

**Syllabus for the Linear Algebra Portion of the Mathematics Comprehensive Exam**  
**Department of Mathematics and Statistics, Amherst College**

**Basic Definitions**

- Vector space
- Subspace
- Span of a subset
- Linear independence
- Basis and dimension
- Linear transformation
- Kernel or null space
- Image or range
- Inverse of a matrix or linear transformation
- Determinant and trace
- Characteristic polynomial
- Eigenvalues and eigenspaces
- Diagonalizability
- Similarity

**Computational Techniques**

- Determine when a subset is a subspace
- Basic matrix manipulations
- Row operations on matrices
- Solving systems of linear equations
- Find the inverse of a matrix
- Find a basis of a given subspace
- Find the nullity, rank, trace and determinant of a matrix
- Find the null space  $N(T)$  and range  $R(T)$  of a linear transformation  $T$
- Given bases of  $V$  and  $W$ , find the matrix of a linear transformation  $T : V \rightarrow W$
- Given a matrix or linear transformation:
  - Compute its characteristic polynomial
  - Find its eigenvalues and eigenspaces

**Basic Results to Know**

- $\dim N(T) + \dim R(T) = \dim V$
- $\text{nullity}(A) + \text{rank}(A) = \text{number of columns of } A$
- Criteria for  $A^{-1}$  to exist
- Criteria for  $A$  to be diagonalizable

**Be able to write simple proofs of problems involving subspaces, linear maps, linear independence, spanning sets, null spaces and ranges.**