

# PHASM/G228 Advanced Topics in Statistical Mechanics

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## Aims and Objectives

- To develop understanding of statistical physics beyond an introductory module.
- To use equilibrium statistical mechanics to deduce the properties of systems of interacting particles, with emphasis on the liquid state and small clusters, and to discuss the nucleation of phase transitions.
- To introduce the mathematics of stochastic processes, including the Langevin equation and Fokker-Planck equation.
- To understand how stochastic dynamics models may be derived from deterministic dynamics.
- To introduce the techniques of stochastic calculus, noting the distinct Itô and Stratonovich rules, and to apply them to solve various problems, such as Brownian motion, including their use in areas outside science such as finance.
- To consider the meaning of entropy and its production in nonequilibrium statistical physics, with reference to the reversibility paradox and Maxwell's demon.
- To develop the concepts of work, heat transfer and entropy production within a framework of stochastic thermodynamics, and to derive fluctuation relations.
- Recommended books: see moodle site.

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