Math 375: Introduction to Representation Theory
Exercise Set 5

1. *Read:* James and Liebeck
   - Ch.12, p.104–113
   - Ch.18, p.179–186

2. In class, we constructed the character table of D_6. Use this table to find all the normal subgroups of D_6.

3. In this problem, we consider the representations of D_4 over $\mathbb{C}$.
   (a) Find the characters of the *tautological* reperesentation of D_4. Use the character to show that this representation is irreducible.
   (b) Find the conjugacy classes of D_4.
   (c) Find the commutator subgroup of D_4.
   (d) Find all the linear characters of D_4.
   (e) Write out the character table for D_4.

4. There exists a group G of order 10 with precisely four conjugacy classes with representatives g_1, g_2, g_3, g_4, and has an irreducible character χ given by

$g_i:$	g_1	g_2	g_3	g_4
χ	2	$\frac{-1+\sqrt{5}}{2}$	$\frac{-1-\sqrt{5}}{2}$	0

   (a) Find the sizes of the conjugacy classes of G. (*Hint:* It would be helpful to also have one other irreducible character for this.)
   (b) Complete the character table of G.

5. Find the character tables of the following groups.
   (a) $G = \langle a, b \mid a^6 = b^3 = 1, a^{-1}ba = b^{-1} \rangle$.
   (b) $G = \langle a, b \mid a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$.