Math 272, Linear Algebra with Applications, Spring 2016
Midterm 2 Practice Test 2

1. Consider the matrix
$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 5 & -4 & 2 \\ -1 & 0 & 3 & -1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$$
   (a) Find $\det(A)$.
   (b) Is A invertible?

2. Consider the matrix
$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 3 & 0 & 4 \\ -1 & 1 & 0 & 3 \end{bmatrix}.$$
   (a) Find a basis for the row space of A.
   (b) Find a basis for the column space of A.
   (c) Find $\operatorname{rank}(A)$.

3. Suppose that the set $\{v_1, v_2, v_3\}$ is a basis for a vector space V. Show that the set $\{v_1, v_2, v_1 + v_3\}$ is also a basis for V.

4. Let A be an $n \times n$ matrix, prove that 0 is an eigenvalue of A if and only if $\operatorname{rank}(A) < n$.

5. Determine whether each of the following statements is true or false, give a brief justification of your answer.
   (a) If A is an $n \times n$ matrix, then $\det(cA) = c\det(A)$.
   (b) The intersection of any two subspaces of a vector space V is a subspace of V.
   (c) If V is a finite dimensional vector space, then any set of vectors that spans V is linearly independent.
   (d) Any subset of a vector space containing the zero vector is linearly dependent.