Your first midterm examination is likely to contain some problems that do not resemble these review problems.

I. Applications of Integration

(1-3) Find the volumes of the solids obtained by rotating the indicated region $\mathcal{R}$ in the xy-plane about the specified axis:

(1) Region: $\mathcal{R}$ is bounded by $y = 1$, $y = \ln x$ and $x = e^2$.
Axes: (a) the line $y = -1$; (b) the line $x = -2$.

(2) Region: $\mathcal{R}$ consists of all points (x, y) with $0 \le x \le \pi$ and $0 \le y \le \sin x$.
Axes: $(2a)$ the line $y = -2$; $(2b)$ the line $x = -1$.

(3) Region: $\mathcal{R}$ is bounded by $x = y(4 - y)$ and the y-axis
Axis: the y-axis.

(4) Write down the integral used to compute the work done against gravity in building a granite pyramid 500 feet high with a square base of side length 800 feet. Take the density of granite as 170 lbs per cubic foot. Explain in detail how the integral is obtained.

(5) There is a point x_0 in the interval $[5, 7]$ where the function $f(x) = (x^2 - 4)^{-1/2}$ takes on its average value over that interval.
(a) How do we know that? (b) Find such a point x_0.

II. Numerical Methods

(6) How many subintervals of $[0, 2]$ should we use to ensure an accuracy within 10^{-6} when we approximate $\int_0^2 4x^3 - x^4\,dx$ using:
(a) the Midpoint Rule?; (b) Simpson's Rule?

(7) A certain integral $\int_1^4 f(x)\,dx$ is approximated by the Trapezoidal Rule using 30 intervals, and the approximation found is 3.14286. The graph of $f''(x)$ is shown here. Find a range of values $[a, b]$ such that the exact value of the integral can be guaranteed to lie within that range, and explain your method. (The numbers a and b should be given to at most 3 decimal places accuracy.)

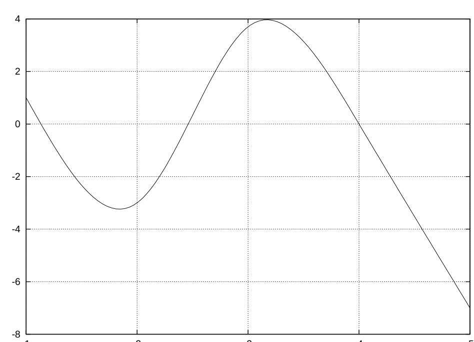

Figure 1: f''

 (More)

III. Techniques of Integration

(8) Evaluate the following integrals.

(a) $\displaystyle\int \sin^3 x \cos^4 x \, dx$ (b) $\displaystyle\int \sec^4 x \, dx$

(c) $\displaystyle\int \tan^5 x \sec^3 x \, dx$ (d) $\int \sec^3 x \, dx$

(9) Evaluate the following integrals. Integral (g) is very difficult unless you are given the following hint: The function $\frac{1}{1+e^x} - \frac{1}{2}$ is an odd function.

(a) $\displaystyle\int x^5 (\ln x)^2 \, dx$ (b) $\displaystyle\int \frac{dx}{x \ln x}$ (c) $\displaystyle\int \cos(\sqrt{x}) \, dx$

(d) $\displaystyle\int x^2 \tan^{-1} x \, dx$ (e) $\displaystyle\int x^{-2} \sin^{-1} x \, dx$ (f) $\displaystyle\int e^{\sqrt{x}} \, dx$

(g) $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos(x)}{1+e^x} \, dx$

(10) Evaluate the following integrals.

(a) $\displaystyle\int \frac{dx}{(25+x^2)^2}$ (b) $\displaystyle\int \frac{x \, dx}{(x^2+36)(x+1)}$ (c) $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$

(d) $\displaystyle\int \frac{x^2-x+4}{(x-5)(x+3)^2} \, dx$ (e) $\displaystyle\int \frac{x^2 \, dx}{(16-x^2)^{3/2}}$ (f) $\displaystyle\int \frac{dx}{x^2+4x+9}$

(11) Evaluate $\displaystyle\int \sin(\ln x) \, dx$ using two integrations by parts. Would another method work?

(12) Evaluate the following improper integrals:

$$\int_5^\infty \frac{dx}{(x-3)(x-4)} \qquad \int_0^1 \ln x \, dx \qquad \int_0^\infty \frac{x^3 \, dx}{e^x} \qquad \int_{-\infty}^\infty \frac{dx}{9+x^2}$$

(13) Show that one of these improper integrals converges, and that one them diverges:

$$\int_7^\infty \frac{dx}{x - |\cos(x)|} \qquad \int_5^\infty \frac{dx}{e^{x^2}}$$