Math 220: Discrete Mathematics
HW for Section 3.4

1. Prove the following:
   (a) If $a \equiv b \mod n$, and $m|n$, then $a \equiv b \mod m$.
   (b) If $a \equiv b \mod n$, and $c > 0$, then $ca \equiv cb \mod cn$.
   (c) If $a \equiv b \mod n$, then $n|a$ if and only if $n|b$.
2. (a) Show that if a, b are relatively prime natural numbers, then n is divisible by the product ab if and only if $a|n$ and $b|n$.
   (b) Show that if a, b are not relatively prime natural numbers, then there exists a natural number n which is divisible by both a and b but not ab.
3. State and prove the divisibility tests for the following numbers:
   (a) 12
   (b) 25
4. The number 25,730 is divisible by 10 and by 2. Is it divisible by 20? Explain why or why not.
5. Suppose a is a natural number such that the sum of the digits of a equals the sum of the digits of $5a$. Show that $9|a$.
6. Let n be a natural number, and let m be formed from n by switching two digits d_i and d_j. (For example, 86231 is formed form 83261 by switching d_3 and d_1. Mixing up two digits like this is sometimes called a *transposition error*.) Show that $9|(m - n)$.