ICM030V-09

ICIAM 2003

Date:
Time:
Venue:

Subject Areas: IC03V
Nonlinear analysis
Dynamical systems
Various

"Integrable Systems"

Chairs: ***Rod Halburd*** **and** ***John Roberts***

Loughborough University
UNSW

Session #1.

Rod G. Halburd: (Loughborough University)
"*Integrable systems: continuous, discrete, and other*"

John Roberts: (UNSW)
"*Integrable maps over finite fields*"

Peter Clarkson: (University of Kent, UK)
"*Rational solutions of the Painlevé equations and associated special polynomials*"

Jonathan Kress: (University of NSW)
"*Superintegrability in 2 and 3 dimensions*"

Session #2.

Nalini Joshi: (University of Sydney)
"*Singularity confinement and Bäcklund transformations*"

Peter Forrester: (University of Melbourne)
"*Painlevé theory and random matrices*"

Yik-Man Chiang: (The Hong Kong University of Science and Technology)
"*On the meromorphic solutions of analytic difference equations of integrable quantum systems*"

Summary:

A differential equation is said to be integrable if it has sufficiently many globally defined conserved quantities. Such equations have many applications in areas such as water waves, optical communications, relativity, and geometry. Integrable differential equations possess a number of remarkable features such as solitons, Bäcklund transformations, and associated spectral problems. Two main themes run through the talks in this minisymposium. One theme is the complex-analytic approach to integrable differential equations. Considered as functions of complex variables, the solutions of most integrable differential equations have very simple singularities. The presence of this simple singularity structure is called the Painlevé property. The other theme in this minisymposium involves the extension of the idea of 'integrability' to different types of equations. Different speakers will discuss notions of integrability for discrete equations, equations over finite fields, and equations over graphs.

Integrable systems: continuous, discrete, and other

Rod G. Halburd

Loughborough University

Abstract. This talk will be an overview of different types of integrable systems and some of the relations between them. It will be aimed at a general mathematical audience and is intended to be an introduction to the integrable systems minisymposium.

The idea of integrability can be applied in a number of different settings. Broadly speaking, an equation (differential, discrete, functional, etc.) is said to be integrable if it has sufficiently many globally defined conserved quantities. Particular topics discussed will include the Painlevé equations, their discrete analogues, and their connections with continuous and discrete soliton equations.

Integrable maps over finite fields

John Roberts

UNSW

ABSTRACT. There is much current interest in characterising and testing for integrability in discrete dynamical systems. Integrable discrete dynamical systems appear in many physical contexts. In this talk, we survey our novel method to detect the presence of rational integrals of motion in symplectic rational maps in two and higher dimensions. The idea is to represent these maps over finite fields and to examine their orbit statistics. We find markedly different orbit statistics depending upon whether the map possesses integrals or not. In effect, it appears we can detect the signature of integrability in the transition from the continuous to the finite phase space. This can be explained partly using results from algebraic geometry over finite fields.

Rational solutions of the Painlevé equations and associated special polynomials

Peter Clarkson

University of Kent, UK

CO-AUTHORS:

Elizabeth Mansfield *(University of Kent at Canterbury)*

ABSTRACT. In this talk I shall discuss rational solutions and associated polynomials for the second, third and fourth Painlevé equations (PII–PIV), and the equations in the PII hierarchy. The Painlevé equations are six nonlinear ordinary differential equations that have been the subject of much interest in the past twenty-five years, which have arisen in a variety of physical applications and may be thought of as nonlinear special functions. Rational solutions of the Painlevé equations are expressible as the logarithmic derivative of special polynomials. For PII these special polynomials are known as the Yablonskii–Vorob'ev polynomials and for PIV as the Okamoto polynomials. The structure of the roots of these Yablonskii–Vorob'ev polynomials is shown to have a highly regular triangular structure and have some interesting, indeed somewhat mysterious, combinatorial properties. Further the properties of the Yablonskii–Vorob'ev polynomials are compared and contrasted with those of classical orthogonal polynomials. The analogous special polynomials for the PII hierarchy, PIII and PIV

are derived and a representation given of the associated rational solutions in the form of determinants through Schur functions.

Further I shall show that the roots of these special polynomials also have a highly regular structure.

Superintegrability in 2 and 3 dimensions

Jonathan Kress

University of NSW

ABSTRACT. In classical mechanics, an n-dimensional Hamiltonian system (having a $2n$-dimensional phase space) is called Liouville integrable if it possesses n functionally independent integrals in mutual involution. When further integrals are known, the system is said to be superintegrable and maximally superintegrable when $2n - 1$ exist. Similar notions exist for quantum systems where integrals are replaced with differential operators commuting with the Hamiltonian. Usually the term superintegrable refers only to systems with integrals that are polynomial in the momenta.

The Coulomb system and the harmonic oscillator are two well known examples. For both of these systems the classical Poisson algebra determines the shape of the classical orbits and the quantum operator algebra determines the energy spectrum of bound states without the need to solve any differential equations. It was in an attempt to discover other systems with similarly 'nice' properties that the search for superintegrable systems began.

In this talk, an overview of known superintegrable systems will be given and some recent results discussed.

Singularity confinement and Bäcklund transformations

Nalini Joshi

University of Sydney

ABSTRACT. Recently, we proved that that difference equations arising from consistent compositions of Bäcklund transformations of the continuous Painlevé equations possess the singularity confinement property. Here we consider examples showing that a restricted converse of this theorem may be possible.

Painlevé theory and random matrices

Peter Forrester

University of Melbourne

ABSTRACT. It has been known since the 1980 paper of the Kyoto school that classical integrable systems theory such as isomonodromy deformation of linear differential equations is relevant to the calculation of spacing distributions in random matrix theory. Moreover this theory leads to exact evaluation of the latter in terms of Painleve transcendents. Through the efforts of a number of research groups these connections have now been thoroughly explored, and new applications found. I will give an overview of these developments.

On the meromorphic solutions of analytic difference equations of integrable quantum systems

Yik-Man Chiang

The Hong Kong University of Science and Technology

ABSTRACT. This is a preliminary report concerning the growth of several classes of meromorphic solutions of certain analytic difference equations which arise naturally in certain integrable quantum systems consideration. Although these are special classes of difference equations in which the coefficients are explicitly given, the results shed light on what we should be looking for in the case of general difference equations.

This is joint work with S. Ruijsenaars.