Math 272, Linear Algebra with Applications, Spring 2017
Final Exam Practice Test

1. Consider the matrix
$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \end{bmatrix}.$$

   (a) Find all eigenvalues of A.

   (b) Find the eigenspace corresponding to each eigenvalue found in part (a).

   (c) Find a formula for A^n. Your answer should consist of a single 3×3 matrix, where the entries may depend on n.

2. (a) Prove that the set $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a + d = 0 \right\}$ is a subspace of $M_{2\times 2}$, the set of 2×2 matrices.

   (b) Find a basis for S.

   (c) What is the dimension of S?

3. Let $T : V \to V$ be a linear transformation that is one to one. Show that if $\{\mathbf{v}_1, \dots \mathbf{v}_n\}$ is a linearly independent set in V, then so is $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_n)\}$.

4. Let A be an $m \times n$ matrix and suppose v is a vector in null(A). Show that v is orthogonal to every vector in row(A). (**Hint:** Find a spanning set for row(A) and show that v is orthogonal to every vector in this set. Then use this to show that v must be orthogonal to every vector in the span of this set.)

5. For each of the statements below, give an example of a 2×2 matrix A that satisfies the condition.

   (a) A has eigenvectors $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ with eigenvalues 2 and 3 respectively.

   (b) A is the matrix representing the transformation $T : \mathcal{P}_2 \to \mathcal{P}_1$ such that $T(ax^2 + bx + c) = (3a + b)x - 2a + 4b$, relative to the bases $\mathcal{B} = \{x^2, x^2 + x, x^2 + x + 1\}$ of P_1 and $\mathcal{C} = \{x, 1\}$ of P_1.

   (c) A is a matrix such that null$(A) = \left\{ r \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ and col$(A) = \left\{ r \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation given by
$$T\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 2x + y \\ x - y \end{bmatrix}.$$

   (a) Show that T is an isomorphism.

   (b) Find the inverse transformation, T^{-1}.

7. Let $S = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 2 \\ 0 \end{bmatrix} \right\}$.

   (a) Find an orthonormal basis for S.

   (b) Find the projection of $\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ onto S.

8. Determine whether each of the following statements is true or false, give a <u>brief</u> justification of your answer.

   (a) If A and B are $n \times n$ matrices then $\det(AB) = \det(BA)$.

   (b) If A is a 3×3 matrix such that $\det(A) = -2$, then $\det(3A^2) = 36$.

   (c) If the reduced row echelon form of a matrix A is the identity matrix I, then A is similar to I.

   (d) If A and B are similar matrices, then they have the same eigenvectors.

   (e) If U is a vector space in which one can find n linearly independent vectors in U, then $\dim(U) = n$.

   (f) If T is a matrix transformation given by a matrix A, then $\dim(\text{range}(T)) = \dim(\text{row}(A))$.

   (g) If A is an $n \times n$ matrix with at least n eigenvectors, then A is diagonalizable.

   (h) The transformation $T : M_{3\times 3} \to M_{3\times 3}$ given by $T(A) = A^t$ is linear.