Quiz 5
Math 250

For each matrix given below, find the inverse, or show that the matrix is not invertible.

(1) $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(2) $B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 2 & -2 & 3 \end{bmatrix}$

1.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-2r_1+r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-r_2+r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & -1 & 1 \end{array}\right]$$

Therefore, $\text{rref}(A) \neq I_3$, so A^{-1} does not exist.

2.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 2 & -2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-2r_1+r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-r_3\to r_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{r_3+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{-2r_3+r_1\to r_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 0 & 2 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{r_2+r_1\to r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right]$$

Therefore, $B^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$.