Workshop 1

1. a) Compute the area of the bounded region enclosed by the curve $y = e^x$, the line $y = 12$, and the y-axis.

b) How does this area compare with the value of the integral $\int_1^{12} \ln x \, dx$? Explain your answer. (A picture may be helpful.)

2. Sketch the region R defined by $1 \le x \le 2$ and $0 \le y \le 1/x^3$.

a) Find (exactly) the number a such that the line $x = a$ divides R into two parts of equal area.

b) Then find (to 3 places) the number b such that the line $y = b$ divides R into two parts of equal area.

3. Suppose R is the region in the plane bounded below by the curve $y = x^2$ and above by the line $y = 1$.

a) Sketch R. Set up and evaluate an integral that gives the area of R.

b) Suppose a solid has base R and the cross-sections of the solid perpendicular to the y-axis are squares. Sketch the solid and find its volume.

c) Suppose a solid has base R and the cross-sections of the solid perpendicular to the y-axis are equilateral triangles. Sketch the solid and find its volume.

4. A freely falling body starting from rest has velocity $v = gt$ and displacement $s = \frac{1}{2}gt^2$ where t is the time elapsed since rest. Suppose the freely falling body starts at rest and falls 1,000 feet.

a) Calculate the time T (in seconds) this takes (here $g = 32$ ft/s^2) and the *time average* of the velocity of the body: $v_{\text{time aver}} = \frac{1}{T}\int_0^T v(t)\, dt$. Draw a graph of the function $v(t)$ for $0 \le t \le T$. Find the time t when $v(t) = v_{\text{time aver}}$ and give a graphical interpretation.

b) Find a formula for the velocity as a function $f(s)$ of displacement s, and calculate the *distance average* of the velocity: $v_{\text{dist aver}} = \frac{1}{1000}\int_0^{1000} f(s)\, ds$. Draw a graph of the function $v = f(s)$ for $0 \le s \le 1000$. Find the distance s that the body has fallen when $f(s) = v_{\text{dist aver}}$ and give a graphical interpretation.

Note: $v_{\text{dist aver}} \neq v_{\text{time aver}}$!