Math 272, Linear Algebra with Applications, Spring 2016
Midterm 1 Practice Test 2

1. Determine all values of c for which the following linear system is consistent.

$$\begin{cases} 2x + 4y - 4z = 12 \\ 3x + 7y - 5z = 20 \\ x + 3y + cz = 7 \end{cases}$$

2. Let $\mathbf{u}$ and $\mathbf{v}$ be solutions to the linear system $A\mathbf{x} = \mathbf{b}$ and let c and d be constants such that $c + d = 1$. Show that $c\mathbf{u} + d\mathbf{v}$ is also a solution to $A\mathbf{x} = \mathbf{b}$.

3. Show that

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \in \mathbb{R}^4 \;\middle|\; x + y - z = 0 \right\}$$

   is a subspace of $\mathbb{R}^4$. Find a basis for W. What is the dimension of W?

4. Consider the set of vectors $\{(1, 0, 3, 1), (0, 1, -1, 1), (1, 2, 1, 0)\}$.

   (a) Is the set linearly independent?

   (b) Does the set span $\mathbb{R}^4$?

5. Prove that if A and B are invertible matrices so is ABA.

6. Determine whether each of the following statements are true or false. Give a brief justification of your answer.

   (a) A homogeneous linear system of 2 equations in 3 variables will always have infinitely many solutions.

   (b) Any three vectors in $\mathbb{R}^3$ will form a basis for $\mathbb{R}^3$.

   (c) A linear system of 3 equations in 2 variables is always inconsistent.

   (d) The composition of two linear transformations is a linear transformation.