Math 272, Linear Algebra with Applications, Spring 2017
Midterm 1 Practice Test

1. Determine all values of c for which the following linear system is consistent.

$$\begin{cases} 2x + 4y - 4z = 12 \\ 3x + 7y - 5z = 20 \\ \quad x + 3y + cz = 7 \end{cases}$$

2. Let $\mathbf{u}$ and $\mathbf{v}$ be solutions to the linear system $A\mathbf{x} = \mathbf{b}$ and let c and d be constants such that $c + d = 1$. Show that $c\mathbf{u} + d\mathbf{v}$ is also a solution to $A\mathbf{x} = \mathbf{b}$.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 5 & -4 & 2 \\ -1 & 0 & 3 & -1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Find $\det(A)$.

   (b) Is A invertible?

4. Consider the set of vectors $\{(1, 0, 3, 1), (0, 1, -1, 1), (1, 2, 1, 0)\}$.

   (a) Is the set linearly independent?

   (b) Does the set span $\mathbb{R}^4$?

5. Prove that if A and B are invertible matrices, then so is $A^t BA$.

6. Determine whether each of the following statements are true or false. Give a brief justification of your answer.

   (a) A homogeneous linear system of 2 equations in 3 variables will always have infinitely many solutions.

   (b) If A and B are $n \times n$ matrices then $\det(A + B) = \det(A) + \det(B)$.

   (c) A set of $n + 1$ vectors in $\mathbb{R}^n$ is linearly dependent.

   (d) A set of $n - 1$ vectors in $\mathbb{R}^n$ is linearly independent.