1B24

WAVES OPTICS AND ACOUSTICS REVISION LECTURE 2 Mainly Optics

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- **1** Reflection of Waves
- **1.1 Joined strings**

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$$\mathbf{r} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \tag{1}$$

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$$\mathbf{r} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$
(1)
$$\mathbf{t} = \frac{2Z_1}{Z_1 + Z_2}.$$
(2)

Reflect from a denser medium, change sign

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Change of sign is phase change of π .

1.2.1 a. directly

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- Typical Part B question

- 1.2 Acoustic waves in dissimilar media reflection and transmission
- 1.2.1 a. directly
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- **1.2.2 b. in terms of impedance**

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Typical Part A question

- 1.2 Acoustic waves in dissimilar media reflection and transmission
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1.2.2 b. in terms of impedance

Typical Part A question Note that the values of R and T give

$$R+T=1, (3)$$

which expresses the conservation of energy.

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2.2 Phase and group velocity

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- 2.2.1 simple two-frequency treatment

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Two waves - mathematics similar to that for beats.
Group velocity

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$$v_{\rm g} = \frac{\partial \omega}{\partial k} = \frac{\partial (v_{\rm p}k)}{\partial k} = v_{\rm p} + k \frac{\partial v_{\rm p}}{\partial k},$$

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for a wave travelling in the direction $\mathbf{k}/|\mathbf{k}|$ and with a wavelength $2\pi/|\mathbf{k}|$.

2.3.2 cylindrical and spherical waves

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2.3.4 Curvature of Wavefront

Negligible at large distances (Fraunhofer limit).

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3.1.1 Moving source, stationary observer

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$$f' = fc/(c - v_s).$$

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taking a positive velocity as being along the source-observer distance.

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Far from array, looking at an angle θ off axis, with sources h apart

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Far from array, looking at an angle θ off axis, with sources h apart, phase difference δ between successive sources will be

$$\delta = \frac{2\pi}{\lambda} h \sin \theta.$$

$$Re^{i(\omega t + \phi)} =$$

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which is a geometric series

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Finally,

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4.2.1 Diffraction grating

Resolving power is

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4.2.1 Diffraction grating

Resolving power is

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where m is the order of diffraction and N is the number of lines in the grating.

5.1 Huygens's principle

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Every point on a primary wavefront serves as the source of spherical secondary wavelets, such that the primary wavelet at a later time is the envelope of these secondary wavelets. The wavelets advance with a speed and frequency which are equal to those of the primary wave at every point in space.

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5.3 Fraunhofer diffraction

The range at which wavefront curvature becomes important, the Rayleigh distance, for an aperture of width d is d^2/λ .

Ignoring the variation of amplitude with distance

Ignoring the variation of amplitude with distance, and absorbing the phase change on the average path length D into the constant term

$$E = \operatorname{constant} \int_{S} e^{-i(k_x x' + k_y y')} \mathrm{d}S',$$

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where α and β are the direction cosines of the ray travelling from the aperture to the point of observation.

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To derive the intensity pattern

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5.5 grating
The grating is a pattern of N slits

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$$I(\theta) = I(0) \left[\frac{\sin\left(\frac{N\pi h}{\lambda}\sin(\theta)\right)}{N\sin\left(\frac{\pi h}{\lambda}\sin(\theta)\right)} \right]^2 \left[\frac{\sin\left(\frac{\pi d}{\lambda}\sin(\theta)\right)}{\frac{\pi d}{\lambda}\sin(\theta)} \right]^2$$

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6.2 Rayleigh for circular aperture

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6.2 Rayleigh for circular aperture

For a circular aperture, the diffraction is circular, and the first zero occurs at an angle of $1.22\lambda/d$, where d is now the diameter of the aperture.

7.1 Snel's law

 $n_1\sin(\theta_1) = n_2\sin(\theta_2)$

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7.2 phase change on reflection

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7.2 phase change on reflection

For small angles of incidence, zero for reflection from a less optically dense material

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 $n_1\sin(\theta_1) = n_2\sin(\theta_2)$

7.2 phase change on reflection

For small angles of incidence, zero for reflection from a less optically dense material, π for reflection from a more optically dense material.

Slab of refractive index n_2 with a medium of refractive index n_1 on each side,

Slab of refractive index n_2 with a medium of refractive index n_1 on each side, light is incident at an angle θ_1 ,

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$$\Delta = \frac{2dn_2}{\cos(\theta_2)}(1 - \sin^2(\theta_2)) = 2n_2d\cos(\theta_2).$$

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where p is an integer.

7.5 Newton's rings

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7.6 wedge

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7.6 wedge

Straight fringes.

Arrangement:

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8.1 compensating plate for white light

Gives equal path lengths through glass for the two optical paths.

8.2 measuring refractive index

Putting a different material (e.g. a different gas) into one arm alters the optical path length.

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8.3 precise measurements of wavelength

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8.3 precise measurements of wavelength

Visibility of fringes varies with path length difference for a doublet source.

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With reflection and transmission coefficients for the two sides of the etalon r, r', t and t',

$$\delta = \frac{2\pi}{\lambda} 2d\cos(\theta) + 2\phi$$

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where

$$T = tt'$$
 $R = rr'$ $F = \frac{4R}{(1-R)^2}.$

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$$\frac{\lambda}{\Delta\lambda_{\min}} = \frac{\pi d}{\lambda}\sqrt{F}$$

10 Geometric Optics

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• origin is located at the vertex of the curved boundary or mirror, and at the centre of a thin lens, with the x axis directed along the optical axis from left to right.

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- object, image, and centre of curvature distances are defined to be the x coordinates of the y, z planes which contain them.

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- origin is located at the vertex of the curved boundary or mirror, and at the centre of a thin lens, with the x axis directed along the optical axis from left to right.
- object, image, and centre of curvature distances are defined to be the x coordinates of the y, z planes which contain them. Thus distances to points or planes to the right of the vertex or lens centre are positive, those to the left are negative.



• light sources and objects are placed to the left of the first surface in the system,

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- light sources and objects are placed to the left of the first surface in the system, so that the light rays travel from left to right, but the object has a negative x coordinate and the object distance will thus be negative.
- angles are taken to be positive or negative dependent on whether their tangents are positive or negative.

11 Refraction at spherical surfaces

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$$rac{n_1}{l_1} - rac{n_2}{l_2} = rac{n_1 - n_2}{r}.$$

We need to insert the signs appropriate to our sign convention.

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11.1 Lenses

$$\frac{1}{v} - \frac{1}{u} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{f}.$$

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• one enters parallel to the optical axis and is refracted so as to pass directly or by projection through the second focal point;

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- one enters parallel to the optical axis and is refracted so as to pass directly or by projection through the second focal point;
- one passes directly or by projection through the first focal point, and emerges parallel to the optical axis;
- one passes undeviated through the centre of the lens (where the lens surfaces are locally parallel).

With the image at infinity

With the image at infinity

with distances in mm.

With the image at infinity

$$M = \frac{h'}{h}$$

with distances in mm.

With the image at infinity

$$M = \frac{h'}{h} = \frac{250}{f},$$

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$$M = \frac{250}{u} = 1 + \frac{250}{f}.$$

Same basic quantities as before – focal points and vertices of lenses

Same basic quantities as before – focal points and vertices of lenses – but where do we measure them from?

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Same basic quantities as before – focal points and vertices of lenses – but where do we measure them from? We define the so-called Cardinal Points or Principal Points as follows:

• two focal points, defined in terms of entry or exit rays parallel to the axis;



• principal planes, defined by the locus of the points of intersection of the incident ray through the focus and the exit ray parallel to the axis (Q₁ and Q₂);

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- the principal points, being the intersections of the principal planes with the axis;

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- the principal points, being the intersections of the principal planes with the axis;
- the nodal points where the ray through the optical centre of the lens (the ray which emerges parallel to its incident direction) intersects the axis.
Simply work through the analysis of the system step by step - as on problem sheet P20.

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Simply work through the analysis of the system step by step - as on problem sheet P20. Take the first lens, find the image it produces, take that image as the object for the second lens and calculate its position relative to the second lens taking into account the distance between the lenses.

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Simply work through the analysis of the system step by step - as on problem sheet P20. Take the first lens, find the image it produces, take that image as the object for the second lens and calculate its position relative to the second lens taking into account the distance between the lenses. Plug this result, adjusted where necessary in accordance with the sign convention, into the lens formula for the second lens – and the job is done. Similarly, the overall magnification is the product of the magnifications produced by the individual lenses.

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12.2 astronomical telescope

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- **12.2** astronomical telescope
- **12.3 terrestrial telescope**

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- **12.2** astronomical telescope
- **12.3 terrestrial telescope**
- 12.4 telephoto lens

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- 12.1 compound microscope
- 12.2 astronomical telescope
- **12.3 terrestrial telescope**
- 12.4 telephoto lens

NOTE: the question will give you information about the arrangement of lenses — you don't have to remember them.