Quiz 7
Math 250

Determine whether each of the sets below is a subspace of $\mathcal{R}^2$. Justify your answer.

(1) $V = \left\{ \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \in \mathcal{R}^2 : v_1 - 3v_2 = 0 \right\}$

(2) $W = \left\{ \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \in \mathcal{R}^2 : w_1(1 - w_2) = 0 \right\}$

(1) (a) Let $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Then $v_1 - 3v_2 = 0 - 3(0) = 0$, so $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is in V.

(b) Suppose $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ are in V. Then $u_1 - 3u_2 = 0$ and $v_1 - 3v_2 = 0$. Therefore,

$$(u_1 + v_1) - 3(u_2 + v_2) = u_1 - 3u_2 + v_1 - 3v_2 = 0 + 0 = 0,$$

and so $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is in V.

(c) Suppose $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is in V. Then $v_1 - 3v_2 = 0$. Therefore

$$cv_1 - 3cv_2 = c(v_1 - 3v_2) = c(0) = 0,$$

and so $c \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is in V.

Thus, V is a subspace of $\mathcal{R}^2$.

(2) The vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ are both in W (since $1 - w_2$ is 0 in each case). However, their sum $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is not in W, since $w_1(1 - w_2) = -3$ in this case. Therefore, W is not closed under addition, and hence is not a subspace of $\mathcal{R}^2$.