Math 272, Linear Algebra with Applications, Spring 2016
Midterm 2 Practice Test 1

1. Let A, B be 3×3 matrices and suppose $\det(A) = 3$ and $\det(B) = 10$. Find $\det(2AB^{-1})$.

2. Consider the matrix
$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}.$$

   (a) Find all eigenvalues of A.

   (b) Find the eigenspace corresponding to each eigenvalue found in part (a).

3. (a) Show that the set $S = \{p(x) \in \mathcal{P}_3 \mid p(0) = 0, p(1) = 0\}$, is a subspace of $\mathcal{P}_3$.

   (b) Find a basis for S.

   (c) What is the dimension of S?

4. Let U and W be subspaces of a vector space V such that $U \cap W = \{\mathbf{0}\}$. Let $\mathbf{u} \in U$ and $\mathbf{w} \in W$ and suppose that $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{w} \neq \mathbf{0}$. Prove that $\{\mathbf{u}, \mathbf{w}\}$ is linearly independent.

5. Determine whether each of the following statements is true or false, give a brief justification of your answer.

   (a) If A and B are $n \times n$ matrices then $\det(A + B) = \det(A) + \det(B)$.

   (b) If $S = \{\mathbf{v}_1, ..., \mathbf{v}_n\}$ is a linearly dependent set of vectors, then any vector in S can be expressed as a linear combination of the other vectors.

   (c) If an $n \times n$ matrix is invertible, then the column vectors of the matrix form a basis for $\mathbb{R}^n$.

   (d) If W is a subspace of a finite-dimensional vector space V then $\dim(W) \leq \dim(V)$.