Assessing Answer Patterns in Questionnaire / Item Response Data Using Mixtures of Rasch Models

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Outline

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- Rasch model
- Mixture models
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- Application: CRAN motivation survey
- Summary
Latent traits

- **Aim:** Measure latent traits.
- **Examples:**
  - Intelligence, abilities (e.g., knowledge, teamwork).
  - Attitudes (e.g., towards strangers, the EU).
  - Responsiveness to stimuli (e.g., advertising).
- **Measurement tool:** Sets of items, e.g., problem solving for measuring ability, agreement with statements for measuring attitudes.
- **Here:** Binary items. Solve a problem yes / no, agree with a statement yes / no.
- **State-of-the-art model for binary items in item response theory:** Rasch model.
Rasch Model

Probability for person $i$ to solve item $j$:

$$P(Y_{ij} = y_{ij} | \theta_i, \beta_j) = \frac{\exp\{y_{ij}(\theta_i - \beta_j)\}}{1 + \exp\{\theta_i - \beta_j\}}.$$ 

- $y_{ij}$: Response by person $i$ to item $j$.
- $\theta_i$: Ability of person $i$.
- $\beta_j$: Difficulty of item $j$.

By construction:

- No covariates, all information is captured by ability and difficulty.
- Both parameters $\theta$ and $\beta$ are on the same scale: If $\beta_1 > \beta_2$, then item 1 is more difficult than item 2 for all subjects.

Central assumption of measurement invariance needs to be checked for both manifest and latent subject groups.
Rasch Model: Estimation

- Joint estimation of $\theta$ and $\beta$ is inconsistent.

- Conditional ML (CML) estimation: Use factorization of the full likelihood on basis of the scores $r_i = \sum_{j=1}^{m} y_{ij}$:

  $$L(\theta, \beta) = f(y|\theta, \beta) = h(y|r, \theta, \beta)g(r|\theta, \beta) = h(y|r, \beta)g(r|\theta, \beta).$$

  Estimate $\beta$ from maximization of $h(y|r, \beta)$.

- Also maximizes $L(\theta, \beta)$ if $g(r|\cdot)$ is assumed to be independent of $\theta$ and $\beta$; but potentially depending on auxiliary parameters $\delta$: $g(r|\delta)$. 
Mixture Model

- Assumption: Data stems from different classes but class membership is unknown.
- Modeling tool: Mixture models.
- Mixture model = $\sum$ weight \times component.
- Components represent the latent classes. They are densities or (regression) models.
- Weights are a priori probabilities for the components / classes, treated either as parameters or modeled through concomitant variables.
Rasch Mixture Model: Framework

Full mixture:

- Weights: Either (non-parametric) prior probabilities $\pi_k$ or weights $\pi(k|x, \alpha)$ based on concomitant variables $x$, e.g., a multinomial logit model.

- Components: Conditional likelihood for item parameters and specification of score probabilities

$$f(y|\alpha, \beta, \delta) = \prod_{i=1}^{n} \sum_{k=1}^{K} \pi(k|x_i, \alpha) h(y_i|r_i, \beta_k) g(r_i|\delta_k).$$

- Estimation of all parameters via ML through the EM algorithm.
Rasch Mixture Model: Score Probabilities

- Original proposition by Rost (1990): Saturated model. Discrete distribution with parameters (probabilities) $g(r) = \Psi_r$.
- Number of parameters necessary is potentially very high: $(\text{number of items} - 1) \times (\text{number of components})$.
- More parsimonious: Assume parametric model on score probabilities, e.g., using mean and variance parameters.
- General approach: Conditional logit model encompassing the original saturated parameterization and a mean / variance parameterization (with only two parameters per component) as special cases

$$g(r|\delta) = \frac{\exp\{z_r^\top \delta\}}{\sum_{j=1}^{m-1} \exp\{z_j^\top \delta\}}.$$
Software

- Available in R in package `psychomix` at http://CRAN.R-project.org/package=psychomix
- Based on package `flexmix` (Grün and Leisch, 2008) for flexible estimation of mixture models.
- Based on package `psychotools` for estimation of Rasch models.
- Frick et al. (2012), provides implementation details and hands-on practical guidance. See also vignette("raschmix", package = "psychomix").
CRAN Motivation: Data

Survey data from 2010 among 663 developers of R packages. Assesses psychological traits like motivation, values, and work design as well as research and R related activities.

Here: Subset of 8 items measuring external regulation of motivation:

- "I can publish the packages in scientific journals."
- "They are part of my master / PhD thesis."
- "I need them for teaching courses."
- "I develop them for clients who pay me."
- "They are a byproduct of my empirical research. If I cannot find suitable existing software to analyze my data, I develop software components myself."
- "They are a byproduct of my methodological research. If I develop / extend methods, I develop accompanying software, e.g., for illustrations and simulations."
- "I expect an enhancement of my career from it."
- "My employer pays me to do so."
CRAN Motivation: Analysis

- First approach: analyze full sample with single Rasch model employing a mean-variance specification of the scores. Question: Is this appropriate?
- Check assumptions via mixture of Rasch models: Is there more than one latent class? Select number of components via BIC.
- Inspect item profiles / answer patterns in latent class(es).
- Covariates: Occupational status, PhD, job in academia. Can they explain class membership? Employ either ex-post or in a concomitant variable model.
CRAN Motivation: Single Rasch Model

Centered item difficulty parameters

journals  thesis  teaching  clients  emp. research  meth. research  career  employer
CRAN Motivation: Mixtures of Rasch Models

Fit model:

```R
R> mix <- raschmix(exReg ~ 1, data = CRAN, k = 1:4, nrep = 5,
+                 scores = "meanvar")
R> mixC <- raschmix(exReg ~ academic + occupation + phd,
+                   data = CRAN, k = 1:4, nrep = 5, scores = "meanvar")
```

Select model:

```R
R> rbind(mix = BIC(mix), mixC = BIC(mixC))
```

```
       1     2      3      4
mix 5249.33 5198.39 5235.96 5234.25
mixC 5249.33 5132.02 5124.72 5140.20
```

```R
R> mix2 <- getModel(mix, which = "2")
R> mixC3 <- getModel(mixC, which = "3")
```

Plot item profiles and effects of concomitant variables:

```R
R> xyplot(mix2)
R> xyplot(mixC3)
R> effectsplot(mixC3)
```
CRAN Motivation: Item Profiles

1 = journals    4 = clients    7 = career
2 = thesis      5 = emp. research    8 = employer
3 = teaching    6 = meth. research
CRAN Motivation: Covariates

- Academic
  - Cluster
    - no
    - yes

- Occupation
  - Cluster
    - other
    - fulltime
    - student

- PhD
  - Cluster
    - no
    - yes
CRAN Motivation: Item Profiles

1 = journals  4 = clients  7 = career
2 = thesis  5 = emp. research  8 = employer
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CRAN Motivation: Effects Displays

**academic effect plot**
- Component 1
- Component 2
- Component 3

**occupation effect plot**
- Component 1
- Component 2
- Component 3

**phd effect plot**
- Component 1
- Component 2
- Component 3
Summary

- The Rasch model is a useful tool to analyze binary questionnaire / item response data.
- Mixtures of Rasch models are a flexible means to check a necessary assumption to provide fair comparisons, i.e., assess if different answer patters are present.
- General framework incorporates concomitant variable models for mixture weights along with various score models.
- Concomitant variable models are a convenient extension to the otherwise covariate-free Rasch model.
- Implementation of all flavors in R package `psychomix` at [http://CRAN.R-project.org/package=psychomix](http://CRAN.R-project.org/package=psychomix)
References


