Math 220, Spring 2017

Chapter 4: Basic Counting Principles

The Sum Rule

If A and B are disjoint finite sets, then

$$|A \cup B| = |A| + |B|.$$

More generally, if $A_1, \dots, A_n$ are mutually disjoint finite sets (i.e., $A_i \cap A_j = \emptyset$ for $i \neq j$), then

$$|A_1 \cup \cdots \cup A_n| = |A_1| + \cdots + |A_n|.$$

Thus, if you are counting objects that can be placed into mutually disjoint categories, then it suffices to count how many objects are in each category.

The Product Rule

If A and B are finite sets, then

$$|A \times B| = |A| \cdot |B|.$$

More generally, if $A_1, \dots, A_n$ are finite sets, then

$$|A_1 \times \cdots \times A_n| = |A_1| \cdots |A_n|.$$

Thus, if you are counting combinations of various types of objects and the choices can be made independently from each other, then the number of combinations is the product of the number of objects of each type.

The Complement Rule

If A is a subset of a finite set B, then

$$|A| = |B| - |B \setminus A|.$$

Thus, if you are working with a known number of objects, then counting how many of them lie in a certain subset is equivalent to counting how many lie in the complement of the subset.

The Bijection Rule

If $f : A \to B$ is a bijection of finite sets, then

$$|A| = |B|.$$

Thus, if you want to count the number of objects of a particular type, it suffices to match these objects in a one-to-one onto manner with objects of another type.