha_j^2} \ln 2\right)$$

whereas for $\alpha > \gamma$ the profile reduces to a Lorentzian:

$$f_v(\tilde{\nu}, \tilde{\nu}_{0j}, \alpha_j, \gamma_j) = \frac{\gamma_j}{\pi (\tilde{\nu} - \tilde{\nu}_{0j})^2 + \gamma_j^2}$$

However a Voigt profile is evaluated for every absorption line in all calculations of cross sections presented in this work. The cross section for each bin is the sum of the contributions from individual lines:

$$\sigma_i = \sum_j \sigma_{ij}$$

where:

$$\sigma_{ij} = S_j f_v(\tilde{\nu}, \tilde{\nu}_{0j}, \alpha_j, \gamma_j).$$

$S_j$ is the line intensity in units of cm per molecule given by:

$$S_j = \frac{A_j \, g_j^\alpha \, \exp\left(-\frac{\gamma_j}{\alpha_j} Q(T)\right)}{\, \tilde{\nu}_{0j}^\alpha \, Q(T)} \left(1 - \exp\left(-\frac{\gamma_j}{\alpha_j} \tilde{\nu}_{0j}\right)\right)$$

Here, $g_j^\alpha$ and $E_j^\alpha$ are the total upper-state degeneracy and lower-state energy respectively, $A_j$ is the Einstein A coefficient for the transition and $\gamma_j = h c/\nu_0$ is the second radiation constant. $\tilde{\nu}_{0j}$ and $A_j$ were taken from the Bf2 line list while the molecular partition function, $Q(T)$, was obtained from the tabulated values of Vidler and Tennyson [71].

When evaluating a Voigt profile it is necessary to select an appropriate profile grid resolution and evaluation width in order to adequately sample the contribution from the profile whilst considering the computational cost. For the profile grid resolution we adopted the staggered wavenumber grid spacings of [33] (see Table 5). These spacings are well below the Voigt width for the $f^\alpha$ parameter set presented on the ExoMol website as this includes both line-by-line parameters where available, which are essential for detailed and high resolution studies, while the current model which allows a width to be generated for every molecular line.

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Table 5
Summary of the grid spacings for the cross sections calculated in difference wavenumber regions.

<table>
<thead>
<tr>
<th>Wavenumber range cm\textsuperscript{-1}</th>
<th>Grid spacing cm\textsuperscript{-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–100</td>
<td>10\textsuperscript{5}</td>
</tr>
<tr>
<td>100–1000</td>
<td>10\textsuperscript{4}</td>
</tr>
<tr>
<td>1000–10,000</td>
<td>10\textsuperscript{3}</td>
</tr>
<tr>
<td>10,000–30,000</td>
<td>10\textsuperscript{2}</td>
</tr>
</tbody>
</table>
half-widths. Their effective Voigt half-width was defined as \[\gamma = 0.5346 \sqrt{0.2166\gamma_j^2 + \gamma_G^2}\] (12)

where \(\gamma_G = \gamma_j = \gamma_j\) and \(\gamma_G \propto (\gamma_j + \gamma_G)\) for comparison to this work. Hedges & Madhusudhan found that both the fixed cut-off method and Lorentzian half-width analysis resulted in significant (>40%) opacity loss at the low end of the pressure range (\(P < 0.01\) bar) compared to the Voigt half-width analysis. As our approach is very similar to Hedges & Madhusudhan we should also avoid this loss of opacity.

4.2. Results

Pressure dependent absorption cross sections for \(\text{H}_2\text{O}\) in a mixed \(\text{H}_2/\text{He}\) environment (85/15% by number) are calculated on a fixed temperature and pressure grid (see Table 6) using the BT2 line list, pressure broadening parameters determined as part of this work (Section 1) and the method described above. However, no attempt is made include contributions to the opacity from the water vapour continuum or water dimer absorption.

The cross sections were calculated between 10 and 30,000 cm\(^{-1}\) using the staggered wavenumber grid given in Table 5 and then binned to a common grid spacing of 0.01 cm\(^{-1}\). Each region was calculated to overlap with its neighbours by at least 1 cm\(^{-1}\), which we find to be sufficient to avoid discontinuities when they are binned to a common grid spacing.

4.3. Interpolation of cross-sections between temperatures and pressures

Cross sections are provided for 12 temperatures and 22 pressures between 300 and 2000 K and 0.001 and 10 bar respectively (see Table 6). A cross section at intermediate conditions may be obtained by first interpolating on the temperature grid using either Eqs. (14) or (16) below [33] and then interpolating on the pressure grid using Eq. (19) below. The interpolation residual is expressed as a percentage of the corresponding absorption cross-section:

\[
\delta\sigma_{\text{max}} = \max \left( \frac{\sigma_{\text{calc}} - \sigma_{\text{interpl}}}{\sigma_{\text{calc}}} \right) \times 100
\] (13)

To obtain cross sections at an intermediate temperature \(T\), interpolation between cross sections computed for a higher temperature \(T_2\) and a lower temperature \(T_1\) may be performed linearly [33]:

\[
\sigma_i = \sigma_i(T_1) + m(T - T_1)
\] (14)

where:

\[
m = \frac{\sigma_i(T_2) - \sigma_i(T_1)}{T_2 - T_1}
\] (15)

Or using a more accurate exponential model [33]:

\[
\sigma_i = \sigma_i(T_1) e^{b_i(T - T_1)}
\] (16)

where:

\[
b_i = \left( \frac{1}{T_2} - \frac{1}{T_1} \right)^{-1} \ln \frac{\sigma_i(T_1)}{\sigma_i(T_2)}
\] (17)

and:

\[
a_i = \sigma_i(T_1) e^{b_i T_1}
\] (18)

The error in the cross sections introduced by the more accurate interpolation scheme does not exceed 1.64% which is less than the estimated uncertainty in the \textit{ab initio} line lists.
To obtain cross sections at an intermediate pressure $P_2$ interpolation between cross sections computed for a higher pressure $P_1$ and a lower pressure $P_1$ may be performed linearly:

$$
\sigma_1 = \sigma_1(P_2) + m(P_2 - P_1)
$$

where:

$$
m = \frac{\sigma_1(P_2) - \sigma_1(P_1)}{P_2 - P_1}
$$

This results in interpolation residuals below 2.6% for the region 6000–30,000 cm$^{-1}$ for all pressures. However, below 6000 cm$^{-1}$ for low pressures and below 1000 cm$^{-1}$ for high pressures, there are spikes in interpolation residual ($>10\%$) for individual wavenumber bins, notably in the wings of strong features. The wings of strong lines add appreciable amounts of opacity to wavenumber bins where there previously was negligible opacity in an non-linear fashion. In reality this effects only a small fraction of the bins ($\sim0.7\%$) and changes the total cross section in the region $10$–$6000$ cm$^{-1}$ by less than 0.002% at worst, hence this should have negligible effect on practical applications.

5. Conclusion

H$_{16}$O line widths pressure-broadened by hydrogen and helium were calculated using the averaged energy difference method and $J$–$J'$-dependence technique. Rotational quantum numbers $J$ up to 50 were considered. The temperature dependence of the widths was derived from calculations made at temperatures in the range 300–2000 K. The calculated data are in reasonable agreement with available experiment.

The widths and temperature exponents are presented in the new ExoMol format as dataset exclusive .

It is our intention to make the cross sections available through the ExoMol cross section service, a web-based interface (www.exomol.com/data/data-types/xsec/) which currently allows astronomers to download zero pressure cross sections for available molecules at user defined temperatures and spectral resolution. The form chosen for representing the pressure-broadening parameters is based on the quantum numbers of the upper and lower levels. This means that the parameters are transferable to other extensive water line lists such as new, complete and more accurate POKAZATEL line list [73] which will be released soon.

Furthermore, the differences in pressure effects between H$_2$ and H$_2$O and H$_2$O should be very small. This means that the broadening file presented here should also be appropriated for the newly computed H$_2$O and H$_2$O hot line lists [74].

Pressure dependent absorption cross sections for other key species in the atmospheres of exoplanets and cool stars (for example NH$_3$, CO, CO$_2$ and CH$_4$) will also be included here in due course [75].

Acknowledgements

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References


[22] Barman TS, Konopacky QM, Macintosh BA, Marois C. Simultaneous detection of