Math 272, Linear Algebra with Applications, Spring 2016
Midterm 1 Practice Test 1

1. Consider the linear system.

$$\begin{cases} x + y - 2z = 3 \\ -x + 2y = -1 \\ -y + z = 1 \end{cases}$$

   (a) Write the linear system in matrix form $A\mathbf{x} = \mathbf{b}$.

   (b) Is A invertible? If so find the inverse.

   (c) Find all solutions to the above linear system.

   (d) Find all solutions to the corresponding homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

2. If A is a 2×2 matrix and the entries on the main diagonal sum to 0 prove that there is some constant c such that $A^2 = cI$.

3. Determine whether the following set S is a subspace of the $\mathbb{R}^3$. If it is a subspace find a basis for S and find its dimension.

   (a) $S = \{(a, b, a + b^2) \mid a, b \in \mathbb{R}\}$

   (b) $S = \left\{ \begin{bmatrix} a+b \\ a \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$

4. A square matrix A is called **symmetric** if $A = A^t$ and **antisymmetric** is $A = -A^t$. Show that if B is any square matrix then $B + B^t$ is symmetric and $B - B^t$ is antisymmetric.

5. Show the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is linear. Find a matrix representation for T.

$$T(x, y) = (x - 2y, x + y, -x).$$

6. Determine whether each of the following statements are true or false. Give a brief justification of your answer.

   (a) If the $n \times n$ matrix A does not have an inverse then the linear system $A\mathbf{x} = \mathbf{b}$ is inconsistent.

   (b) Homogeneous linear systems always have at least one solution.

   (c) A set of $n + 1$ vectors in $\mathbb{R}^n$ is linearly dependent.

   (d) A set of $n - 1$ vectors in $\mathbb{R}^n$ is linearly independent.