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Citation: J. Chem. Phys. 122, 104317 (2005); doi: 10.1063/1.1862620
View online: https://doi.org/10.1063/1.1862620
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Published by the American Institute of Physics

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Dipole moment and rovibrational intensities in the electronic ground state of NH₃: Bridging the gap between ab initio theory and spectroscopic experiment

Sergei N. Yurchenko
Max-Planck-Institut für Kohlenforschung, Kaiser-Wilhelm-Platz 1, D-45470 Mülheim an der Ruhr, Germany

Miguel Carvajal
Departamento de Física Aplicada, Facultad de Ciencias Experimentales, Avenida de las FF.AA. s/n, Universidad de Huelva, 21071 Huelva, Spain

Hai Lin
Department of Chemistry, University of Minnesota, Minneapolis, Minnesota 55455

Jingjing Zheng and Walter Thiel
Max-Planck-Institut für Kohlenforschung, Kaiser-Wilhelm-Platz 1, D-45470 Mülheim an der Ruhr, Germany

Per Jensen
FB C-Theoretische Chemie, Bergische Universität, D-42097 Wuppertal, Germany

(Received 22 December 2004; accepted 6 January 2005; published online 15 March 2005)

We report theoretical values for the transition moments of an extensive set of vibrational bands in the electronic ground state of ¹⁴NH₃. For selected bands, we have further made detailed simulations of the rotational structure. The calculations are carried out by means of recently developed computational procedures for describing the nuclear motion and are based on a high-level ab initio potential energy surface, and high-level dipole moment surfaces, for the electronic ground state of NH₃. The reported theoretical intensity values are compared to, and found to agree very well with, corresponding experimental results. It is believed that the computational method, in conjunction with high-quality ab initio potential energy and dipole moment surfaces, can simulate rotation-vibration spectra of XY₃ pyramidal molecules prior to observation with sufficient accuracy to facilitate the observation of these spectra. By degrading the accuracy of selected elements of the calculations, we have also investigated the influence of customary approximations on the computed intensity values. © 2005 American Institute of Physics. [DOI: 10.1063/1.1862620]

I. INTRODUCTION

Modern ab initio calculations of molecular potential energy surfaces, coupled with high-level treatments of the nuclear motion, provide very accurate descriptions of the properties of isolated molecules. In particular, the theoretical calculations often yield molecular energy levels in very good agreement with the results of spectroscopic experiments. One could think that owing to the progress in theoretical methods, the assignment of molecular spectra would become increasingly trivial: The spectrum, or at least the transition frequencies, can be predicted from first principles with sufficient accuracy that the assignment of each individual transition can be straightforwardly obtained from the theoretical calculation. There are examples, such as the water molecule,¹² where such a “first-principles assignment” can be made, but present-day assignment of spectra is still largely based on the more traditional methods of general pattern recognition, search for combination differences, and the least-squares fitting of the frequencies for the assigned transitions, with subsequent prediction of the frequencies for unassigned lines, by means of models involving effective rotation-vibration parameters (see, for example, Ref. 3).

In recent papers⁴–⁸ we have described the development and application of a theoretical model for simulating rotation-vibration spectra for isolated electronic states of XY₃ pyramidal molecules. The applications, so far, have been to the NH₃ (Refs. 4, 6, and 8) and PH₃ (Refs. 5 and 7) molecules. Initially,⁴,⁶ we developed a computational procedure for calculating rotation-vibration energies and wave functions of XY₃ molecules from an ab initio potential energy surface. With energies for ¹⁴NH₃, obtained with this procedure from a high-quality ab initio potential energy surface,⁴,⁹ we were able to assist the assignment of high-resolution molecular spectra, in that we could verify—for the most part—the tentative assignment to the 4ν₂ band⁴ of 55 weak transitions observed in an experimental study of the ν₁, ν₃, and 2ν₃ bands of ¹⁴NH₃.

Our theoretical model for the rotation and vibration of an XY₃ molecule (for details, see Ref. 6) is a variational (i.e., perturbation-theory-free) implementation of the Hougen–Bunker–Johns (henceforth HBJ) approach.¹¹,¹² The HBJ approach is designed to provide maximum separation of the rotational and vibrational motions in the quantum-

⁴Author to whom correspondence should be addressed. Electronic mail: yurchenko@mpi-muelheim.mpg.de

⁵Author to whom correspondence should be addressed. Electronic mail: thiel@mpi-muelheim.mpg.de
mechanical description, and so our model is particularly suitable for calculating the energies and wave functions of highly excited rotational states. So, recently we could generate energies and wave functions for states with \( J \leqslant 80 \) in the vibrational ground state of \( \text{PH}_3 \) and study the formation of sixfold clusters of rotational energies; these clusters are analogous to the fourfold clusters extensively discussed for \( \text{XH}_2 \) molecules (see Refs. 12 and 13 and references therein).

Since we can treat highly excited rotational states of \( \text{XY}_3 \) molecules, we can generate the energies and wave functions necessary to simulate realistic rotation-vibration spectra of them. Such simulations obviously help “bridging the gap” between \textit{ab initio} theory and experimental spectroscopy; they are a prerequisite for the first-principles assignments mentioned above. We have already described\(^8\) the extension of our theoretical model to the computation of line strengths (see below) and intensities for electric dipole transitions within an isolated electronic state of an \( \text{XY}_3 \) pyramidal molecule. We have also reported\(^8\) the calculation of the vibrational ground state of \( \text{PH}_3 \) and study the formation of rotational energies and wave functions for states with

\[
\rho = \sum_{j=1}^{3} (A^{-1})_{ij} (\tilde{\mu} \cdot e_j),
\]

6D-1 grid in Ref. 4) consisting of 14 400 unique geometries that form a regular grid in the range \( 0.85 \, \text{Å} \leq r_i \leq 1.20 \, \text{Å} \) and \( 80^\circ \leq \alpha_i \leq 120^\circ \). Here, \( r_i \) is the instantaneous value of the internuclear distance \( \text{N}–\text{H}_i \), \( i = 1, 2, 3 \), and the bond angles are given as \( \alpha_1 = \angle (\text{H}_2\text{NH}_3) \), \( \alpha_2 = \angle (\text{H}_1\text{NH}_3) \), and \( \alpha_3 = \angle (\text{H}_1\text{NH}_2) \) [see Fig. 1].

In the \textit{ab initio} calculations the components of the molecular dipole moment are given in a right-handed Cartesian axis system \( x'y'z' \) with origin in the nitrogen nucleus. The \( \text{H}_1 \) nucleus lies on the \( z' \) axis with a positive value of the \( z' \) coordinate, and the \( y'z' \) plane is defined by the nitrogen nucleus and the protons \( \text{H}_1 \) and \( \text{H}_2 \).

B. A general analytical representation of the dipole moment

In the so-called molecular bond (MB) representation,\(^8,12\) the electronically averaged dipole moment vector \( \overline{\mu} \) for \( \text{NH}_3 \) is given by

\[
\overline{\mu} = \mu_1 e_1 + \mu_2 e_2 + \mu_3 e_3,
\]

where the three functions \( \mu_i \), \( i = 1, 2, 3 \), depend on the vibrational coordinates, and \( e_i \) is the unit vector along the \( \text{N}–\text{H}_i \) bond,

\[
e_i = \frac{\mathbf{r}_i - \mathbf{r}_4}{|\mathbf{r}_i - \mathbf{r}_4|},
\]

with \( \mathbf{r}_i \) as the position vector of nucleus \( i \) (the protons are labeled 1, 2, 3, and the nitrogen nucleus is labeled 4, see Fig. 1) in the axis system \( x'y'z' \) defined above. As discussed in Ref. 8, the representation of \( \overline{\mu} \) in Eq. (1) is “body fixed” in the sense that it relates the dipole moment vector directly to the instantaneous positions of the nuclei (i.e., to the vectors \( \mathbf{r}_i \)). In consequence, we can use Eq. (1) to obtain the coordinates of \( \overline{\mu} \) in any axis system.

Our implementation of the MB representation for the electronic-ground-state dipole moment of \( \text{NH}_3 \) is detailed in Ref. 8; here we give only a brief outline. We express the three functions \( \mu_i \), \( i = 1, 2, 3 \), as

\[
\mu_i = \sum_{j=1}^{3} (A^{-1})_{ij} (\tilde{\mu} \cdot e_j),
\]

with \( (A^{-1})_{ij} \) is an element of the nonorthogonal 3 × 3 matrix \( A^{-1} \) obtained as the inverse of

II. THE MOLECULAR DIPOLE MOMENT

A. The \textit{ab initio} calculation

As discussed in Ref. 8, the \textit{ab initio} dipole moment values employed in the present work were computed with the MOLPRO2000 (Refs. 14 and 15) package at the CCSD(T)/aug-cc-pVTZ level of theory (i.e., coupled cluster theory with all single and double substitutions\(^{16}\) and a perturbative treatment of connected triple excitations\(^{17,18}\) with the augmented correlation-consistent triple \( \zeta \) basis\(^{19,20}\) in the frozen-core approximation (abbreviated as ATZFc level). Dipole moments were computed in a numerical finite-difference procedure with an added external dipole field of 0.005 a.u. The convergence thresholds were \( 10^{-10} \) for density and \( 10^{-7} \) a.u. for energy in Hartree–Fock calculations, and \( 10^{-10} \) a.u. for energy and \( 10^{-10} \) for coefficients in CCSD(T) computations.

The \textit{ab initio} dipole moment surface used in the present study has been determined on a six-dimensional grid (the

FIG. 1. The labeling of the nuclei, the molecule-fixed axis system \( xyz \), and selected coordinates employed for \( \text{NH}_3 \) (see text).
\[
A = \begin{pmatrix}
1 & \cos \alpha_3 & \cos \alpha_2 \\
\cos \alpha_3 & 1 & \cos \alpha_1 \\
\cos \alpha_2 & \cos \alpha_1 & 1
\end{pmatrix}.
\]

(4)

When the molecule is planar, i.e., when \(\alpha_1 + \alpha_2 + \alpha_3 = 2\pi\), the determinant \(|A| = 0\) and \(A\) cannot be inverted. For planar geometries \(e_1, e_2, e_3\) and \(e_i\) are linearly independent and there are infinitely many possible values of \((\bar{\mu}_1^\text{Bond}, \bar{\mu}_2^\text{Bond}, \bar{\mu}_3^\text{Bond})\). In this case we set \(\bar{\mu}_3^\text{Bond} = 0\) in Eq. (1) and express \(\bar{\mu}\) in terms of \(e_1\) and \(e_2\) only, i.e., we determine \(\bar{\mu}_1^\text{Bond}\) and \(\bar{\mu}_2^\text{Bond}\) in terms of \(\bar{\mu} \cdot e_1\) and \(\bar{\mu} \cdot e_2\).

We have shown in Ref. 8 that the projections \(\bar{\mu} \cdot e_j, j = 1, 2, 3\), in Eq. (3) can be expressed in terms of the geometrically defined coordinates \(r_1, r_2, r_3, \alpha_1, \alpha_2\), and \(\alpha_3\). By utilizing that a permutation of the protons in \(\text{NH}_3\) does not change the molecular dipole moment \(\bar{\mu}\) we further derived\(^8\) that all three projections are given in terms of a single function \(\bar{\mu}_0(r_1, r_2, r_3, \alpha_1, \alpha_2, \alpha_3)\):

\[
\bar{\mu} \cdot e_1 = \bar{\mu}_0(r_1, r_2, r_3, \alpha_1, \alpha_2, \alpha_3) = \bar{\mu}_0(r_1, r_3, r_2, \alpha_1, \alpha_3, \alpha_2),
\]

(5)

\[
\bar{\mu} \cdot e_2 = \bar{\mu}_0(r_2, r_3, r_1, \alpha_2, \alpha_3, \alpha_1) = \bar{\mu}_0(r_2, r_1, r_3, \alpha_2, \alpha_1, \alpha_3),
\]

(6)

\[
\bar{\mu} \cdot e_3 = \bar{\mu}_0(r_3, r_1, r_2, \alpha_3, \alpha_1, \alpha_2) = \bar{\mu}_0(r_3, r_2, r_1, \alpha_3, \alpha_2, \alpha_1).
\]

(7)

This function is expressed as an expansion

\[
\bar{\mu}_0 = \sum_k \mu_k(0) \bar{\xi}_k + \sum_{k,l} \mu_{k,l}(0) \bar{\xi}_k \bar{\xi}_l + \sum_{k,l,m} \mu_{k,l,m}(0) \bar{\xi}_k \bar{\xi}_l \bar{\xi}_m + \cdots,
\]

(8)

in the variables

\[
\bar{\xi}_k = r_k \exp(-\beta^2 r_k^2), \quad k = 1, 2, 3,
\]

(9)

\[
\bar{\xi}_l = \cos(\alpha_{l-3}) - \cos \left(\frac{2\pi}{3}\right) = \frac{1}{2} + \cos(\alpha_{l-3}), \quad l = 4, 5, 6,
\]

(10)

which are chosen such that \(\bar{\mu}_0 = 0\) for \((r_1 = r_2 = r_3 = 0, \alpha_1 = \alpha_2 = \alpha_3 = 2\pi/3)\). Following Marquardt et al.\(^{22}\) we have introduced the factor \(\exp(-\beta^2 r_k^2)\) in order to keep the expansion in Eq. (8) from diverging at large \(r\).

The function \(\bar{\mu}_0(r_1, r_2, r_3, \alpha_1, \alpha_2, \alpha_3)\) is invariant to the simultaneous interchanges \(r_2 \leftrightarrow r_3\) and \(\alpha_2 \leftrightarrow \alpha_3\) [Eq. (5)] and, therefore, the expansion coefficients \(\mu_k(0), \mu_{k,l}(0), \mu_{k,l,m}(0)\), and \(\mu_{k,l,m,n}(0)\) in Eq. (8) are subject to constraints. In general, we have

\[
\mu_{k,l,m}^{(0)} = \mu_{k,l,m}^{(0)} = \mu_{k,l,m}^{(0)}.
\]

(11)

if the indices \(k', l', m', \ldots\) are obtained from \(k, l, m, \ldots\) by replacing all indices 2 by 3, all indices 3 by 2, all indices 5 by 6, and all indices 6 by 5. For instance, \(\mu_2 = \mu_3, \mu_5 = \mu_6, \mu_{2,2} = \mu_{3,3}, \mu_{2,3} = \mu_{3,2}, \mu_{5,6} = \mu_{6,5}, \mu_{2,2} = \mu_{1,3,3},\) and \(\mu_{1,2,6} = \mu_{1,3,5}\).

We have determined the values of the expansion parameters in Eq. (8), which we take to fourth order, in a least-squares fitting to the \(3 \times 14,440\) \textit{ab initio} dipole moment projections \(\bar{\mu} \cdot e_j, j = 1, 2, 3\), calculated \textit{ab initio} for \(\text{NH}_3\) at the CCSD(T)/aug-cc-pVTZ level of theory (Sec. II A). Details of this fitting are given in Ref. 8. We could usefully vary 91 parameters in the final fitting, which had a root-mean-square (rms) deviation of 0.0006 D. Table I lists the optimized parameter values. Parameters, whose absolute values were determined to be less than their standard errors in initial fittings, were constrained to zero in the final fitting and omitted from the table. Furthermore, we give in the table only one member of each parameter pair related by Eq. (11).

As already reported,\(^4\) the ATZfc dipole moment surface gives rise to an “equilibrium” moment of \(\mu_e = 1.5198\) D at the ATZfc \textit{ab initio} equilibrium geometry of \(r_1 = r_2 = r_3 = r_e = 1.0149\) Å and \(\alpha_1 = \alpha_2 = \alpha_3 = 106.4^\circ\). The experimental value\(^23\) for \(\mu_e\) is \((1.561 \pm 0.005)\) D.

### III. COMPUTATIONAL DETAILS

#### A. Rotation-vibration wave functions

We wish to calculate the intensities of electric dipole transitions within the ground electronic state of \(\text{NH}_3\). We consider a transition from an initial state \(i\) with rotation-vibration wave function \(|\Phi_i^{(w)}\rangle\) to a final state \(f\) with rotation-vibration wave function \(|\Phi_f^{(w)}\rangle\). As discussed at length in Ref. 6, the rotation-vibration wave functions \(|\Phi_i^{(w)}\rangle\) and \(|\Phi_f^{(w)}\rangle\) are obtained, together with the associated rotation-vibration energies, in variational calculations, i.e., by diagonalization of a matrix representation of the rovibrational Hamiltonian. Consequently, the wave functions are expressed as linear combinations of basis functions [see Eq. (65) of Ref. 6]:

\[
|\Phi_{rv}^{(w)}\rangle = \sum_{\nu \kappa r_{\text{tot}}} C_{\nu \kappa r_{\text{tot}}}^{(w)} |J_u K_m \nu_{\text{rot}}\rangle |V\rangle, \quad w = i \text{ or } f,
\]

(12)

where \(C_{\nu \kappa r_{\text{tot}}}^{(w)}\) are expansion coefficients, \(|J_u K_m \nu_{\text{rot}}\rangle\) is a symmetrized rotational basis function, and the vibrational function \(|V\rangle\) is given by

\[
|V\rangle = |n_{1}\rangle|n_{2}\rangle|n_{3}\rangle|n_{b, l_b, \tau_{\text{bend}}}\rangle|n_{1, J_u, K, \tau_{\text{inv}}}\rangle.
\]

(13)

All the functions \(|n_{1}\rangle, |n_{2}\rangle, |n_{3}\rangle, |n_{b, l_b, \tau_{\text{bend}}}\rangle, |n_{1, J_u, K, \tau_{\text{inv}}}\rangle\), and the quantum numbers labeling them are defined in detail in Ref. 6: \(|n_{1}\rangle, |n_{2}\rangle, \text{ and } |n_{3}\rangle\) are one-dimensional Morse-oscillator eigenfunctions describing the stretching motion of the \(\text{XY}_3\) molecule, \(|n_{b, l_b, \tau_{\text{bend}}}\rangle\) is a symmetrized eigenfunction of the two-dimensional harmonic oscillator modeling the small-amplitude bending motion, and \(|n_{1, J_u, K, \tau_{\text{inv}}}\rangle\) is a symmetrized inversion basis function obtained by numerical solution of a zero-order inversion Schrödinger equation. All three quantum numbers \(\nu_{\text{rot}}, \tau_{\text{bend}}, \text{ and } \nu_{\text{inv}}\) assume values of 0 or 1 in such a way that the parity \((-1)^{2\nu_{\text{rot}}+\nu_{\text{bend}}+\nu_{\text{inv}}}\) of \(|J_u K_m \nu_{\text{rot}}\rangle\) is \((-1)^{\nu_{\text{rot}}}\), the parity of \(|n_{b, l_b, \tau_{\text{bend}}}\rangle\) is \((-1)^{\tau_{\text{bend}}}\), and the parity of \(|n_{1, J_u, K, \tau_{\text{inv}}}\rangle\) is \((-1)^{\nu_{\text{inv}}}\).

The rotational basis functions \(|J_u K_m \nu_{\text{rot}}\rangle\) depend on three Euler angles \(\theta, \phi, \chi\), which define the orientation of the molecule-fixed axis system \(xyz\) relative to the space-fixed (or laboratory-fixed) axis system \(XYZ\). The \(xyz\) axis is
system follows the rotation of the molecule. In our HBJ-based theory, it is defined in terms of Eckart and Sayvetz conditions;\textsuperscript{6,12} these conditions minimize the coupling between different types of molecular motion, in particular, between rotation and vibration.

Each stretching basis function \( |n_j, j = 1, 2, 3 \) depends on one coordinate \( \Delta r_j = r_j - r_{eq} \), where \( r_j \) is the linearized version\textsuperscript{6} of the internuclear distance \( r_j \) and \( r_{eq} \) is the equilibrium value of \( r_j \). The bending basis function \( |n_b, l_b, \tau_{bend} \) depends on the coordinates \( (S^c_{4a}, S^c_{4b}) \), the linearized versions\textsuperscript{6} of

\[
S^c_{4a} = \frac{1}{\sqrt{6}} (2 \alpha_1 - \alpha_2 - \alpha_3),
\]

\[
S^c_{4b} = \frac{1}{\sqrt{2}} (\alpha_2 - \alpha_3).
\]

Finally, the inversion basis function \( |n_1, J, K, \tau_{inv} \) depends on the HBJ inversion coordinate\textsuperscript{6,25,26} \( \rho \). In HBJ theory,\textsuperscript{11} the small-amplitude vibrations [described by the coordinates \( (r_1, r_2, r_3, S^c_{4a}, S^c_{4b}) \) here] are viewed as displacements from a flexible reference configuration replacing the rigid equilibrium structure of customary rotation-vibration theory (see, for example, Ref. 3). For the \( XY_3 \) molecules considered here, we define the reference configuration to have three equal, constant bond lengths \( r_1 = r_2 = r_3 = r_e \) and three equal but variable bond angles \( \alpha_1 = \alpha_2 = \alpha_3 \). Thus, the reference configuration has \( C_3v \) or \( D_3h \) geometrical symmetry and we define the inversion coordinate \( \rho \) as the angle between the \( C_3 \) rotational symmetry axis and any one of the \( N-H \) bonds as shown in Fig. 1. That is, \( 0 \leq \rho \leq \pi \) and the reference configuration is planar for \( \rho = \pi/2 \).

In the variational calculations the expansions of the kinetic energy factors\textsuperscript{6} \( G_{\alpha \beta} \) and the pseudopotential\textsuperscript{6} \( U \) are taken to fourth order, and the potential energy \( V \) is expanded through sixth order. In the numerical integration of the inversion Schrödinger equation a grid of 1000 points is used. The size of the vibrational basis set is controlled by the parameter \( P_{\max} \), where

\[
P = 2(n_1 + n_2 + n_3) + n_1 + n_2 \leq P_{\max}.
\]
B. Line strengths and intensities

In the present work, we neglect hyperfine structure (i.e., the effect of the nuclear spins on the molecular energies). In this approximation, the line strength\textsuperscript{12,24} \(S(f \rightarrow i)\) of the rotation-vibration transition \(f \rightarrow i\) is obtained from Eq. (4) of Ref. 8:

\[
S(f \rightarrow i) = g_m \sum_{m_f, m_i} \sum_{A=X,Y,Z} |\langle \Phi_{n_f}^{(f)} | \tilde{\mu}_A | \Phi_{n_i}^{(i)} \rangle|^2,
\]

where \(g_m\) is the nuclear spin statistical weight factor\textsuperscript{12,24} and \(\tilde{\mu}_A\) is the electronically averaged component of the molecular dipole moment along the space-fixed axis \(A=X, Y,\) or \(Z\). The quantum numbers \(m_f\) and \(m_i\) are the projections of the total angular momentum, in units of \(\hbar\), on the \(Z\) axis in the initial and final states, respectively.

The intensity of absorption spectra is determined by the absorption coefficient\textsuperscript{12,24} \(\varepsilon(\bar{v})\) which depends on the absorption wave number \(\bar{v}\). If we assume the absorbing molecules to be in thermal equilibrium at an absolute temperature \(T\), the integral of \(\varepsilon(\bar{v})\) over an absorption line is related to the line strength as

\[
I(f \rightarrow i) = \int I_{\text{Line}} d\bar{v} = \frac{8 \pi^3 N_A \bar{v}_f \frac{e^{-E_{f}/kT}}{Q}}{(4 \pi e_0)^3 3 \hbar c} \times [1 - \exp(-\hbar c \bar{v}_{f}/kT)] S(f \rightarrow i).
\]

This expression is valid for the transition from the state \(i\) with energy \(E_i\) to the state \(f\) with energy \(E_f\), where \(\hbar c \bar{v}_{f} = E_f - E_i, N_A\) is the Avogadro constant, \(h\) is Planck's constant, \(c\) is the speed of light in vacuum, \(k\) is the Boltzmann constant, \(e_0\) is the permittivity of free space, and, finally, \(Q\) is the partition function defined as \(Q = \Sigma g_j \exp(-E_j/kT)\), where \(g_j\) is the total degeneracy of the state with energy \(E_j\) and the sum runs over all energy levels of the molecule.

Experimental values of \(I(f \rightarrow i)\) are obtained by numerical integration of experimentally determined \(\varepsilon(\bar{v})\) values.

A detailed expression for the line strength of an individual rotation-vibration transition within an isolated electronic state of an \(XY_3\) pyramidal molecule is given in Eq. (21) of Ref. 8. This expression is used in the intensity calculations reported in the present work. It is obtained by inserting Eq. (12) in Eq. (17) and expressing \(\bar{v}_{f}, A=X, Y, Z,\) in terms of \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\), the dipole moment components along the molecule-fixed axes \(xyz\). The line strength is expressed in terms of the vibronic matrix elements \(\langle V^{\alpha} | \bar{\mu}_a | V^{\alpha'} \rangle\), \(\alpha=x, y, z,\) and in terms of the expansion coefficients \(C_{\alpha}^{(f)} V_{\alpha}^{(f)} \) and \(C_{\alpha}^{(i)} V_{\alpha}^{(i)}\) from Eq. (12). The transformation \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z) \rightarrow (\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) is carried out by means of standard techniques described, for example, in Chapter 14 of Ref. 12. Examples of the application of these techniques are given for triatomic molecules in Refs. 28 and 29.

C. The ab initio potential energy surface

The calculations of the present work are made with the CBS**−5 \textit{ab initio} potential energy surface\textsuperscript{4,8,9} at the first step towards determining this surface, ATZfc energies were computed for 51 816 nuclear geometries at the CCSD(T)/aug-cc-pVTZ level of theory also used in the dipole moment calculations described in Sec. II A. At 3814 selected nuclear geometries, more accurate energies (CBS+) were determined by extrapolating the CCSD(T) results to the complete basis set limit and including corrections for relativistic effects and core-valence correlation.\textsuperscript{4} The differences between the ATZfc and CBS+ energies were fitted by a sixth-order polynomial in geometrically defined, internal coordinates, and the CBS**−5 surface (which is close to CBS+ quality) was generated by adding corrections, computed from the sixth-order polynomial, to the ATZfc energies at all 51 816 grid points. An analytical representation of this surface was obtained by fitting a sixth-order expansion, given in Eq. (57) of Ref. 4, through all CBS**−5 data points. The resulting 181 potential parameters will be published elsewhere,\textsuperscript{9} they are presently available from the authors on request. The CBS**−5 surface provides a complete description of the electronic-ground-state potential energy surface of \(NH_3\) for energies up to 20 000 cm\(^{-1}\) above equilibrium.

D. The representation of the dipole moment in the \textit{xyz} axis system

In order to compute the matrix elements \(\langle V^{\alpha} | \bar{\mu}_a | V^{\alpha'} \rangle\) \((\alpha=x, y, z,\) that enter into the expression for the line strength in Eq. (21) of Ref. 8, we must determine, from the \textit{ab initio} results discussed in Sec. II A, the dipole moment components \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) in the molecule-fixed axis system \(xyz\). As discussed extensively in Ref. 8, we aim at representing \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) as expansions with \(\rho\)-dependent expansion coefficients

\[
\bar{\mu}_a(\xi_1, \xi_2, \xi_3, \xi_{4a}, \xi_{4b}, \rho) = \mu_a^0(\rho) + \sum_k \mu_a^{kl}(\rho) \xi_k \xi_l + \sum_{k=1}^{l=1} \mu_a^{klm} \xi_k \xi_l \xi_m + \sum_{k=1}^{l=1} \mu_a^{klmn} \xi_k \xi_l \xi_m \xi_n + \cdots
\]

in the linearized variables

\[
\xi_k = 1 - \exp(-a \Delta r_k^2), \quad k = 1, 2, 3
\]

\[
\xi_{4a} = S_{4a}, \quad \xi_{4b} = S_{4b}, \quad S_{4a} = S_{4b} = \alpha \Delta r, \quad \alpha = x, y
\]

The range parameter \(\alpha\) occurs in the analytical representation for the potential energy function.\textsuperscript{6} The \(\rho\)-dependent functions \(\mu_a^{kl} (\rho) (\alpha=x\ or\ y)\) in Eq. (19) are chosen as

\[
\mu_a^{kl} (\rho) = \sum_{s=0}^{S_{max}} \mu_a^{kl}(s) (\sin \rho_0 - \sin \rho)^s, \quad 0 \leq \rho \leq \pi
\]

where we take \(\rho_0 = \pi/2\), corresponding to the planar configuration. To represent \(\bar{\mu}_z\), we choose
The dipole moment components \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) have \(E'\) symmetry in \(D_{3h}(M)\), the molecular symmetry group \(^{6,8,12,24}\) of \(\text{NH}_3\), and \(\bar{\mu}_z\) has \(A''\) symmetry. The irreducible representations of \(D_{3h}(M)\) are given in Table 1 of Ref. 8 and in Table A-10 of Ref. 12. The functions in Eqs. (21)–(23) are chosen so as to ensure that \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) transform correctly under \(E'\), the inversion operation in \(D_{3h}(M)\). The transformation properties under the nuclear permutation operations in \(D_{3h}(M)\) impose symmetry relations between the expansion parameters \(\mu_{kl}^{(s)}(\alpha = x, y, or z)\) in Eqs. (21)–(23); these relations have been derived analytically by means of MAPLE VI (Ref. 30) as described in Ref. 8.

We have already expressed the electronically averaged dipole moment in the electronic ground state of \(\text{NH}_3\) in the MB representation [Eqs. (1)–(11) in conjunction with Table I] and we could now, in principle, obtain the dipole moment components \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) as analytical functions of the coordinates \(\rho, \xi^t_1, \xi^t_2, \xi^t_3, \xi^t_4, \) and \(\xi^t_{4b}\) from the expression in Eq. (1). However, as detailed in Ref. 8, we have chosen instead the conceptually more complicated, but numerically simpler approach of obtaining, at each \(ab\) \(initio\) point, numerical values of \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) by carrying out, at the \(ab\) \(initio\) point in question, the coordinate transformation from the body-fixed axis system \(x'y'z'\) used in the \(ab\) \(initio\) calculation (Sec. II A) to the \(xyz\) axis system. We then determine the values of the expansion parameters \(\mu_{kl}^{(s)}\) in Eqs. (21)–(23) by fitting Eq. (19) through the computed values of \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\). The expansion parameters \(\mu_{kl}^{(s)}\) and \(\mu_{kl}^{(s)}\) in Eq. (21) are connected by symmetry relations since \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) have \(E'\) symmetry in \(D_{3h}(M)\), and so these two dipole moment components must be fitted together. The component \(\bar{\mu}_z\), with \(A''\) symmetry, can be fitted separately. Taking Eq. (19) to sixth order, we fitted the \(3 \times 14\ 400\) \(ab\) \(initio\) data points using 174 parameters for the \(\bar{\mu}_z\) component and 271 parameters for \((\bar{\mu}_x, \bar{\mu}_y)\). The rms deviations attained were 0.000 076 D and 0.0002 D, respectively.

IV. APPLICATIONS

A. Transition moments

With the dipole moment components \((\bar{\mu}_x, \bar{\mu}_y, \bar{\mu}_z)\) represented as given in Eq. (19) and the vibrational wave functions \(|\Phi_{ij}^{(s)}\rangle\), \(w = f\) or \(f\), given by Eq. (12) for \(J = 0\), we can compute the vibrational transition moments defined as

\[
\mu_{ji} = \sqrt{\sum_{\alpha, \beta, \gamma, \delta} \mid \langle \Phi_{ij}^{(s)} | \bar{\mu}_\alpha | \Phi_{kl}^{(s)} \rangle \mid^2}
\]

for vibrational transitions in the electronic ground state of \(^{14}\text{NH}_3\); the matrix elements required are generated by techniques described in Ref. 6 for matrix elements of the potential energy function. In calculating the vibrational wave functions, we use the \(ab\) \(initio\) potential energy surface CBS**-5 (Sec. III C) and a basis set with \(P_{\text{max}} = 14\) [Eq. (16)]. With this basis set, the \(J = 0\) matrix blocks corresponding to \(A\) and \(E\) symmetries in the group \(D_{3h}(M)\) have the dimensions \(N(A) = 1455\) and \(N(E) = 2571\), respectively.

The values of the vibrational transition moments are, to a large extent, determined by the variation of the dipole moment components with the vibrational coordinates. By comparing our theoretical values for the vibrational transition moments to experimental values and other theoretical values available in the literature\(^{6,12,22,23,31–41}\) for \(^{14}\text{NH}_3\), we can assess the quality of the ATZfc dipole moment surface. We make this comparison in Tables II and III. In these tables, the vibrational states are labeled such that a superscript \(+\) indicates the lower (symmetric) inversion component; the upper (antisymmetric) component is indicated by a superscript \(–\). The symmetric and antisymmetric components of the vibrational ground state are labeled \(0^+\) and \(0^–\), respectively. In addition to the transition moment values obtained theoretically in the present work, Tables II and III give the available experimental values and the results of two other theoretical calculations by Marquardt \(et\ al.\) and Pracna \(et\ al.\) respectively. These latter results are analogous to ours in that they are pure \(ab\) \(initio\) predictions made without fitting to experiment. The work of Pracna \(et\ al.\) also contains the results of such fittings, but we present here their “potential function I” results, obtained directly from \(ab\) \(initio\) data.

Our results agree very satisfactorily with the experimental transition moment values; the over-all agreement is better than that obtained in the two other theoretical calculations considered here. The most remarkable agreement is found for the fundamental transitions in Table II and for transitions to excited inversion states. The inversion motion in \(\text{NH}_3\) involves tunneling between two equivalent minima on the potential energy surface and cannot be reasonably modeled by the motion of a one-dimensional harmonic oscillator. It is gratifying that, as shown in Tables II and III, our theoretical model describes well the intensities of transitions involving changes in the excitation of this complex motion, at least at low and moderate excitation. The transition moment results suggest that the ATZfc \(ab\) \(initio\) dipole moment surface (Sec. II A) is the most accurate dipole moment surface currently available for \(\text{NH}_3\).

B. Intensity simulations

As discussed in Sec. I, it is desirable to be able to simulate molecular spectra (i.e., to compute transition wave numbers and intensities) in order to assist their detection and subsequent first-principles assignment. In the present section, we report such simulations for \(^{14}\text{NH}_3\) absorption bands starting in the lowest vibrational states \(0^n\) and ending in the states \(2v_2, v_4, v_1, v_3,\) and \(2v_4\). The simulated spectra are drawn as stick diagrams where the height of the stick representing a line is the integrated absorption coefficient from Eq. (18). The line strengths entering into this equation are computed from Eq. (21) of Ref. 8 with the spin statistical weight factors \(g_{ns}\) from Table 2 of Ref. 8. The simulations are made...
with the ATZfc \textit{ab initio} dipole moment surface (Sec. II A) and the CBS$$^+$$–5 potential energy surface (Sec. III C). We generate rovibrational wave functions with \( J \leq 18 \) and, in order to make the necessary variational calculations feasible, the size of the vibrational basis set is reduced to \( P_{\text{max}}=8 \) [Eq. (16)] relative to the \( P_{\text{max}}=14 \) basis set employed for calculating the vibrational transition moments in Sec. IV A. The largest matrix blocks obtained with the \( P_{\text{max}}=8 \) basis set are the \( E \) symmetry blocks around \( J=18 \) with the dimension \( N(E) = 9150 \).

We compare our theoretical intensities with the results of two recent experimental studies of \( ^{14}\text{NH}_3 \) absorption spectra: the \( 2\nu_2/\nu_3 \) bands in the 5–7 \( \mu \)m region\(^{34}\) and the \( \nu_1/\nu_3/2\nu_2/4\nu_2 \) bands near 3 \( \mu \)m.\(^{10}\) Both spectra have been recorded at room temperature. The experimental data (wave numbers and intensities) for the \( 2\nu_2/\nu_3 \) bands, listed in the Appendix of Ref. 34, include 1203 rovibrational transitions between states with \( J \leq 18 \). The experimental data for the \( 2\nu_2/\nu_1/\nu_3 \) bands\(^{32}\) are given in Appendix 1 of Ref. 10 and comprise 975 transitions between states with \( J \leq 11 \). We simulate spectra in the wave number regions 1300–2000 cm\(^{-1}\) (for the \( 2\nu_2/\nu_3 \) bands) and 3000–3800 cm\(^{-1}\) (for the \( 2\nu_2/\nu_1/\nu_3 \) bands) at \( T=295 \) K. In computing the integrated absorption coefficient from Eq. (18), we use the partition function value \( q=1713.33 \), which is obtained from the \( J \leq 18 \) term values calculated variationally below 6000 cm\(^{-1}\), and the spin statistical weight factors for \( ^{14}\text{NH}_3 \) from Table 2 of Ref. 8. We assign to each calculated eigenstate the vibrational quantum numbers \( V \) and the rotational quantum number \( K \) of the basis function with the largest contribution to the eigenfunction, i.e., with the largest value of \( |C_{VK}|^2 \) in Eq. (12). We discard transitions for which the calculated value of \( I(f \leftarrow i) < 10^{-4} \) cm mol\(^{-1}\). This threshold is much lower than the integrated absorption coefficient of the weakest observed line [a \( 2\nu_2 \) band line with \( I(f \leftarrow i) = 0.7 \) cm mol\(^{-1}\)] listed in Refs. 10 and 34.

In Figs. 2 and 3 we show simulations of the \( 2\nu_2, \nu_4, 2\nu_4, \nu_1, \) and \( \nu_3 \) absorption bands of \( ^{14}\text{NH}_3 \). The bands are artificially separated according to the assignment of the upper state. For example, in the 1300–2000 cm\(^{-1}\) wave number region (Fig. 2) we plot in separate displays the transitions to the \( \nu_4 \) and \( 2\nu_2 \) states, respectively. Each simulated band is compared to an “experimentally derived” stick spectrum drawn with experimental values for transition wave numbers and intensities,\(^{10,34}\) these transitions are also artificially separated according to the “experimental” assignment of the upper state. The experimentally derived spectra show only the

---

**Table II.** Band centers \( v_i \) and vibrational transition moments \( \mu_i \) for \( ^{14}\text{NH}_3 \); Transitions originating in the vibrational ground state.

<table>
<thead>
<tr>
<th>( f )</th>
<th>( i )</th>
<th>( v_i ) (cm(^{-1}))</th>
<th>Obs.</th>
<th>Reference(^a)</th>
<th>Reference(^b)</th>
<th>Reference 39(^c)</th>
<th>Reference 22</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>0−</td>
<td>0+</td>
<td>0.79</td>
<td>1.471 93(1)(^d)</td>
<td>31</td>
<td>1.536</td>
<td>1.574</td>
<td>1.4564</td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>931.64</td>
<td>0.248(7)</td>
<td>32</td>
<td>0.269</td>
<td>0.2183</td>
<td>0.2445</td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>968.12</td>
<td>0.236(4)</td>
<td>32</td>
<td>0.258</td>
<td>0.2075</td>
<td>0.2347</td>
<td></td>
</tr>
<tr>
<td>2( \nu_2 )</td>
<td>0−</td>
<td>1596.68</td>
<td>0.020 36(25)</td>
<td>34</td>
<td>0.027</td>
<td>0.0091</td>
<td>0.0202</td>
<td></td>
</tr>
<tr>
<td>2( \nu_2 )</td>
<td>0−</td>
<td>1882.18</td>
<td>0.003 256(35)</td>
<td>34</td>
<td>0.007</td>
<td>0.0261</td>
<td>0.0026</td>
<td></td>
</tr>
<tr>
<td>3( \nu_2 )</td>
<td>0−</td>
<td>2383.36</td>
<td>0.004 96(13)</td>
<td>33</td>
<td>0.004</td>
<td>0.0261</td>
<td>0.0054</td>
<td></td>
</tr>
<tr>
<td>3( \nu_2 )</td>
<td>0−</td>
<td>2895.51</td>
<td>0.002 856(40)</td>
<td>33</td>
<td>0.003</td>
<td>0.0155</td>
<td>0.0027</td>
<td></td>
</tr>
<tr>
<td>4( \nu_2 )</td>
<td>0−</td>
<td>3461</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4( \nu_2 )</td>
<td>0−</td>
<td>4055</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>3335.23</td>
<td>0.026 2(1)</td>
<td>10</td>
<td>0.0366</td>
<td>0.0269</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>3337.08</td>
<td>0.026 2(1)</td>
<td>10</td>
<td>0.0366</td>
<td>0.0270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(( \nu_2 + \nu_3 ))(^+)</td>
<td>0−</td>
<td>4293.72</td>
<td>0.0079</td>
<td>35</td>
<td>0.0067</td>
<td>0.0087</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(( \nu_2 + \nu_3 ))(^−)</td>
<td>0−</td>
<td>4320.06</td>
<td>0.0079</td>
<td>35</td>
<td>0.0066</td>
<td>0.0083</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>3443.68</td>
<td>0.018 2(1)</td>
<td>10</td>
<td>0.0915</td>
<td>0.0181</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>3443.20</td>
<td>0.018 2(1)</td>
<td>10</td>
<td>0.0915</td>
<td>0.0180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(( \nu_2 + \nu_3 ))(^+)</td>
<td>0−</td>
<td>4416.91</td>
<td>0.020 6</td>
<td>35</td>
<td>0.0128</td>
<td>0.0246</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(( \nu_2 + \nu_3 ))(^−)</td>
<td>0−</td>
<td>4434.61</td>
<td>0.020 6</td>
<td>35</td>
<td>0.0127</td>
<td>0.0244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>1626.28</td>
<td>0.084 08(34)</td>
<td>34</td>
<td>0.0839</td>
<td>0.0828</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_2 )</td>
<td>0−</td>
<td>1626.58</td>
<td>0.084 08(34)</td>
<td>34</td>
<td>0.0838</td>
<td>0.0827</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2( \nu_4 )(^+)</td>
<td>0−</td>
<td>3215.23</td>
<td>0.009 20(6)(^f)</td>
<td>10</td>
<td>0.0073</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2( \nu_4 )(^−)</td>
<td>0−</td>
<td>3217.59</td>
<td>0.009 20(6)(^f)</td>
<td>10</td>
<td>0.0074</td>
<td></td>
<td></td>
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<tr>
<td>2( \nu_4 )(^+)+</td>
<td>0−</td>
<td>3240.18</td>
<td>0.009 20(6)(^f)</td>
<td>10</td>
<td>0.0091</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2( \nu_4 )(^−)+</td>
<td>0−</td>
<td>3240.82</td>
<td>0.009 20(6)(^f)</td>
<td>10</td>
<td>0.0090</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(( \nu_4 + \nu_3 ))(^+)</td>
<td>0−</td>
<td>2540.53</td>
<td>0.002 358(36)</td>
<td>33</td>
<td>0.0077</td>
<td>0.0091</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(( \nu_4 + \nu_3 ))(^−)</td>
<td>0−</td>
<td>2585.34</td>
<td>0.002 182(82)</td>
<td>33</td>
<td>0.0080</td>
<td>0.0094</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Derived from the experimental data collected in Ref. 6.

\(^b\)Reference for the observed value given under the heading “Obs.”

\(^c\)\textit{Ab initio} predictions, see text.

\(^d\)Experimental uncertainties are given in parentheses (in units of the last digit quoted) where available.

\(^e\)The experimental value (Ref. 10) corresponds to the total \( 2\nu_2 \) transition moments and should not be directly compared to the theoretical values for \( 2\nu_2 \) and \( 2\nu_2 \) bands, respectively.
transitions assigned in Refs. 10 and 34 and this explains why the experimental spectra generally contain less lines than the simulated ones. In the simulated and the experimentally derived stick spectra, the intensities are given as integrated absorption coefficients in cm mol⁻¹. The experimental values, originally given in cm⁻² atm⁻¹, were converted to cm mol⁻¹ at T=295 K. Note that the same absolute intensity scale is used for the two members of each theoretical/experimental spectrum pair.

It should be mentioned that in the wave number intervals 1300–2000 cm⁻¹ and 3000–3800 cm⁻¹, there are more vibrational bands than those assigned in Refs. 10 and 34. For example, the hot bands (3ν₂−ν₂)/(ν₂+ν₄−ν₂) lie in the 5–7 μm region and overlap with the 2ν₂/ν₄ bands. This, of course, makes the experimental assignment more complicated. Also, the “theoretical assignment” based on the eigen-vector coefficients v_v_{2,1}^{(n)} in Eq. (12) can differ from the “experimental assignment” in the event of strong accidental mixing of different basis functions in an eigenfunction. However, we do not expect such effects to be very important in the present study as we are not aiming at a line-by-line comparison of theory with experiment, but rather at a qualitative band-structure comparison. Therefore, we disregard possible misassignments and plot each simulated band separately as described above.

For the 2ν₂/ν₄ band system shown in Fig. 2 there is significant similarity between the simulated and experimentally derived stick spectra, in particular, in the case of the ν₄ fundamental band, for which a large amount of experimental data (935 transitions) is available. Also for the 2ν₂/ν₁/ν₅ band system in Fig. 3 do the simulated fundamental bands ν₁ and ν₅ look highly similar to their experimental counterparts.

### Table III. Band centers νᵢ and vibrational transition moments μᵢ for ¹⁴NH₃: Transitions originating in vibrationally excited states.

<table>
<thead>
<tr>
<th>States</th>
<th>νᵢ (cm⁻¹)</th>
<th>Obs.</th>
<th>Reference¹</th>
<th>Reference 2²</th>
<th>Reference 3⁴</th>
<th>Reference 22</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν₂⁺</td>
<td>ν₂</td>
<td>35.69</td>
<td>1.244 8(23)³</td>
<td>36</td>
<td>1.305</td>
<td>1.355</td>
<td>1.2376</td>
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<tr>
<td>2ν₂⁻</td>
<td>ν₂</td>
<td>629.35</td>
<td>0.557</td>
<td>0.5084</td>
<td>0.5144</td>
<td></td>
<td></td>
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<tr>
<td>2ν₂⁺</td>
<td>ν₂</td>
<td>949.75</td>
<td>0.285 (10)</td>
<td>37</td>
<td>0.313</td>
<td>0.2698</td>
<td>0.2855</td>
</tr>
<tr>
<td>3ν₂⁻</td>
<td>ν₂</td>
<td>1416.03</td>
<td>0.073 7(16)</td>
<td>38</td>
<td>0.089</td>
<td>0.0288</td>
<td>0.0738</td>
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<td>3ν₂⁺</td>
<td>ν₂</td>
<td>1963.08</td>
<td>0.000 81(11)</td>
<td>38</td>
<td>0.004</td>
<td>0.0416</td>
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<tr>
<td>4ν₂⁻</td>
<td>ν₂</td>
<td>2494</td>
<td>0.012</td>
<td>0.0413</td>
<td>0.0088</td>
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<td>4ν₂⁺</td>
<td>ν₂</td>
<td>3123</td>
<td>0.004</td>
<td>0.0184</td>
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<td>ν₁⁻</td>
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<td>2367.90</td>
<td>0.004 0</td>
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<td>0.004 0</td>
<td>38</td>
<td>0.0159</td>
<td>0.0008</td>
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<tr>
<td>(ν₁+ν₂⁻)⁺</td>
<td>ν₂</td>
<td>3326.39</td>
<td>0.0337</td>
<td>0.0244</td>
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<tr>
<td>(ν₁+ν₂⁺)⁻</td>
<td>ν₂</td>
<td>3387.63</td>
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<td>0.0249</td>
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<td>ν₁⁻</td>
<td>ν₁</td>
<td>2511.25</td>
<td>0.022 86(29)</td>
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<td>0.0178</td>
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<td>ν₁⁺</td>
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<td>2475.87</td>
<td>0.022 86(29)</td>
<td>38</td>
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<td>0.0181</td>
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<tr>
<td>(ν₁+ν₂⁻)⁺</td>
<td>ν₂</td>
<td>3484.48</td>
<td>0.0913</td>
<td>0.0320</td>
<td></td>
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<tr>
<td>(ν₁+ν₂⁺)⁻</td>
<td>ν₂</td>
<td>3467.28</td>
<td>0.0907</td>
<td>0.0277</td>
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<tr>
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<td>ν₁</td>
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<td>0.0022</td>
<td>0.0125</td>
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<td>ν₁⁺</td>
<td>ν₁</td>
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<td>0.0022</td>
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<td>2ν₂⁺</td>
<td>ν₂</td>
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<td>0.006 72(15)</td>
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<td>2ν₂⁻</td>
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¹Derived from the experimental data collected in Ref. 6.
²Reference for the observed value given under the heading Obs.
³Ab initio predictions, see text.
⁴Experimental uncertainties are given in parentheses (in units of the last digit quoted) where available.
FIG. 2. Comparison of simulated and observed (Ref. 34) spectra near 5–7 μm: the $\nu_4$ and $2\nu_2$ absorption bands of $^{14}$NH$_3$.

FIG. 3. Comparison of simulated and observed (Ref. 10) spectra near 3 μm: the $\nu_1$, $\nu_3$, $2\nu_4^0$, and $2\nu_4^{\pm2}$ absorption bands of $^{14}$NH$_3$. 
The similarity is less pronounced for the weaker transitions to the overtone levels $2\nu_4$. It is obvious that for the $2\nu_4$ bands, the experimentally derived stick spectra contain significantly less lines than the simulations, and some of this dissimilarity may be due to the fact that not all $2\nu_4$ lines were assigned in Ref. 10, possibly because in the real spectrum, they overlap with stronger $\nu_1/\nu_3$ lines. The trend of the vibrational transition moments in Tables II and III is reflected in the line intensities: The vibrational transition moment of the $\nu_1$ fundamental band is approximately twice as large as the $\nu_1$ transition moment, and therefore the $\nu_1$ band is approximately four times stronger than the $\nu_1$ band.

In Table IV we compare theoretical and experimentally determined values for the integrated absorption coefficients of the strongest rotation-vibration transition in each of the five vibrational bands considered here. Also for these intensities of individual rotation-vibration lines, there is good agreement between theory and experiment. Each of the theoretical intensity values is relatively close to its experimentally observed counterpart. If we sort the lines in Table IV in order of increasing intensity, we obtain the sequence $2\nu_4$, $2\nu_2$, $\nu_1$, $\nu_3$, $2\nu_1$, which is correctly reproduced by the theoretical calculation. We now present the calculated elements of the computational scheme in order to investigate how these elements influence the computed wave number regions 1300–2000 cm$^{-1}$ [Fig. 2] and 3000–3800 cm$^{-1}$ [Fig. 3] considered here, the simulations of the weakest bands $2\nu_2$ in the 1300–2000 cm$^{-1}$ region and $2\nu_4$ in the 3000–3800 cm$^{-1}$ region show the largest discrepancies from the experimentally derived stick spectra. The most noticeable discrepancy is that, as mentioned above, the simulated spectra of weak bands contain significantly more lines than the experimentally derived ones. This indicates that in the weak bands, many lines were not assigned in the experimental studies of Refs. 10 and 34 even though their intensities appear significant in our simulations. Since in the experiment, all bands in a given wave number region are superimposed, it is plausible that many of these “extraneous” simulated weak-band lines overlap with strong-band lines in the observed spectrum, so that it is impossible to assign them.

### C. Simplified intensity calculations

The simulations shown in Figs. 2 and 3 aim at high accuracy in each stage of the calculation. We now present the results of calculations where we degrade the accuracy of selected elements of the computational scheme in order to investigate how these elements influence the computed wave numbers and intensities. We focus on four elements of the calculation.

1. The basis set. In the “standard” calculations producing Figs. 2 and 3, we used a basis set with $P_{\text{max}}=8$ [Eq. (16)].

2. The expansion of the vibrational-coordinate dependent functions $G_{\alpha\beta}$ in the kinetic energy operator. In the standard calculations, these expansions are truncated after fourth order in the linearized coordinates $\xi_n$, $n = 1, 2, \ldots, 5$.

3. The expansion of the potential energy function $V$ in
terms of the $\xi^i$ coordinates, taken to sixth order in the standard calculation.

(d) The expansion of the dipole moment components $\bar{\mu}_{\alpha}$, $\alpha=x,y,z$ in the $\xi^i$ coordinates; these expansions are taken to sixth order in the standard calculation.

The simplest model for rovibrational intensities describes the rotation as that of a rigid symmetric rotor (taken to be the molecule at the equilibrium geometry) and the vibrational motion as that of uncoupled harmonic oscillators. That is, the wave function is a product of a rigid-rotor function and harmonic-oscillator eigenfunctions, while the electronically averaged dipole moment is represented as a first-order expansion in normal coordinates.\textsuperscript{12,24} This is known as the “doubly harmonic” approximation. As remarked in Sec. III A, it is inappropriate to model the inversion motion by a harmonic oscillator since the inversion involves the tunneling between two potential minima separated by a superable barrier. Therefore, in the “degraded” calculations, we maintain the numerically generated inversion basis functions also used in the standard calculations. These can be thought of as rigid inverter wave functions.\textsuperscript{25,26} We also maintain in these test calculations the Morse-oscillator functions of Sec. III A as basis functions for the stretching motion, and so, in these calculations, we do not neglect the anharmonicity of the stretching motion.

We make test simulations of the $\nu_4$ and $2\nu_2$ bands at $T=295$ K by degrading the calculations in the following manner.

(A) \textit{Basis set reduction}: We reduce the basis set to have $P_{\text{max}}=2$ [Eq. (16)]; this is the smallest basis set capable of producing the upper states of the $\nu_4/2\nu_2$ transitions. The simulated spectra, shown in the “A” displays of Fig. 4, are drastically changed relative to the simulations in Fig. 2. Both the absolute intensities and the line positions undergo serious changes.

(B) \textit{Simplification of the kinetic energy operator}: To assess the importance of the kinetic energy operator for the quality of the spectrum simulations, we calculate intensities with the vibrational-coordinate-dependent functions $G_{\alpha,\beta}$ in the kinetic energy operator\textsuperscript{6} set equal to

![Figure 4: The $\nu_4$ and $2\nu_2$ bands of $^{14}\text{NH}_3$ simulated at $T=295$ K with (A) a reduced basis set, (B) a simplified kinetic-energy operator, (C) a simplified potential energy function, and (D) a simplified dipole moment representation (see text).]
FIG. 5. The standard simulations of the $2\nu_2$/$\nu_4$ bands of $^{14}$NH$_3$ from Fig. 2 (labeled “Standard”), compared to simulations obtained in the approximation of making all four simplifications (A)–(D) simultaneously (labeled “Approximate”) and in the doubly harmonic approximation (labeled “Doubly harmonic”).

the zero-order term $G_{a,b}^{(0)}(\rho) = G_{a,b}(\rho, 0, 0, 0)$, which depends on $\rho$ only. $G_{a,b}(\rho)$ is the value of $G_{a,b}$ in the reference configuration. All other aspects of the calculation are kept in their standard form. The resulting simulated spectra are shown in the “B” displays of Fig. 4. The effect of the kinetic energy simplification is drastic: The simulated $2\nu_2$ band changes so much that it is hardly recognizable and the $\nu_4$ band is also altered significantly from the standard simulation in Fig. 2.

(C) Simplification of the potential energy function: We truncate the expression for the potential energy function after the second-order terms in the five linearized coordinates $\xi_n$:

$$V = V(0)(\rho) + \sum_k V_k^{(1)}(\rho) \xi_k + \sum_{kl} V_{kl}^{(2)}(\rho) \xi_k \xi_l$$

while keeping the basis set and the functions $G_{a,b}$ in their standard forms. The effect of the potential energy truncation (shown in the “C” displays of Fig. 4) is noticeable but minor for the $\nu_4$ band while very pronounced in the case of the $2\nu_2$ band.

(D) Simplification of the dipole moment expansion: We simplify the analytical representation of the dipole moment components as much as possible by considering only the leading terms in Eq. (19):

$$\bar{\mu}_x = \mu_{4a} \xi_4$$

$$\bar{\mu}_y = \mu_{4b} \xi_4$$

$$\bar{\mu}_z = \mu_{4c} \cos \rho$$

all other parameters of the calculation are kept at their standard values. In this rather customary approximation, the $\nu_4$ intensities derive solely from $\bar{\mu}_x$ and $\bar{\mu}_y$, whereas the intensity of the $2\nu_2$ band originates in $\bar{\mu}_z$. The simulated spectra are given in the “D” displays of Fig. 4. When we compare with the standard results in Fig. 2, there is no noticeable change in the structure of the $\nu_4$ band. However, the $2\nu_2$ band, whose upper state is an excited inversion state, has completely lost its original structure.

When we compare Figs. 2 and 4, we see that for the $\nu_4$ fundamental band, the degradation of the kinetic energy operator (B) has the largest influence on the intensity pattern. This is to be expected since the $\nu_4$ band is a strong band that remains allowed (in the sense of Chapter 12 of Ref. 24) when we make the approximations (A), (C), (D) described above. The much weaker $2\nu_2$ band would be forbidden (in the sense of Chapter 12 of Ref. 24) if the vibrational wave functions were approximated by those of uncoupled harmonic oscillators and we use first-order expansions of the molecule-fixed molecular dipole moment components. This band is heavily influenced by all four approximations (A)–(D), and so, conversely, in order to obtain a correct description of it we must avoid these approximations.

As demonstrated in Fig. 4, the kinetic energy coupling between the different vibrational modes plays the most critical role in the determination of the rotational structure. An inaccurate description of this coupling fails to give correct intensity patterns for the $P$, $Q$, and $R$ branches. The analogous potential energy coupling is also very important, especially for weak transitions that are forbidden in the doubly harmonic approximation. The intensities, and the wave number dependence on rotational excitation, of these transitions are crucially dependent on the anharmonic terms in the potential energy function. The electric anharmonicity also affects to some extent the rotational structure of transitions that
are forbidden in the doubly harmonic approximation, and moreover it has a pronounced effect on the vibrational intensity patterns.

In Fig. 5, we compare the standard simulations of the $2\nu_2/\nu_4$ bands (the corresponding displays, headed “Standard” in the figure, are repetitions of the simulations shown in Fig. 2) with the simulations obtained by making all four simplifications (A)–(D) simultaneously (labeled “Approximate”) and those obtained in the doubly harmonic approximation (labeled “ Doubly harmonic”). The Approximate simulations differ from the Doubly harmonic ones in that in the Approximate calculation, the inversion motion is treated in the rigid inverter approximation, and the stretching motion is described by Morse-oscillator basis functions which account for anharmonicity. In addition, the Approximate dipole moment function is not strictly “electrically harmonic” because we express $\bar{\mu}_2$ in terms of the curvilinear coordinate $\rho$. Figure 5 shows that both in the doubly harmonic approximation and in the approximation defined by the four simplifications (A)–(D), the simulations are altered significantly relative to those obtained in the standard approximation, and the effect is much larger for the weak $2\nu_2$ band. Comparison of the simulated spectra under Approximate and Doubly harmonic shows that there are only minor changes in the appearance of the $\nu_4$ band, in spite of the fact that there are no inversion splittings in the double harmonic approximation (these splittings are not visible on the scale of Fig. 5). There are drastic changes for the $2\nu_2$ band, of course, which carries no intensity in the doubly harmonic approximation.

V. SUMMARY AND CONCLUSION

We have presented extensive first-principles simulations of rotation-vibration spectra of $^{14}$NH$_3$. The calculations are carried out by means of recently developed computational methods for calculating rotation-vibration energies and intensities for isolated electronic states of XY$_3$ pyramidal molecules. By degrading the accuracy of selected elements of the calculations, and by doing calculations in the so-called doubly harmonic approximation, we have investigated the influence of customary approximations on the intensity values obtained. The result of these numerical experiments is an illustration of the properties of allowed and forbidden bands. The $\nu_1$ band of $^{14}$NH$_3$ is allowed even in the crude approximation of rigid-rotor/harmonic-oscillator wave functions and an electrically harmonic dipole moment function. Therefore, this band is strong and its intensities are relatively insensitive to improvements in the intensity calculation. As strikingly demonstrated in Fig. 5, the $2\nu_2$ band requires electrical anharmonicity (i.e., an expansion of the dipole moment at least to second order in the normal coordinates) to gain intensity. It is forbidden in the doubly harmonic approximation. Consequently it is much weaker than the $\nu_1$ band, and its intensity pattern is critically dependent on the level of theory in the nuclear-motion calculation.

As witnessed by the vibrational transition moments in Tables II and III, the simulated spectra in Figs. 2 and 3, and the line strengths of individual rotation-vibration transitions in Table IV, the theoretical intensity results of the present work are generally in very satisfactory agreement with the experimentally measured intensity values available for the electronic ground state of $^{14}$NH$_3$. Not only do we predict vibrational transition moments that explain the relative intensities of the observed vibrational bands, we also compute absolute line-strength values for individual rotation-vibration lines in good agreement with experimental results. Consequently, we expect that our computational method, in conjunction with high-quality ab initio potential energy and dipole moment surfaces, can simulate rotation-vibration spectra of XY$_3$ pyramidal molecules prior to observation with sufficient accuracy to facilitate the observation and assignment of these spectra.

ACKNOWLEDGMENTS

The initial stages of this work were supported by the European Commission through Contract No. HPRN-CT-2000-00022 “Spectroscopy of Highly Excited Rovibrational States.” The work of P.J. is supported in part by the Deutsche Forschungsgemeinschaft and the Fonds der chemischen Industrie.

14MOLPRO2000 is a package of ab initio programs written by H.-J. Werner and P. J. Knowles, with contributions from R. D. Amos, A. Bernhardsson, A. Berning et al.
15C. Hampel, K. Peterson, and H.-J. Werner, Chem. Phys. Lett. 190, 1 (1992), and references therein. The program to compute the perturbative triples corrections has been developed by M. J. O. Deegan and P. J. Knowles, ibid. 227, 321 (1994).
Here, we only discuss the intensities of the vibrational transitions to the ν1, ν3, and 2ν4 states; the amount of experimental data available for the 4ν2 band is very limited.